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A Fuzzy Inventory Model for Deteriorating Items with Stock and Time Dependent Demand and Partial Backlogging Under Trade Credit period

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Abstract

In the present paper, a Fuzzy inventory model for deteriorating items has been developed with stock and time dependent demand for which the supplier permits a fixed trade credit period. This paper also assumes that excess demand is partially backlogged. All the inventory costs involved in this model are taken as triangular fuzzy numbers. Replenishment cycle length, the time at which shortage begins and the optimal total inventory cost are taken as decision variables. Sensitivity for this model is also studied. Graded mean representation is used to defuzzify the model. The advantage of proposed approach is that it is simple, gives the better result in relatively less computational work.

Key words: Triangular fuzzy numbers, Stock and Time Dependent Demand, Deteriorating Items, Trade Credit Period.

1. Introduction

In real life situation, inventory control plays an important role as the total investment in inventories of various kinds is quite substantial. Almost every business must carry out some inventory for smooth and efficient running of its operation. The problem is to take decisions that how much should be stocked and when should be stocked for smooth running of any business. Also many products in real life subject to significant rate of deterioration. Hence the impact of product deterioration should not be neglected in the decision process. The decrease or loss of utility due to decay is usually a function on hand inventory. Retail firms, wholesalers manufacturing companies and even blood banks generally have a stock of goods on hand.

Usually the demand rate is decided by the amount of stock level. The motivational effect on the people may be caused by the presence of stock at times. Large quantities of goods displayed in markets lure the customers to buy more. If the stock is insufficient the customers may prefer some other brands, as a result the shortage will fetch loss to the producer. There are two trade options in markets.1 the supplier gets his amounts immediately after delivering the goods. 2.The supplier he himself gives some credit period to the retailer to market his products or to boost the trade. Some goods like fresh vegetables deteriorates in due course of time. So the demand decreases with time .Hence in real life situation it is apt to take demand as a function of on-hand inventory and time (exponentially declining function with respect to time).

Many complicated problem arising in the field of engineering, social science, economics, medical science and management involving uncertainties dealt with classical methods are found to be inadequate in recent times. So the fuzzy set theory can be used in decision making process for better understanding of uncertainty situation.

2. Literature Survey :

Keeping an inventory for future sale or use is very common in business. Retail firms, wholesalers, manufacturing companies and even blood banks generally have a stock of goods on hand. As Levin's *et al*.[10] view, usually the demand rate is decided by the amount of stock level. The motivational effect on the people may be caused by the presence of stock at times. Large quantities of goods displayed in markets according to seasons lure the customers to buy more. If the stock is insufficient the customers may prefer some other brands, as a result the shortage will fetch loss to the producer. There are two trade options in markets. 1. The supplier gets his amount immediately after delivering the goods. 2. The supplier he himself gives some credit period to the retailer either to market his products or to boost the trade. Some goods are deteriorating in due course of time.

More researchers are attracted by stock-dependent demand rate patterns. Silver and Peterson[15] noted that sales at the retail level tend to be proportional to the stock level. Gupta and Vrat[8] assumed that the demand rate was a function of initial stock level. Mandal and Phaujdar[11] developed a production inventory model for deteriorating items with uniform rate of production and linearly stock dependent demand. Backer and Urban[1], Datta and Pal[6] and Goh[7] concentrated on the situation that defined the demand rate depends on the instantaneous stock level.

In real life, suppliers offer some credit periods to the buyers to stimulate the demand. Chung[4] presented the DCF (Discounted Cash Flow) approach for the analysis of the optimal policy in the presence of trade credit. Jamal *et al*.[9].

The acceptance of the backlogged demand depends on the waiting time. Hence the waiting time for the next replenishment plays a vital role in this acceptance. Chang and Dye[3] developed a model for deteriorating items with time varying demand and shortages in which the backlogging rate is assumed to be inversely proportional to the waiting time for the next replenishment.

Chung – Yuan Dye[5] developed an inventory model for deteriorating items with stock dependent demand and partial backlogging under conditions of permissible delay in payments. R. Uthayakumar *et al*.[12] have developed A deterministic inventory model with stock and time dependent demand under permissible delay in payment. In this paper it is assumed the demand occurs during a specified period i.e occurs according to poisons distribution, so automatically distribution of time must follow exponential distribution. The pro=posed fuzzy model now involves the demand dependent on stock and time which is taken as exponential distribution. Also inventory cost are taken as triangular fuzzy numbers.

3. Fuzzy Preliminaries

Definition 1 :

Let X denotes a universal set. Then the fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1], to each element x $\in$ X where the value of $\mu_{\tilde{A}}(x)$ at x shows the grade of membership of x

Definition 2 :

A fuzzy set $\tilde{A}$ on R is convex if $\tilde{A}$ $(\lambda x_1 + (1-\lambda)x_2) \geq min\left[\tilde{A}(x_1), \tilde{A}(x_2)\right]$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$.

Definition 3:

A fuzzy set $\tilde{A}$ in the universe of discourse X is called as a fuzzy number in the universe of discourse X.

Triangular fuzzy number:

We consider the situation where fuzzy numbers are represented by triangular membership functions. The fuzzy number $\tilde{A}$ is said to be triangular fuzzy number if it is fully determined by (a_1, a_2, a_3,) of crisp numbers such that ($a_1 < a_2 < a_3$,) whose membership function, representing triangle, can be denoted by

$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{x-a_1}{a_2-a_1} a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} a_2 \leq x \leq a_3 \\ 0 \quad otherwise \end{cases}$$

The Function Principle :

The function principle was introduced by Chen[6] to treat fuzzy arithmetical operations. This principle is used for the operation for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$ Where

$T=(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$, c_1=min T, $c_2=a_2b_2$, c_3=max T if a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then $\tilde{A}\times\tilde{B}=(a_1b_1, a_2b_2, a_3b_3)$.

(iii) $-\tilde{B}=(-b_3,-b_2,-b_1)$ then the subtraction of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}-\tilde{B}=(a_1-b_3, a_1-b_2, a_3-b_1)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(iv) $\frac{1}{\tilde{B}}=\tilde{B}^{-1}=(\frac{1}{b_3},\frac{1}{b_2},\frac{1}{b_1})$ where b_1, b_2, b_3 are all non zero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}}=(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1})$

(v) For any real number K,

$K\tilde{A}=(Ka_1, Ka_2, Ka_3) if K>0$

$K\tilde{A}=(Ka_3, Ka_2, Ka_1) if K<0$

Graded Mean Integration Representation Method :

Suppose $\tilde{A}=(a_1, a_2, a_3)$ then

$$d(\tilde{A})=\frac{(a_1+4a+a_3)}{6}$$

4. Notations And Assumptions :

The proposed fuzzy mathematical model of the inventory problem considered here is developed on the basis of the following notations and assumptions.

4.1 Notations :

$\tilde{K}$ - Fuzzy Ordering cost of inventory, \$ per order.

$I(t)$ - The inventory level at time t.

θ - Deterioration rate, a fraction of the onhand inventory

$\tilde{P}$ - Fuzzy Purchase cost, \$ per unit

$\tilde{h}$ - Fuzzy Holding cost excluding interest charges, \$ per unit/ year.

$\tilde{s}$ - Fuzzy Shortage cost, \$ per unit/year

λ - Constant demand with respect to time (follows exponential distribution)

$\tilde{\pi}$ - Fuzzy Opportunity cost due to lost sales, \$ per unit

$\tilde{I}_e$ - Fuzzy Interest which can be earned,\$/year

$\tilde{I}_r$ - Fuzzy Interest charges which invested in inventory, \$/year $\tilde{I}_r \geq \tilde{I}_e$.

M - Permissible delay in settling the accounts and 0<M<1.

T - The length of replenishment cycle.

T_1 - Time at which shortage starts, $0\leq T_1\leq T$.

$T\tilde{V}C$ - The Fuzzy average total inventory cost per unit time.

$T\tilde{V}C_1(T_1,T)$- The Fuzzy average total inventory cost per unit time for $T_1 \geq M$ in case 1.

$T\tilde{V}C_2(T_1,T)$-The Fuzzy average total inventory cost per unit time for $T_1< M$ in case 2.

4.2 Assumptions :

1. The inventory system involves only one item.
2. The replenishment occurs instantaneously at an infinite rate.

3. There is no replacement or repair of deteriorated units.
4. The demand rate function D(t) is deterministic and is a known function of instantaneous stock level I(t) and time t and it is given by

$$D(t) = \begin{cases} \alpha + \beta I(t) + \lambda e^{-\lambda t} \ ; 0 \le t < T_1 \\ \alpha + \lambda e^{-\lambda t} \ ; T_1 \le t < T \end{cases}$$

where $\alpha > 0$ & $0 < \beta < 1$, $\lambda > 0$.

5. Shortages are allowed and the backlogged rate is defined to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant and $T_1 \le t < T$.

5. Model Formulation :

Due to the combined effects of the demand and deterioration in the interval $[0, T_1)$ the inventory is exhausted. The excess demand is partially backlogged in the interval $[T_1, T)$ as shown in figure.

The differential equation of I(t) with respect to time can be given as

$$\frac{dI(t)}{dt} = \begin{cases} -\alpha - \beta I(t) - \lambda e^{\lambda t} - \theta I(t), & ;0 \le t \le T_1 \\ -\lambda e^{\lambda t} - \dfrac{\alpha}{1+\delta(T-t)}, & ;T_1 \le t < T \end{cases} \tag{1}$$

with boundary condition $I(T_1) = 0$

Considering the first case ie., $0 \le t < T_1$

$$\frac{dI(t)}{dt} = -\alpha - \beta I(t) - \lambda e^{-\lambda t} - \theta I(t)$$

$$\frac{dI(t)}{dt} + (\beta + \theta) I(t) = -\alpha - \lambda e^{-\lambda t}$$

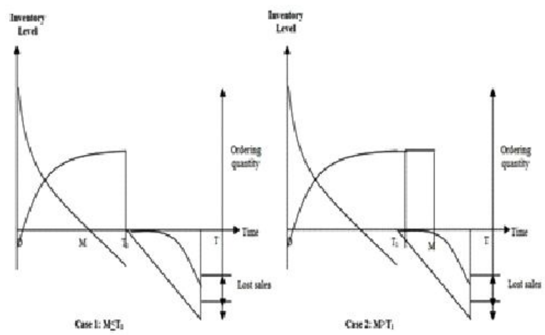
It is linear equation in I(t) and its solution is

$$I(t) e^{(\beta+\theta)t} = \int \left(-\alpha - \lambda e^{-\lambda t}\right) e^{(\beta+\theta)t} dt + c,$$

where c is the constant of integration.

Using boundary conditions and simplifying we get

$$I(t) = \frac{\alpha}{\beta+\theta}\left(e^{(\beta+\theta)(T_1-t)} - 1\right) + \frac{\lambda}{\beta+\theta-\lambda}\left(e^{(\beta+\theta)(T_1-t)-\lambda T_1} - e^{-\lambda t}\right) \tag{2}$$



Graphical Representation of Inventory system

The stock Fuzzy holding cost in the interval $[0, T_1)$ denoted by HC can be written as HC

$$= \tilde{h} \int_0^{T_1} I(t)\, dt$$

$$= \tilde{h}\left\{ \frac{-\alpha}{(\beta+\theta)^2}\left[1 + T_1(\beta+\theta) + e^{(\beta+\theta)T_1}\right] + \frac{1}{(\beta+\theta)(\beta+\theta-\lambda)}\left[-\lambda e^{-\lambda T_1} + -\lambda e^{(\beta+\theta)T_1} + \left(e^{-\lambda T_1} - 1\right)(\beta+\theta)\right]\right\} \tag{3}$$

Fuzzy Deterioration cost in $(0, T_1)$ denoted by DC is given by

$$DC = \tilde{P}\theta \int_0^{T_1} I(t) dt$$

$$= \tilde{P}\theta \left\{ \frac{-\alpha}{(\beta+\theta)^2}\left[1 + T_1(\beta+\theta) + e^{(\beta+\theta)T_1}\right] + \right.$$

$$\frac{1}{(\beta+\theta)(\beta+\theta-\lambda)}\left[-\lambda e^{-\lambda T_1}+-\lambda e^{(\beta+\theta)T_1}+\left(e^{-\lambda T_1}-1\right)(\beta+\theta)\right]\Bigg\} \tag{4}$$

We have to consider two costs in the shortage period. The first is to derive the shortage cost for the backlogged items, and the second is finding the opportunity cost due to lost sales.

The Fuzzy shortage cost in the interval $[T_1, T)$ denoted by SC is given by

$$SC = \tilde{s}\int_{T_1}^{T} I(t)dt$$

$$=\tilde{s}\left[\left(\frac{e^{-\lambda T}}{\lambda}-Te^{-\lambda T_1}\right)+\left(\frac{e^{-\lambda T_1}}{\lambda}+T_1 e^{-\lambda T_1}\right)\right]+$$

$$\frac{\tilde{s}\,\alpha}{\delta^2}\left[\delta\left(T-T_1\right)-\log\left(1+\delta\left(T-T_1\right)\right)\right] \tag{5}$$

The Fuzzy opportunity cost due to lost sales denoted by OC is given by

$$OC = \tilde{\pi}\int_{T_1}^{T}(\alpha+\lambda e^{-\lambda t}-\frac{\alpha+\lambda e^{-\lambda t}}{1+\delta(T-t)})dt$$

$$OC=\frac{\tilde{\pi}}{\delta^2}\left\{\left(\alpha\delta^2-\delta\lambda^2\right)(T-T_1)+\delta^2\left(e^{-\lambda T_1}-e^{-\lambda T}\right)+\right.$$

$$\left.\log\left(1+\delta(T-T_1)\right)\left(\alpha\delta+\lambda\delta-\lambda^2-T\delta\lambda^2\right)\right\} \tag{6}$$

Now consider the supplier's credit period M in settling the accounts. There are two cases. Case 1:$M\le T_1$ or Case 2:$M>T_1$ we shall discuss these two cases one by one.

5.1 Case 1:M≤T₁

Since the length of the period with positive inventory stock of the items is greater than the credit period, the buyer can use the sale revenue with an Fuzzy annual rate $\tilde{I}_e$ in $[0, T_1)$.

The interest earned denoted by IE_1 is

$$IE_1 = \tilde{P}\tilde{I}_e\int_0^{T_1}(T_1-t)D(t)dt$$

$$=\tilde{P}\tilde{I}_e\left\{\left(\frac{\alpha}{2}+\frac{\alpha\beta}{\beta+\theta}\right)\frac{T_1^{\,2}}{2}+\left(\frac{\alpha\beta}{\beta+\theta}+\frac{\lambda\beta e^{-\lambda T_1}}{\beta+\theta-\lambda}\right)\left(\frac{1}{(\beta+\theta)^2}+\right.\right.$$

$$\left.\left.\left(\frac{T_1 e^{(\beta+\theta)T_1}}{\beta+\theta}\right)-\frac{e^{(\beta+\theta)T_1}}{(\beta+\theta)^2}\right)+\left(\lambda-\frac{\lambda\beta}{\beta+\theta-\lambda}\right)\left(\frac{T_1}{\lambda}+\frac{1}{\lambda^2}\left(e^{-\lambda T_1}-1\right)\right)\right\} \tag{7}$$

After the credit period the buyer has to pay the interest for the goods still in stock with Fuzzy annual rate $\tilde{I}_r$ We can find the interest payable denoted by IP as follows.

$$IP = \tilde{P}\tilde{I}_r\int_M^{T_1} I(t)dt$$

$$=\tilde{P}\tilde{I}_r\left\{\left(\frac{\alpha}{(\beta+\theta)^2}\left(e^{(\beta+\theta)(T_1-M)}-1-(\beta+\theta)(T_1-M)\right)\right)\right.$$

$$\left.+\frac{\lambda}{\beta+\theta-\lambda}\left[e^{-\lambda T_1}\left(-\frac{1}{\beta+\theta}+e^{(\beta+\theta)(T_1-M)}\right)+\left(\frac{e^{-\lambda T_1}}{\lambda}-\frac{e^{-\lambda M}}{\lambda}\right)\right]\right\} \tag{8}$$

According to this change all the costs were found (with fuzzy cost).

Therefore the total average cost in case 1 in given by

$$\tilde{TVC}_1=\frac{K+HC+DC+SC+OC+IP-IE_1}{T}$$

$$\tilde{TVC}_1=\frac{1}{T}\left\{\tilde{K}+(\tilde{h}+\tilde{p}\theta)\left[\frac{-\alpha}{(\beta+\theta)^2}\left[1+T_1(\beta+\theta)+e^{(\beta+\theta)T_1}\right]+\right.\right.$$

$$\left.\left.\frac{1}{(\beta+\theta)(\beta+\theta-\lambda)}\left[-\lambda e^{-\lambda T_1}+-\lambda e^{(\beta+\theta)T_1}+\left(e^{-\lambda T_1}-1\right)(\beta+\theta)\right]\right]\right\}$$

$$+\frac{\tilde{s}}{T}\left[\left(\frac{e^{-\lambda T}}{\lambda}-Te^{-\lambda T_1}\right)+\left(\frac{e^{-\lambda T_1}}{\lambda}+T_1e^{-\lambda T_1}\right)\right]+$$
$$\frac{\tilde{s}\,\alpha}{\delta^2}\left[\delta\left(T-T_1\right)-\log\left(1+\delta\left(T-T_1\right)\right)\right]+$$
$$\frac{\tilde{\pi}}{\delta^2 T}\left\{\left(\alpha\delta^2-\delta\lambda^2\right)\left(T-T_1\right)+\delta^2\left(e^{-\lambda T_1}-e^{-\lambda T}\right)+\right.$$
$$\left.\log\left(1+\delta(T-T_1)\right)\left(\alpha\delta+\lambda\delta-\lambda^2-T\delta\lambda^2\right)\right\}+$$
$$\frac{\tilde{p}\tilde{I}_r}{T}\left\{\left(\frac{\alpha}{(\beta+\theta)^2}\left(\mathrm{e}^{(\beta+\theta)(T_1-M)}-1-(\beta+\theta)(T_1-M)\right)\right)+\right.$$
$$\frac{\lambda}{\beta+\theta-\lambda}\left(\left[e^{-\lambda T_1}\left(-\frac{1}{\beta+\theta}\right)+e^{(\beta+\theta)(T_1-M)}\right]+\left(\frac{e^{-\lambda T_1}}{\lambda}-\frac{e^{-\lambda M}}{\lambda}\right)\right)-$$
$$\frac{\tilde{P}\tilde{I}_e}{T}\left\{\left(\frac{\alpha}{2}+\frac{\alpha\beta}{\beta+\theta}\right)\frac{T_1^2}{2}+\left(\frac{\alpha\beta}{\beta+\theta}+\frac{\lambda\beta e^{-\lambda T_1}}{\beta+\theta-\lambda}\right)\left(\frac{1}{(\beta+\theta)^2}+\right.\right.$$
$$\left.\left.\left(\frac{T_1e^{(\beta+\theta)T_1}}{\beta+\theta}\right)-\frac{e^{(\beta+\theta)T_1}}{(\beta+\theta)^2}\right)+\left(\lambda-\frac{\lambda\beta}{\beta+\theta-\lambda}\right)\left(\frac{T_1}{\lambda}+\frac{1}{\lambda^2}\left(e^{-\lambda T_1}-1\right)\right)\right\}$$
(9)

$$\frac{\partial T\tilde{V}C_1\left(T_1,T\right)}{\partial T_1}=0 \qquad (10)$$

and $$\frac{\partial T\tilde{V}C_1\left(T_1,T\right)}{\partial T}=0 \qquad (11)$$

provided they satisfy the sufficient conditions

$$\left[\frac{\partial^2 T\tilde{V}C_1\left(T_1,T\right)}{\partial T_1^{\,2}}\right]_{at\ (T_1^*,T^*)}>0,$$
$$\left[\frac{\partial^2 T\tilde{V}C_1\left(T_1,T\right)}{\partial T^2}\right]_{at\ (T_1^*,T^*)}>0$$

and

$$\left[\left(\frac{\partial^2 T\tilde{V}C_1(T_1,T)}{\partial T^2{}_1}\right)\left(\frac{\partial^2 T\tilde{V}C_1(T_1,T)}{\partial T^2}\right)-\left[\frac{\partial^2 T\tilde{V}C_1(T_1,T)}{\partial T_1\partial T}\right]^2\right]_{at\ (T_1^*,T^*)}>0$$

Using (10),(11) solve for T_1, T using algorithm (1) and their relevant total fuzzy cost using (9).

To obtain the optimal values of T_1& T, we use the following algorithm.

5.2 Algorithm – 1 :

Step 1: Perform (i) – (iv)

i. Start with $T_{1,(1)} = M$
ii. Substituting $T_{1,(1)}$ in equation (10) to find $T_{(1)}$
iii. Using $T_{(1)}$ in equation (11) we can find $T_{1,(2)}$
iv. Repeat (ii) and (iii) until no change occurs in the values of T_1 and T.

Step 2: compare T_1 and M

1. If $M \le T_1$, T_1 is feasible. Then go to step (3).
2. If $M > T_1$, T_1 is not feasible. Set $T_1 = M$ in the equation (11) to calculate T and then go to step 3.

Step 3: Calculate the corresponding $T\tilde{V}C_1$ (T_1*,T*).

5.3 Case 2: $T_1< M$

In this case, the buyer need not pay interest and he earns the interest with annual fuzzy interest rate $\tilde{I}_e$. The interest earned denoted by IE_2 is given by

$IE_2 = P$

$$IE_2=P\tilde{I}_e\left\{\frac{\alpha\theta}{\beta+\theta}\left(MT_1-\frac{T_1^2}{2}\right)+\left(\frac{\alpha\beta}{\beta+\theta}+\frac{\lambda\beta e^{-\lambda T_1}}{\beta+\theta-\lambda}\right)\right.$$
$$\left[\frac{1}{(\beta+\theta)^2}+\frac{2T_1e^{(\beta+\theta)T_1}}{\beta+\theta}-\frac{e^{(\beta+\theta)T_1}}{(\beta+\theta)^2}+\frac{M-T_1}{\beta+\theta}-\frac{Me^{(\beta+\theta)T_1}}{\beta+\theta}\right]+$$

$$+\frac{\lambda(\theta-\lambda)}{\beta+\theta-\lambda}\left[\frac{M}{\lambda}+\frac{(M-T_1)}{\beta+\theta}e^{-\lambda T_1}-\frac{1}{\lambda^2}\left(e^{-\lambda T_1}-1\right)\right]\Bigg\}$$

We have the total average fuzzy cost in this case as

$$T\tilde{V}C_2=\frac{\text{K + HC + DC + SC + OC - IE}_2}{\text{T}}$$

$$T\tilde{V}C_2=\frac{1}{T}\left\{K+(\tilde{h}+\tilde{p}\theta)\left[\frac{-\alpha}{(\beta+\theta)^2}\left[1+\text{T}_1(\beta+\theta)+\text{e}^{(\beta+\theta)\text{T}_1}\right]\right.\right.$$

$$\left.+\frac{1}{(\beta+\theta)(\beta+\theta-\lambda)}\left[-\lambda e^{-\lambda T_1}+-\lambda e^{(\beta+\theta)T_1}+\left(e^{-\lambda T_1}-1\right)(\beta+\theta)\right]\right]\Bigg\}$$

$$+\frac{\tilde{s}}{T}\left[\left(\frac{e^{-\lambda T}}{\lambda}-Te^{-\lambda T_1}\right)+\left(\frac{e^{-\lambda T_1}}{\lambda}+T_1e^{-\lambda T_1}\right)\right]+$$

$$\frac{\tilde{s}\,\alpha}{\delta^2}\left[\delta(T-T_1)-\log\left(1+\delta(T-T_1)\right)\right]$$

$$+\frac{\tilde{\pi}}{\delta^2T}\left\{\left(\alpha\delta^2-\delta\lambda^2\right)(T-T_1)+\delta^2\left(e^{-\lambda T_1}-e^{-\lambda T}\right)+\right.$$

$$\left.\log\left(1+\delta(T-T_1)\right)\left(\alpha\delta+\lambda\delta-\lambda^2-T\delta\lambda^2\right)\right\}$$

$$-\left\{\frac{\alpha\theta}{\beta+\theta}\left(MT_1-\frac{T_1^2}{2}\right)+\left(\frac{\alpha\beta}{\beta+\theta}+\frac{\lambda\beta e^{-\lambda T_1}}{\beta+\theta-\lambda}\right)\left[\frac{1}{(\beta+\theta)^2}\right.\right.$$

$$+\frac{2T_1e^{(\beta+\theta)T_1}}{\beta+\theta}-\frac{e^{(\beta+\theta)T_1}}{(\beta+\theta)^2}+\frac{M-T_1}{\beta+\theta}-\frac{Me^{(\beta+\theta)T_1}}{\beta+\theta}$$

$$+\frac{\lambda(\theta-\lambda)}{\beta+\theta-\lambda}\left[\frac{M}{\lambda}+\frac{(M-T_1)}{\beta+\theta}e^{-\lambda T_1}-\frac{1}{\lambda^2}\left(e^{-\lambda T_1}-1\right)\right]\Bigg\} \tag{12}$$

$$\frac{\partial T\tilde{V}C_2\,(T_1,T)}{\partial T_1}=0 \tag{13}$$

and $\frac{\partial T\tilde{V}C_2(T_1,T)}{\partial T}=0$ (14)

provided they satisfy the sufficient conditions

$$\left[\frac{\partial^2 T\tilde{V}C_2(T_1,T)}{\partial T_1^2}\right]_{at\ (T_1^*,T^*)}>0,$$

$$\left[\frac{\partial^2 T\tilde{V}C_2(T_1,T)}{\partial T^2}\right]_{at\ (T_1^*,T^*)}>0 \quad \text{and}$$

$$\left[\left(\frac{\partial^2 T\tilde{V}C_2(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 T\tilde{V}C_2(T_1,T)}{\partial T^2}\right)-\left[\frac{\partial^2 T\tilde{V}C_2(T_1,T)}{\partial T_1\partial T}\right]^2\right]_{at\ (T_1^*,T^*)}>0$$

Using (13),(14) solve for T_1, T using algorithm (2) and their relevant total cost using (12).

To obtain the optimal values of T_1 & T, we use the following algorithm.

5.4 Algorithm 2 :

Step 1: Perform (i) – (iv)

i. Start with $T_{1,(1)} = M$
ii. Substituting $T_{1,(1)}$ in equation (13) to find $T_{(1)}$
iii. Using $T_{(1)}$ in equation (14) we can find $T_{1,(2)}$
iv. Repeat (ii) and (iii) until no change occurs in the values of T_1 and T

Step 2: compare T_1 and M

1. If $T_1 < M$, T_1 is feasible. Then go to step (3).
2. If $T_1 \geq M$, T_1 is not feasible. Set $T_1 = M$ in the equation (17) to calculate T and then go to step 3.

Step 3: Calculate the corresponding $T\tilde{V}C_2$ (T_1^*,T^*)

Our main aim is to find the optimal values of T_1 and T which minimize TVC (T_1,T), we find that

$\tilde{TVC}(T_1^*,T^*)=\text{Min}\{\tilde{TVC}_1(T_1^*,T^*), \tilde{TVC}_2(T_1^*,T^*)\}$.

6. Numerical Examples :

To illustrate the preceding theory the following examples are presented.

Example-1: Let $\tilde{K}$=(100,200,300), $\tilde{P}$=(30,40,50), $\tilde{h}$=(10,12,14), $\tilde{s}$=(20,25,30), $\tilde{\pi}$=(10,15,20), $\tilde{I}_e$=(0.15,0.20,0.25), $\tilde{I}_r$=(0.11, 0.12,0.13), λ=0.03, θ=0.08, α=1000,β=0.3, M is taken as 5/365, 10/365,15/365 and 20/365 for

δ=1,2,3,4

In this example four different values of δ are adopted. For each value of δ four different values of M are tested. The results are given in Table 1. From Table 1, we observe that for each value of M the fuzzy average total inventory cost increases as δ (the backlogging parameter) increases. Hence to minimize the average inventory cost the retailer should control the backlogging parameter. * Now we fix δ and check it with distinct value of M. The results are given in Table 2. Table 2 informs us that for a fixed δ, the optimal fuzzy average inventory cost decreases as M increases. It is natural in our daily life.

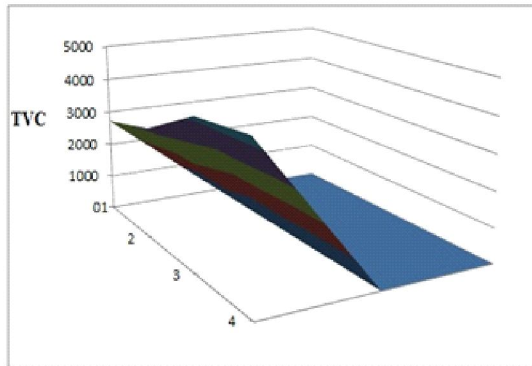
Table 1

M		δ=1	δ=2	δ=3	δ=4
5	TVC	2722.53	3193.83	4202.02	4432.38
	T_1*	0.2449	0.0533	0.0217	0.0133
	T*	0.2698	0.0540	0.0290	0.0135

Table 2

δ		M=5	M=10	M=15	M=20
1	TVC	2722.53	2474.78	2256.50	1535.37
	T_1*	0.2449	0.0872	0.0879	0.0190
	T*	0.2698	0.0947	0.0946	0.0233

Convexity of TVC with respect to T_1* and T* for Table 1



Example-2:

Let $\tilde{K}$=(200,300,400), $\tilde{P}$=(30,40,50), $\tilde{h}$=(11,12,13), $\tilde{s}$=(20,25,30), $\tilde{\pi}$=(10,15,20), $\tilde{I}_e$=(0.13,0.14,0.15), $\tilde{I}_r$=(0.11,0.12,0.13), λ=0.04, θ=0.07, α=1000, β=0.3, M is taken as 5/365, 10/365 ,15/365 and 20/365 for δ=1,2,3,4

In this example four different values of δ are adopted. For each value of δ four different values of M are tested. The results are given in Table 1. From Table 1, we observe that for each value of M the fuzzy average total inventory cost increases as δ (the backlogging parameter) increases. Hence to minimize the average inventory cost the retailer should control the backlogging parameter. * Now we fix δ and check it with distinct value of M. The results are given in Table 2. Table 2 informs us that for a fixed δ, the optimal fuzzy average inventory cost decreases as M increases. It is natural in our daily life.

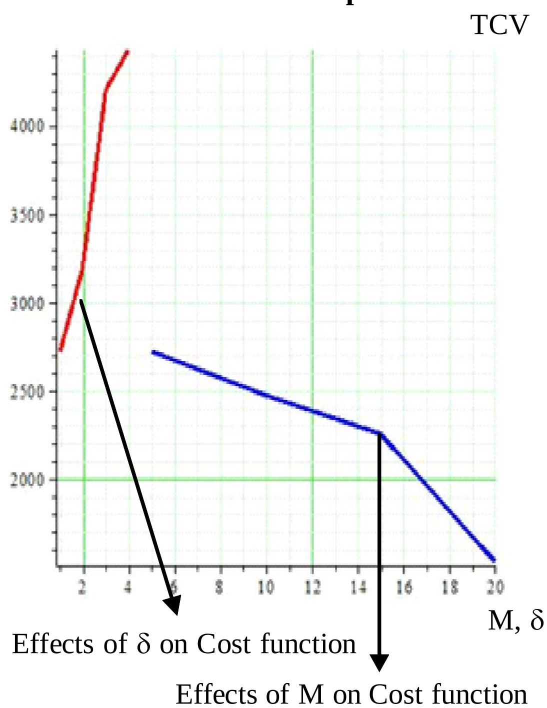
Table 3

M		δ=1	δ=2	δ=3	δ=4
5	TVC	2757.19	3066.27	3326.02	3551.36
	T_1*	0.0274	0.0689	0.0688	0.0687
	T*	0.0491	0.1219	0.1103	0.1052

Table 4

δ		M=5	M=10	M=15	M=20
1	TVC	2822.17	2618.78	2416.50	2199.18
	T_1*	0.3118	0.0961	0.0779	0.0190
	T*	0.3220	0.0989	0.0801	0.0194

Effects of Total Cost function over Backlogging parameter and trade credit period



6.1: Managerial Implications :

Based on the numerical examples considered above we now study the effects of change in M and δ on the optimal values of T_1,T and $T\tilde{V}C$ (T_1,T).

1. Each δ is tested with fixed value of M. From the computed results in Table 1, it is implied that the retailer should restrict the backlogging parameter with the aim of reducing the fuzzy average total inventory cost.

2. We analyze for fixed value of δ with varied values of M. Table 2 portrays a clear picture that supplier's permissible delay makes the retailer very lucrative, *ie.*, the retailer is the most beneficiary if he gets longer permissible delay period from the supplier.

7. Conclusion

In this paper, we developed a fuzzy inventory model with stock and time dependent demand with shortages under the condition of permissible delay in payments, inventory costs in Triangular fuzzy numbers. The backlogging rate is considered to be a decreasing function of the waiting time for the next replenishment. In the proposed model.we show that the minimized objective fuzzy cost function is jointly convex. Considering the fuzzy set theory in inventory modeling Rendors authenticity to the model formulated since fuzziness is the closet possible approach to reality. As reality is imprecise and only can be approximated to the certain extent, same way fuzzy theory helps one to incorporate uncertainties in the formulation of model, thus bringing it closer to the reality.

Numerical Examples are presented to illustrate the model. The results of the sensitivity analysis are also consistent. From the tables 1&2 we conclude that, for the proportion of customers who would like to accept backlogging at time t decreases as δ increases and for fixed δ, the increasing value of M will result in a significant decrease in the optimal average inventory cost.

This paper may be extended to octagonal fuzzy costs. Another possible extension of this work may consider the assumption of variable deterioration rate.

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