Ultra Scientist Vol. 27(3)A, 205-216 (2015).

On Weak forms of Continuous and Irresolute functions in Fuzzy Bitopological Spaces

T. THANGAM[1] and K. BAGEERATHI[2]

[1]Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur-628215, Tamilnadu (India)
[2]Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur-628216, Tamilnadu (India)

(Acceptance Date 16th October, 2015)

Abstract

Focus of this paper is to introduce the concept of pairwise fuzzy $\mathfrak{C}$- semi continuous, pairwise fuzzy $\mathfrak{C}$- totally continuous, pairwise fuzzy $\mathfrak{C}$- totally semi continuous and pairwise fuzzy $\mathfrak{C}$- irresolute mappings of a fuzzy bitopological space where $\mathfrak{C} : [0,1] \rightarrow [0,1]$ is a complement function. Several examples are given to illustrate the concepts introduced in this paper.

Key words : Fuzzy complement function $\mathfrak{C}$, fuzzy $\mathfrak{C}$-(τ_i, τ_j)–semi closure, fuzzy $\mathfrak{C}$-(τ_i, τ_j)–semi interior subsets, pairwise fuzzy $\mathfrak{C}$- semi continuous, pairwise fuzzy $\mathfrak{C}$- totally continuous, pairwise fuzzy $\mathfrak{C}$- totally semi continuous, pairwise fuzzy $\mathfrak{C}$- irresolute pairwise contra fuzzy $\mathfrak{C}$- irresolute mappings and fuzzy bitopological spaces.

1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh[10] in the year 1965. The theory of fuzzy topological space was introduced and developed by C. L. Chang[3]. A. Kandil[5] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological space. The concept of complement function $\mathfrak{C} : [0,1] \rightarrow [0,1]$ was introduced by K. Bageerathi and P. Thangavelu in[2]. The concept of pairwise fuzzy semi continuous and pairwise fuzzy irresolute was introduced and studied by S.S. Thakur, R. Malvia in[6,7] In this paper the concept of pairwise fuzzy $\mathfrak{C}$ -semicontinuous, pairwise fuzzy $\mathfrak{C}$- totally continuous, pairwise fuzzy

$\mathfrak{C}$- totally semi continuous, pairwise fuzzy $\mathfrak{C}$ - irresolute and pairwise contra fuzzy $\mathfrak{C}$ - irresolute mappings open sets, in fuzzy bitopological spaces is introduced and several examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries :

In this section we list some definitions and results that are needed. Any function $\mathfrak{C}:[0, 1]\to[0, 1]$ defined from the interval [0, 1] to itself is called a complement function. Throughout the paper $\mathfrak{C}$ denotes an arbitrary complement function and (X, τ_i, τ_j) is a fuzzy bitopological space in the sense of A. Kandil[5]. Throughout this paper,for fuzzy set λ of a fuzzy bitopological space (X, τ_i, τ_j), τ_i- int λ and τ_j-$cl_{\mathfrak{C}}\,\lambda$ means, respectively, the interior and closure of λ with respect to fuzzy topologies τ_i and τ_j.

Definition 2.1[2]*:*

If λ is a fuzzy subset of X then the complement $\mathfrak{C}\lambda$ of a fuzzy set λ is a fuzzy subset with membership function defined by $\mu_{\mathfrak{C}}\lambda\,(x) = \mathfrak{C}(\mu\lambda\,(x))$ for all $x\in X$.

A subset λ of a fuzzy topological space is fuzzy closed if its standard complement λ', where $\lambda'(x) = 1-\lambda(x)$ is fuzzy open. Several fuzzy topologists used this type of complement while extending the concepts in general topological spaces to fuzzy topological spaces. But there are other complements available in the fuzzy literature.

The properties of fuzzy complement function $\mathfrak{C}$ and $\mathfrak{C}\lambda$ are given in George Klir[4] and Bageerathi *et al.*,[2]. The following lemma will be useful in sequel. Some of the complement functions are given below.

Examples 2.2[4]*:*

(i) The standard complement function: $\mathfrak{C}_1(x) = 1- x$.

(ii) The Threshold type complement function for any $t\in [0, 1)$:

$$\mathfrak{C}_t(x) = \begin{cases} 1 & \text{for } 0 \le x \le t \\ 0 & \text{for } t < x \le 1 \end{cases}.$$

(iii) Sugeno class complement function for any $\lambda \in (1, \infty)$:

$$\mathfrak{C}_{S\lambda}(x) = \frac{1-x}{1+\lambda x}, \text{ for } x\in [0, 1].$$

(iv) Yagor class of complement function for $\omega \in (o, \infty)$:

$$\mathfrak{C}_{Y\omega}(x) = (1 - x^{\omega})^{1/\omega}, \text{ for } x\in [0, 1].$$

The next lemma can be easily established.

Lemma 2.3[4]*:*

The complement functions $\mathfrak{C}_1$, $\mathfrak{C}_t$, $\mathfrak{C}_{S\lambda}$ and $\mathfrak{C}_{Y\omega}$ satisfy the following conditions.

(i) Boundary condition: $\mathfrak{C}(0) = 1$ and $\mathfrak{C}(1)= 0$;

(ii) Monotonicity: for all x, y $\in$ [0, 1], $x \le y \Rightarrow \mathfrak{C}(x) \ge \mathfrak{C}(y)$;

(iii) $\mathfrak{C}$ is continuous and

(iv) Involutive: $\mathfrak{C}(\mathfrak{C}(x)) = x$ for all $x \in[0, 1]$.

Definition 2.4[1] *:*

For a family $\{A_\alpha: \alpha\in\Delta\}$ of fuzzy sub sets of X, the union, $A= \cup\{A_\alpha: \alpha\in\Delta\}$ and the intersection, $B = \cap\{A_\alpha : \alpha \in \Delta\}$, are defined with membership functions respectively

$\mu_A(x) = \sup\{\mu_{A\alpha}(x) : \alpha \in \Delta\}$ and $\mu_B(x) = \inf\{\mu_{A\alpha}(x) : \alpha \in \Delta\}$, $x \in X$.

Lemma 2.5[2] *:*

Let $\mathfrak{C} : [0, 1] \to [0, 1]$ be a complement function that satisfies the involutive and monotonicity properties. Then for any family $\{A_\alpha : \alpha \in \Delta\}$ of fuzzy subsets of X we have

(i) $\mathfrak{C}(\sup\{\mu_{A\alpha}(x) : \alpha \in \Delta\} = \inf\{\mathfrak{C}(\mu_{A\alpha}(x)) : \alpha \in \Delta\} = \inf\{(\mu_{\mathfrak{C}A\alpha}(x)) : \alpha \in \Delta\}$ and
(ii) $\mathfrak{C}(\inf\{\mu_{A\alpha}(x) : \alpha \in \Delta\} = \sup\{\mathfrak{C}(\mu_{A\alpha}(x)) : \alpha \in \Delta\} = \sup\{(\mu_{\mathfrak{C}A\alpha}(x)) : \alpha \in \Delta\}$.

Lemma 2.6[4] *:*

Let $\mathfrak{C} : [0, 1] \to [0, 1]$ be a complement function that satisfies involutive and monotonicity properties. Then for any family $\{A_\alpha : \alpha \in \Delta\}$ of fuzzy subsets of X. we have

(i) $\mathfrak{C}(\cup\{A_\alpha : \alpha \in \Delta\}) = \cap\{\mathfrak{C}A_\alpha : \alpha \in \Delta\}$ and
(ii) $\mathfrak{C}(\cap\{A_\alpha : \alpha \in\}) = \cup\{\mathfrak{C}A_\alpha : \alpha \in \Delta\}$.

Proposition 2.7[8]*:*

If the complement functions $\mathfrak{C}$ satisfies the monotonicity and involutive properties, then for any fuzzy subset λ of a fuzzy bitopological space, we have

(i) $\mathfrak{C}(\tau_i - \text{int}\lambda) = \tau_i - \text{cl}_{\mathfrak{C}}(\mathfrak{C}\lambda)$ and
(ii) $\mathfrak{C}(\tau_i - \text{cl}_{\mathfrak{C}}\lambda) = \tau_i - \text{int}(\mathfrak{C}\lambda)$.

Theorem 2.8[9] *:*

Let (X, τ_i, τ_j) be a fuzzy bitopological space and $\mathfrak{C}$ be any complement function. Then for a fuzzy subset λ and μ of a fuzzy bitopological space X, we have

(i) $\tau_i - \text{SInt}_{\mathfrak{C}}(\lambda) \le \lambda$.
(ii) λ is fuzzy $\mathfrak{C} - \tau_i$ - semi open $\Leftrightarrow \tau_i - \text{SInt}_{\mathfrak{C}}(\lambda) = \lambda$.
(iii) $\tau_i - \text{SInt}_{\mathfrak{C}}(\tau_i - \text{SInt}_{\mathfrak{C}}(\lambda)) = \tau_i - \text{SInt}_{\mathfrak{C}}(\lambda)$.
(iv) If $\lambda \le \mu$ then $\tau_i - \text{SInt}_{\mathfrak{C}}(\lambda) \le \tau_i - \text{SInt}_{\mathfrak{C}}(\mu)$.

Theorem 2.9[9]*:*

Let $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive properties. Then for a fuzzy subset λ of a fuzzy bitopological space X, we have(i) $\mathfrak{C}((\tau_i, \tau_j) - \text{SInt}_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j) - \text{S cl}_{\mathfrak{C}}(\mathfrak{C}\lambda)$. (ii) $\mathfrak{C}((\tau_i, \tau_j) - \text{S cl}_{\mathfrak{C}}(\lambda)) = (\tau_i, \tau_j) - \text{SInt}_{\mathfrak{C}}(\mathfrak{C}\lambda)$.

Theorem 2.10[9] *:*

Let (X, τ_i, τ_j) be a fuzzy bitopological space and $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive properties. Then for a fuzzy subset λ and μ of a fuzzy bitopological space X, we have(i) $\lambda \le \tau_i - \text{S cl}_{\mathfrak{C}}(\lambda)$.
(ii) λ is fuzzy $\mathfrak{C} - \tau_i$ - semi closed $\Leftrightarrow \tau_i\text{-S cl}_{\mathfrak{C}}(\lambda) = \lambda$.
(iii) $\tau_i - \text{S cl}_{\mathfrak{C}}(\tau_i - \text{S cl}_{\mathfrak{C}}(\lambda)) = \tau_i - \text{S cl}_{\mathfrak{C}}(\lambda)$.
(iv) If $\lambda \le \mu$ then $\tau_i - \text{S cl}_{\mathfrak{C}}(\lambda) \le \tau_i - \text{S cl}_{\mathfrak{C}}(\mu)$.

Lemma 2.11[2] *:*

Suppose f is a function from X to Y. Then $f^{-1}(\mathfrak{C}B) = \mathfrak{C}(f^{-1}(B))$ for any fuzzy sub set B of Y.

Lemma 2.12[2] *:*

Suppose f is a function from X to Y and $\mathfrak{C}$ satisfies monotonicity and involutive properties. Then $f(\mathfrak{C} A) \supseteq \mathfrak{C}(f(A))$ for any fuzzy sub set A of X.

Definition 2.13[7]*:*

A mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be a pairwise fuzzy continuous (resp.pairwise fuzzy open) if $f : (X, \tau_1) \to (Y, \sigma_1)$ and $f : (X, \tau_2) \to (Y, \sigma_2)$ are fuzzy continuous (resp.fuzzy open).

Theorem 2.14[8] *:*

A mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a pairwise fuzzy continuous iff $f^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - τ_i –closed in X for each fuzzy $\mathfrak{C}$ - σ_i -closed in Y, where $\mathfrak{C}$ satisfies the monotonicity and involutive properties.

3. Pairwise fuzzy $\mathfrak{C}$ *- semi continuous:*

In this section we define the notion of pairwise fuzzy $\mathfrak{C}$ - semi continuous set and discussed some of their properties.

Definition 3.1:

A mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise fuzzy $\mathfrak{C}$ - semi continuous if the inverse image of every fuzzy σ_i - open in Y is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X.

Definition 3.2:

Let $f : (X, t_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function then f is said to be (a) pairwise fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open if $f(\lambda)$ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open for each fuzzy τ_i - open in X.

(b) pairwise fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed if $f(\lambda)$ is pairwise fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi closed for each fuzzy $\mathfrak{C}$ - τ_i - closed in X.

Definition 3.3 :

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function. Then f is called pairwise fuzzy $\mathfrak{C}$ - closed if $f(\lambda)$ is fuzzy $\mathfrak{C}$ - σ_i - closed subset of Y for each fuzzy $\mathfrak{C}$ - τ_i - closed subset λ of X.

Theorem 3.4 :

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a pairwise fuzzy continuous and pairwise fuzzy open function. Let $\mathfrak{C}$ be any complement function. Then (a) If λ is fuzzy $\mathfrak{C}$ -(τ_i, τ_j)- semi open in X, then $f(\lambda)$ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open in Y. (b) If λ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open in Y, then $f^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X.

Proof :

(a) Let λ be a fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X, then there exists fuzzy τ_i - open μ such that $\mu \le \lambda \le \tau_j\text{-}cl_{\mathfrak{C}}(\mu)$. Since f is fuzzy pairwise open and fuzzy pairwise continuous, $f(\mu)$ is fuzzy σ_i - open. we have $f(\mu) \le f(\lambda) \le f(\tau_j\text{-}cl_{\mathfrak{C}}\ \mu) \le \sigma_j\text{-}cl_{\mathfrak{C}}\ f(\mu)$. This shows that $f(\lambda)$ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open in Y.

(b) Let λ be a fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open in Y, then there exists fuzzy σ_i - open μ such that $\mu \le \lambda \le \sigma_j\text{-}cl_{\mathfrak{C}}(\mu)$. Since f is fuzzy pairwise continuous and fuzzy pairwise open, $f^{-1}(\mu)$ is fuzzy τ_i - open in X. we have $f^{-1}(\mu) \leq f^{-1}(\lambda) \leq f^{-1}(\sigma_j - cl_{\mathfrak{C}}\ \mu) \leq \tau_j - cl_{\mathfrak{C}}\ f^{-1}(\mu)$. This shows that $f^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$-(τ_i, τ_j)- semi open in X.

Remark 3.5:

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function and $\mathfrak{C}$ be a complement function, then Every pairwise fuzzy continuous is pairwise fuzzy $\mathfrak{C}$- semi continuous. But the converse may not be true.

Example 3.6:

Let X = Y = {a, b}. The mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by f (a) = a, f (b) = b. Let $\tau_1 = \{0, \{a_{.5}, b_{.3}\}, 1\}$ and $\tau_2 = \{0, \{a_{.3}, b_{.5}\}, 1\}$. Let $\sigma_1 = \{0, \{a_{.6}, b_{.4}\}, 1\}$ and $\sigma_2 = \{0, \{a_{.2}, b_{.4}\}, 1\}$. Let $\mathfrak{C}(x) = \dfrac{1-x}{1+2x}$ be the complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}(\tau_1)=\{1, \{a_{.25}, b_{.44}\}, 0\}$ and $\mathfrak{C}(\tau_2) = \{1, \{a_{.44}, b_{.25}\}, 0\}$. Let $\lambda = \{a_{.95}, b_{.99}, c_{.99}\}$. The family of all fuzzy $\mathfrak{C}$ - σ_i- closed sets are $\mathfrak{C}(\sigma_1) = \{1, \{a_{.18}, b_{.33}\}, 0\}$ and $\mathfrak{C}(\sigma_2)= \{1, \{a_{.31}, b_{.33}\}, 0\}$. Let $\mu = \{a_{.6}, b_{.4}\}$. Then $f^{-1}(\mu)=\{a_{.6}, b_{.4}\}$. It can be evaluated that $\tau_2 - cl_{\mathfrak{C}}\ (\tau_1 - Int\ (f^{-1}(\mu)) = 1 \geq f^{-1}(\mu)$. That is $f^{-1}(\mu)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open but this is not fuzzy τ_i- open.

Remark 3.7:

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function and $\mathfrak{C}$ be a complement function then

(a) Every pairwise fuzzy open is pairwise fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open.

(b) Every pairwise fuzzy $\mathfrak{C}$- closed is pairwise fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi closed.

The reverse implications are not true as shown by following examples.

Example 3.8:

Let X = Y = {a, b}. The mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by f (a) = a, f (b) = b. Let $\tau_1 = \{0, \{a_{.6}, b_{.6}\}, 1\}$ and $\tau_2 = \{0, \{a_{.3}, b_{.5}\}, 1\}$. Let $\sigma_1 = \{0, \{a_{.6}, b_{.4}\}, 1\}$ and $\sigma_2 = \{0, \{a_{.2}, b_{.4}\}, 1\}$. Let $\mathfrak{C}(x) = \dfrac{1-x}{1+3x}$, $0 \leq x \leq 1$ be the complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}(\tau_1) = \{1, \{a_{.18}, b_{.18}\}, 0\}$ and $\mathfrak{C}(\tau_2) = \{1, \{a_{.44}, b_{.25}\}, 0\}$. The family of all fuzzy $\mathfrak{C}$ - σ_i - closed sets are $\mathfrak{C}(\sigma_1) = \{1, \{a_{.18}, b_{.33}\}, 0\}$ and $\mathfrak{C}(\sigma_2) = \{1, \{a_{.31}, b_{.33}\}, 0\}$. Let $\mu = \{a_{.6}, b_{.6}\}$. Then $f(\mu)=\{a_{.6}, b_{.6}\}$. It can be evaluated that σ_2- $cl_{\mathfrak{C}}$ $(\sigma_1 \text{-Int}\ (f(\mu)) = 1 \geq f(\mu)$. That is $f(\mu)$ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open but this is not fuzzy $s_{i\text{-}}$ open.

Example 3.9 :

Let X = Y = {a, b}. The mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by f (a) = a, f (b) = b. Let $\tau_1 = \{0, \{a_{.6}, b_{.6}\}, 1\}$ and $\tau_2 = \{0, \{a_{.3}, b_{.5}\}, 1\}$. Let $\sigma_1 = \{0, \{a_{.6}, b_{.4}\}, 1\}$ and $\sigma_2 = \{0, \{a_{.2}, b_{.4}\}, 1\}$. Let $\mathfrak{C}(x) = \dfrac{1-x}{1+3x}$, $0 \leq x \leq 1$ be the complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}(\tau_1) = \{1, \{a_{.18}, b_{.18}\}, 0\}$ and $\mathfrak{C}(\tau_2) = \{1, \{a_{.44}, b_{.25}\}, 0\}$. The family of all fuzzy $\mathfrak{C}$ - σ_i - closed sets are $\mathfrak{C}(\sigma_1)=\{1, \{a_{.18}, b_{.33}\}, 0\}$ and $\mathfrak{C}(\sigma_2)=$

{1, {$a_{.31}$, $b_{.33}$}, 0}. Let μ = {$a_{.18}$, $b_{.18}$}. Then f (μ)={$a_{.18}$, $b_{.18}$}. It can be evaluated that σ_2 - Int (σ_1 - $cl_{\mathfrak{C}}$ (f (μ)) = {0} $\leq$ f (μ). That is f (μ) is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi closed but this is not fuzzy σ_i- closed.

Theorem 3.10:

Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) and $\mathfrak{C}$ be a complement function that satisfies the monotonic and involutive properties then the following are equivalent :

(a) For every fuzzy $\mathfrak{C}$ - σ_i - closed set μ in Y, $f^{-1}(\mu)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed in X.
(b) f ((τ_i, τ_j) - S $cl_{\mathfrak{C}}$ (μ)) $\leq$ σ_i - $cl_{\mathfrak{C}}$ f (μ) for every fuzzy set μ of X.
(c) (τ_i , τ_j) - S $cl_{\mathfrak{C}}$ ($f^{-1}(\lambda)$) $\leq$ f^{-1}(σ_i - $cl_{\mathfrak{C}}$ λ) for every fuzzy set λ of Y.
(d) f^{-1} (σ_i - Int l) $\leq$ (τ_i, τ_j) - $SInt_{\mathfrak{C}}$ ($f^{-1}(\lambda)$) for every fuzzy set l of Y.

Proof :

(a) $\Rightarrow$ (b)

Let μ be any fuzzy subset of X, then σ_i - $cl_{\mathfrak{C}}$ f(μ) is fuzzy $\mathfrak{C}$ - σ_i - closed in Y. By our assumption f^{-1}(σ_i - $cl_{\mathfrak{C}}$ f (μ)) is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed in X. since $\mathfrak{C}$ satisfies the monotonic and involutive properties, (τ_i, τ_j) - S $cl_{\mathfrak{C}}$ (μ) = (τ_i, τ_j) - S $cl_{\mathfrak{C}}$ f^{-1}(f (μ))
$\leq$ (τ_i, τ_j) - S $cl_{\mathfrak{C}}$ f^{-1}(σ_i - $cl_{\mathfrak{C}}$ f (μ))By Theorem 2.10, (τ_i, τ_j) - S $cl_{\mathfrak{C}}$ (μ) = f^{-1} (σ_i - $cl_{\mathfrak{C}}$ (f (μ)). Therefore f ((τ_i, τ_j) - S $cl_{\mathfrak{C}}$ (μ) dd σ_i - $cl_{\mathfrak{C}}$ f (μ).

(b) $\Rightarrow$ (c)

Let λ be any fuzzy subset of Y. By (b), f ((τ_i , τ_j) - S $cl_{\mathfrak{C}}$ f^{-1} (λ)) dd σ_i - $cl_{\mathfrak{C}}$ f (f^{-1} (λ)) = σ_i - $cl_{\mathfrak{C}}$ (λ). This implies that (τ_i, τ_j) - S $cl_{\mathfrak{C}}$ ($f^{-1}(\lambda)$) dd f^{-1}(σ_i - $cl_{\mathfrak{C}}$ λ).

(c) $\Rightarrow$ (d)

Taking complement on both sides of (c) and since $\mathfrak{C}$ satisfies the monotonic and involutive properties, we get $\mathfrak{C}$ ((τ_i, τ_j) - S $cl_{\mathfrak{C}}$ (f^{-1}($\mathfrak{C}$ λ))) $\geq$ $\mathfrak{C}$ (f^{-1}(σ_i - $cl_{\mathfrak{C}}$ $\mathfrak{C}$ λ)). By theorem 2.7, f^{-1} (σ_i - Int λ) $\leq$ (τ_i, τ_j) - $SInt_{\mathfrak{C}}$ ($f^{-1}(\lambda)$).

(d) $\Rightarrow$ (a)

Let m be any fuzzy $\mathfrak{C}$ - σ_i - closed set in Y.Since $\mathfrak{C}$ satisfies the monotonic and involutive properties , $\mathfrak{C}$ μ is fuzzy σ_i - open in Y. Therefore $\mathfrak{C}$ μ = σ_i - Int($\mathfrak{C}$ μ). According to the assumption f^{-1}($\mathfrak{C}$ μ) = f^{-1}(σ_i - Int $\mathfrak{C}$ μ) $\leq$ (τ_i, τ_j) - $SInt_{\mathfrak{C}}$ (f^{-1}($\mathfrak{C}$ μ)) $\leq$ f^{-1}($\mathfrak{C}$ μ). Hence f^{-1}($\mathfrak{C}$ μ) = (τ_i , τ_j) - $SInt_{\mathfrak{C}}$ (f^{-1}($\mathfrak{C}$ μ)). Therefore (f^{-1}($\mathfrak{C}$ μ)) is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. This implies that $f^{-1}(\mu)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed in X.

The following example shows that if the complement function does not satisfies the involutive condition then the conclusion of the above Theorem is not true.

Example 3.11:

Let X = Y = { a, b}. Consider the identity map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1 , σ_2) defined by f(a) = a and f(b) = b. Let τ_1 = {0, {$a_{.2}$, $b_{.3}$}, 1}, τ_2 = {0, {$a_{.3}$, $b_{.2}$},1}, σ_1 = {0, {$a_{.4}$, $b_{.3}$}, 1} and σ_2 = {0, {$a_{.2}$, $b_{.1}$},1}. Let $\mathfrak{C}$ (x) =$\sqrt{1-x}$, 0 $\leq$ x $\leq$ 1, be a complement function that does not satisfy the involutive property. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = { 0, {a.$_{89}$, b.$_{83}$}, 1} and $\mathfrak{C}$ (τ_2) = {0, {a.$_{83}$, b.$_{89}$}, 1}. The family of all fuzzy $\mathfrak{C}$ - σ_i - closed sets are given by $\mathfrak{C}$ (σ_1) = {0, {a.$_8$, b.$_5$ }, 1} and $\mathfrak{C}$ (σ_2) = {0, {a.$_{89}$, b.$_{95}$},1}. Let λ = {a.$_5$, b.$_4$ }. Then $f^{-1}(\lambda)$ = {a.$_5$, b.$_4$}. It can be calculated that is fuzzy (τ_1, τ_2) - S $cl_{\mathfrak{C}}$ $(f^{-1}(\lambda))$ ={a.$_8$, b.$_8$ } and $f^{-1}(\sigma_1 - cl_{\mathfrak{C}}\ \lambda)$ = {a.$_8$, b.$_5$}. Therefore (τ_1, τ_2) - S $cl_{\mathfrak{C}}$ $(f^{-1}(\lambda))$ = {a.$_8$, b.$_8$ } $\nleq f^{-1}(\sigma_1 - cl_{\mathfrak{C}}\ \lambda)$. But for every fuzzy $\mathfrak{C}$ - σ_i - closed set μ in Y, $f^{-1}(\mu)$ is fuzzy $\mathfrak{C}$-(τ_i, τ_j) - semi closed in X.

4. Pairwise fuzzy $\mathfrak{C}$ - totally continuous and Pairwise fuzzy $\mathfrak{C}$ - totally semi continuous functions:

In this section we introduce the concept of pairwise fuzzy $\mathfrak{C}$ - totally continuous and pairwise fuzzy $\mathfrak{C}$ - totally semi continuous funtions and discussed some of their properties.

Definition 4.1:

A function $f : (X, \tau_i, \tau_j) \rightarrow (Y , \sigma_i , \sigma_j)$ is said to be pairwise fuzzy $\mathfrak{C}$ - totally continuous if the inverse image of every fuzzy σ_i - open subset of Y is both fuzzy $\mathfrak{C}$ - τ_i - closed and fuzzy τ_i - open subset of X.

Definition 4.2:

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y , \sigma_1, \sigma_2)$ is said to be pairwise fuzzy $\mathfrak{C}$ - totally semi continuous if the inverse image of every fuzzy σ_i - open subset of Y is both fuzzy $\mathfrak{C}$ - (τ_i,τ_j) - semi closed and fuzzy $\mathfrak{C}$ - (τ_i ,τ_j) - semi open subset of X.

Every pairwise fuzzy $\mathfrak{C}$ - totally continuous function is pairwise fuzzy $\mathfrak{C}$ - totally semi continuous but the converse is not true as shown by the following example.

Example 4.3:

Let X = Y = { a, b, c}. Consider the identity function $f : (X , \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ defined by f(a) = a, f(b) = b and f(c) = c. Let τ_1 = { 0, {a.$_3$, b.$_5$, c.$_4$}, {.2, .1, .1}, 1}, τ_2 = {0, 1}, σ_1 = {0, {a.$_4$, b.$_6$, c.$_4$}, 1} and σ_2 = {0, 1}. Let $\mathfrak{C}(x) = \dfrac{1-x}{1+2x}$, be a complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = {1, {a.$_{4375}$, b.$_{25}$, c.$_{33}$}, {.6, .8, .8}, 0}, $\mathfrak{C}$ (τ_2) = {0, 1}, $\mathfrak{C}$ (σ_1) = {1, {a.$_{33}$, b.$_{18}$, c.$_{33}$}, 0} and $\mathfrak{C}$ (σ_2) = {0, 1}. Then f is pairwise fuzzy $\mathfrak{C}$ - totally semi continuous but not pairwise fuzzy $\mathfrak{C}$ - totally continuous.

Every pairwise fuzzy $\mathfrak{C}$ - totally semi continuous function is pairwise fuzzy $\mathfrak{C}$ - semi continuous but the converse is not true as shown by the following example.

Example 4.4:

Let X = Y = { a, b, c}. Consider the function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ defined by f(a) = a , f(b) = b and f(c) = c. Let τ_1 = {0, 1, {a.$_3$, b.$_5$, c.$_4$}}, τ_2= { 0, 1}, σ_1 = {0, 1, {a.$_4$, b.$_6$, c.$_4$}} and σ_2 = {0, 1}. Let $\mathfrak{C}(x) = \dfrac{1-x}{1+2x}$, be a complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are given by $\mathfrak{C}$ (τ_1) = {0, {a.$_{4375}$, b.$_{25}$, c.$_{33}$}, 1}, $\mathfrak{C}$ (τ_2) = {0, 1}, $\mathfrak{C}$ (σ_1) = {1, {a.$_{33}$, b.$_{18}$, c.$_{33}$}, 0} and $\mathfrak{C}$ (σ_2) = {0, 1}. Then f is pairwise fuzzy $\mathfrak{C}$ - semi continuous but not pairwise fuzzy $\mathfrak{C}$ - totally semi continuous.

Theorem 4.5:

If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise fuzzy $\mathfrak{C}$ - totally semi continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \delta_1, \delta_2)$ is pairwise fuzzy $\mathfrak{C}$ - continuous then $g \circ f : (X, \tau_1, \tau_2) \to (Z, \delta_1, \delta_2)$ is pairwise fuzzy $\mathfrak{C}$ - totally semi continuous function.

Proof :

Let λ be a fuzzy δ_{i} - open set in Z. Since g is pairwise fuzzy $\mathfrak{C}$ - continuous, $g^{-1}(\lambda)$ is fuzzy σ_i - open in Y. Since f is pairwise fuzzy $\mathfrak{C}$ - totally semi continuous, $f^{-1}(g^{-1}(\lambda))$ is both fuzzy $\mathfrak{C}$ - (τ_i,τ_j) - semi open and fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed in X. This implies that $(g \circ f)^{-1}(\lambda)$ is both fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed and fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X. Hence $g \circ f$ is pairwise fuzzy $\mathfrak{C}$ - totally semi continuous.

5. Pairwise fuzzy $\mathfrak{C}$ - irresolute and Pairwise contra fuzzy $\mathfrak{C}$ - irresolute functions :

In this section we introduce the concept of pairwise fuzzy $\mathfrak{C}$ - irresolute and pairwise contra fuzzy $\mathfrak{C}$ - irresolute functions and discussed some of their properties.

Definition 5.1:

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$. Then f is said to be

(i) pairwise fuzzy $\mathfrak{C}$ - irresolute if the inverse image of every fuzzy $\mathfrak{C}$ - (σ_i, σ_j)- semi open in Y is fuzzy $\mathfrak{C}$ - (τ_i,τ_j) - semi open in X.

(ii) pairwise contra fuzzy $\mathfrak{C}$ - irresolute if the inverse image of every fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open in Y is fuzzy $\mathfrak{C}$ - (τ_i,τ_j) - semi closed in X.

Theorem 5.2:

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $\mathfrak{C}$ be a complement function that satisfies the involutive condition. Then the following conditions are equivalent:

(i) f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute.

(ii) The inverse image of each fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi closed in Y is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open set in X.

Proof :

Let μ be a fuzzy $\mathfrak{C}$ - (σ_{i}, σ_j) - semi closed in Y. Since $\mathfrak{C}$ satisfies the involutive condition , $\mathfrak{C}\,\mu$ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open. Since f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute, by Definition 5.1 , $f^{-1}(\mathfrak{C}\,\mu)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j)- semi open in X. By Theorem 2.11, $\mathfrak{C}(f^{-1}(\mu))$ is fuzzy $\mathfrak{C}$ -(τ_i, τ_j) - semi closed in X. Therefore $f^{-1}(\mu)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X.

Conversely, let μ be a fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open in Y. Since $\mathfrak{C}$ satisfies the involutive condition , $\mathfrak{C}\,\mu$ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi closed in Y. By our assumption, $f^{-1}(\mathfrak{C}\,\mu)$ is fuzzy $\mathfrak{C}$ -(τ_i, τ_j) - semi open in X. By Theorem 2.11, $\mathfrak{C}(f^{-1}(\mu))$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X. By Theorem 2.11, $f^{-1}(\mu)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed in X. By Definition 5.1, f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute.

The following example shows that if the complement function does not satisfies the involutive condition then the conclusion of the above Theorem is not true.

Example 5.3:

Let X = Y = { a, b}. Consider the identity map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by f(a)=a and f(b) = b. Let τ_1={0, {a.2, b.3},1}, τ_2 = { 0, {a.3 , b.2},1}, σ_1 = {0, {a.4 , b.3}, 1} and σ_2 = {0, {a.2 , b.1},1}. Let $\mathfrak{C}(x)= \sqrt{1-x}$, $0 \leq x \leq 1$, be a complement function that does not satisfy the involutive property. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = {0, {a.89 , b.83}, 1} and -$\mathfrak{C}$ (τ_2) = {0, {a.83 , b.89},1}. The family of all fuzzy $\mathfrak{C}$ - σ_i - closed sets are given by $\mathfrak{C}$ (σ_1)={0, {a.8 , b.5}, 1} and $\mathfrak{C}$ (σ_2)= {0, {a.89 , b.95},1}. Let λ = {a.5, b.4}. Then λ is fuzzy $\mathfrak{C}$ - (σ_1, σ_2) - semi open in Y and $f^{-1}(\lambda)$ = {a.5 , b.4} is fuzzy $\mathfrak{C}$ - (σ_1,σ_2) - semi closed in X. Let μ = {a.4 , b.2 } is fuzzy $\mathfrak{C}$ - (σ_{1-} ,σ_2) - semi closed but $f^{-1}(\mu)$ = {a.4 , b.2} is not fuzzy $\mathfrak{C}$ - (τ_1, τ_2) - semi open.

Theorem 5.4:

Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \delta_1, \delta_2)$ be two maps such that $g \circ f : (X, \tau_1, \tau_2) \to (Z, \delta_1, \delta_2)$. Then

(i) g o f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute if g is pairwise fuzzy $\mathfrak{C}$ - irresolute and f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute.
(ii) g o f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute if g is pairwise contra fuzzy $\mathfrak{C}$ - irresolute, f is pairwise fuzzy $\mathfrak{C}$ - irresolute and $\mathfrak{C}$ satisfies the involutive property.

Proof :

Let l be a fuzzy $\mathfrak{C}$ - (δ_i,δ_j) - semi open in Z. Since g is pairwise fuzzy $\mathfrak{C}$ -irresolute, $g^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - $(\sigma_{i,j})$ - semi open in Y. Since f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute, $f^{-1}(g^{-1}(\lambda))$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j)- semi closed in X. That is $(g \circ f)^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi closed in X. By Definition 5.1, g o f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute.

Let λ be a fuzzy $\mathfrak{C}$ - (δ_i, δ_j) - semi closed in Z. Since g is pairwise contra fuzzy $\mathfrak{C}$ - irresolute, $\mathfrak{C}$ satisfies the involutive property and by Theorem 5.2, we have $g^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open in Y. Since f is pairwise fuzzy $\mathfrak{C}$ - irresolute, $f^{-1}(g^{-1}(\lambda))$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X. That is $(g \circ f)^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X. By Theorem 5.2, g o f is pairwise contra fuzzy $\mathfrak{C}$ - irresolute.

Remark 5.5:

The concept of pairwise fuzzy $\mathfrak{C}$ - irresolute and pairwise fuzzy continuous are independent as shown by the following example.

Example 5.6:

Let X = Y = {a, b}. Consider the map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by f(a) = a and f(b) = b. Let τ_1 = {0, 1, {a.3 , b.5}}, τ_2 ={0, 1}, σ_1 = {0, 1, {a.4 , b.6}} and σ_2 = {0, 1}. Let $\mathfrak{C}(x) = \dfrac{1-x}{1+x}$, be a complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}$ (τ_1) = {0, 1, {a.5 , b.3}} and $\mathfrak{C}$ (τ_2) = {0, 1}. The family of all fuzzy $\mathfrak{C}$ - σ_i - closed sets are given by $\mathfrak{C}$ (σ_1) = {0, 1, {a.4 , b.25 }} and $\mathfrak{C}$ (σ_2)={0, 1}. Let λ={a.4 , b.6}. Then $f^{-1}(\lambda)$=

$\{a_{.4}, b_{.6}\}$. Then it can be computed that λ is fuzzy $\mathfrak{C}$ - (σ_2, σ_1) - semi open and $f^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - (τ_2, τ_1)-semi open. But λ is fuzzy σ_i-open but $f^{-1}(\lambda)$ is not fuzzy τ_i - open. This shows that f is pairwise fuzzy $\mathfrak{C}$ - irresolute but not pairwise fuzzy $\mathfrak{C}$ - continuous.

Example 5.7:

Let X = Y = {a, b}. Consider the map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by f(a) = a and f(b) = b. Let $\tau_1 = \{0, 1, \{a_{.3}, b_{.5}\}, \{a_{.4}, b_{.6}\}\}$, $\tau_2 = \{0, 1, \{a_{.4}, b_{.1}\}\}$, $\sigma_1 = \{0, 1, \{a_{.4}, b_{.6}\}\}$ and $\sigma_2 = \{0, 1\}$. Let $\mathfrak{C}(x) = \frac{1-x}{1+x}$, be a complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}(\tau_1) = \{0, 1, \{a_{.5}, b_{.3}\}, \{a_{.4}, b_{.25}\}\}$ and $\mathfrak{C}(\tau_2) = \{0, 1, \{a_{.4}, b_{.8}\}\}$. The family of all fuzzy $\mathfrak{C}$ - σ_i - closed sets are given by $\mathfrak{C}(\sigma_1) = \{0, 1, \{a_{.4}, b_{.25}\}\}$ and $\mathfrak{C}(\sigma_2) = \{0, 1\}$. Let $\lambda = \{a_{.6}, b_{.6}\}$. Then λ is fuzzy $\mathfrak{C}$ - (σ_2, σ_1) - semi open but $f^{-1}(\lambda)$ is not fuzzy $\mathfrak{C}$ - (τ_2, τ_1) - semi open. Also inverse image of every σ_i - open is τ_i - open. This implies that f is pairwise fuzzy $\mathfrak{C}$ - continuous but not pairwise fuzzy $\mathfrak{C}$ - irresolute.

Remark 5.8 :

Every pairwise fuzzy $\mathfrak{C}$ - irresolute is pairwise fuzzy $\mathfrak{C}$ - semi continuous but the converse need not be true as shown by the following example.

Example 5.9:

Let X = Y = {a, b}. Consider the map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by f(a) = a and f(b) = b. Let $\tau_1 = \{0, 1, \{a_{.3}, b_{.5}\}, \{a_{.4}, b_{.6}\}\}$, $\tau_2 = \{0, 1, \{a_{.4}, b_{.1}\}\}$, $\sigma_1 = \{0, 1, \{a_{.4}, b_{.6}\}\}$ and $\sigma_2 = \{0, 1\}$. Let $\mathfrak{C}(x) = \frac{1-x}{1+x}$, be a complement function. Then the family of all fuzzy $\mathfrak{C}$ - τ_i - closed sets are $\mathfrak{C}(\tau_1) = \{0, 1, \{a_{.5}, b_{.3}\}, \{a_{.4}, b_{.25}\}\}$ and $\mathfrak{C}(\tau_2) = \{0, 1, \{a_{.4}, b_{.8}\}\}$. The family of all fuzzy $\mathfrak{C}$ - σ_i - closed sets are given by $\mathfrak{C}(\sigma_1) = \{0, 1, \{a_{.4}, b_{.25}\}\}$ and $\mathfrak{C}(\sigma_2) = \{0, 1\}$. Let $\lambda = \{a_{.6}, b_{.6}\}$. Then λ is fuzzy $\mathfrak{C}$ - (σ_2, σ_1) - semi open but $f^{-1}(\lambda)$ is not fuzzy $\mathfrak{C}$ - (τ_2, τ_1) - semi open. Also inverse image of every σ_i - open is $\mathfrak{C}$ - (τ_i, τ_j) - semi open . This implies that inverse image of every σ_i - open is $\mathfrak{C}$ - (τ_i, τ_j) - semi open. But inverse image of every $\mathfrak{C}$ - (σ_i, σ_j) - semi open need not be $\mathfrak{C}$ - (τ_i, τ_j) - semi open.

Theorem 5.10:

Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be fuzzy bitopological spaces and $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$. Then the following are equivalent:

(i) $f((\tau_i, \tau_j)\text{ - S cl}_{\mathfrak{C}}(\lambda)) \leq (\tau_i, \tau_j)\text{ - S cl}_{\mathfrak{C}}\, f(\lambda)$ for every subset λ of X.

(ii) $(\tau_i, \tau_j)\text{ - S cl}_{\mathfrak{C}}(f^{-1}(\mu)) \leq f^{-1}((\tau_i, \tau_j)\text{ - S cl}_{\mathfrak{C}}(\mu))$ for every subset m of Y.

Proof (i) $\Rightarrow$ (ii)

Let μ be a fuzzy set in Y. Then $f^{-1}(\mu)$ is fuzzy set in X. Therefore by our assumption, $f((\tau_i, \tau_j)\text{ - S cl}_{\mathfrak{C}}(f^{-1}(\mu))) \leq (\tau_i, \tau_j)\text{-S cl}_{\mathfrak{C}}(f(f^{-1}(\mu))) = (\tau_i, \tau_j)\text{ - S cl}_{\mathfrak{C}}(\mu)$. This implies that $(\tau_i, \tau_j)\text{ - S cl}_{\mathfrak{C}}(f^{-1}(\mu)) \leq f^{-1}((\tau_i, \tau_j)\text{ - S cl}_{\mathfrak{C}}(\mu))$.

(ii) $\Rightarrow$ (i) Let $\mu = f(\lambda)$ where λ is s subset of X. Then $(\tau_i, \tau_j)\text{-S cl}_{\mathfrak{C}}(\lambda) = (\tau_i, \tau_j)\text{ -S cl}_{\mathfrak{C}}(f^{-1}(\mu))$

$\leq f^{-1}((\tau_i, \tau_j) - S\ cl_{\mathfrak{C}}(\mu)) = f^{-1}((\tau_i, \tau_j) - S\ cl_{\mathfrak{C}}\ f(\lambda))$. This implies that $f((\tau_i, \tau_j) - S\ cl_{\mathfrak{C}}(\lambda)) \leq (\tau_i, \tau_j) - S\ cl_{\mathfrak{C}}(f(\lambda))$.

Theorem 5.11:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$. Then the following are equivalent:
(i) f is pairwise fuzzy $\mathfrak{C}$ - irresolute.
(ii) $f^{-1}((\tau_i, \tau_j)\text{-}S\ Int_{\mathfrak{C}}(\mu)) \leq (\tau_i, \tau_j) - S\ Int_{\mathfrak{C}}(f^{-1}(\mu))$ for every subset μ of Y.

Proof :

Let μ be a fuzzy $\mathfrak{C}$ - (τ_i,τ_j) - semi open in Y. By (i) $f^{-1}((\tau_i, \tau_j) - S\ Int_{\mathfrak{C}}(\mu))$ is fuzzy $\mathfrak{C}$ - (τ_i,τ_j) - semi open. Therefore by Theorem 2.9, $f^{-1}((\tau_i, \tau_j) - S\ Int_{\mathfrak{C}}(\mu)) = (\tau_i, \tau_j)\text{-}S\ Int_{\mathfrak{C}}(f^{-1}((\tau_i, \tau_j) - S\ Int_{\mathfrak{C}}(\mu))) \leq (\tau_i, \tau_j) - S\ Int_{\mathfrak{C}}(f^{-1}(\mu))$.

Conversely let μ be any fuzzy $\mathfrak{C}$ - (τ_i,τ_j) - semi open in Y. Then $(\tau_i, \tau_j) - S\ Int_{\mathfrak{C}}(\mu) = \mu$ and $f^{-1}(\mu) = f^{-1}((\tau_i, \tau_j) - S\ Int_{\mathfrak{C}}(\mu)) \leq (\tau_i, \tau_j) \text{-}S\ Int_{\mathfrak{C}}(f^{-1}(\mu))$. This implies that $f^{-1}(\mu) = (\tau_i, \tau_j) - S\ Int_{\mathfrak{C}}(f^{-1}(\mu))$. This shows that $f^{-1}(\mu)$ is fuzzy $\mathfrak{C}$ - (τ_i,τ_j) - semi open. Therefore f is pairwise fuzzy $\mathfrak{C}$ - irresolute.

Theorem 5.12:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \delta_1, \delta_2)$. Then $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \delta_1, \delta_2)$ is

(i) pairwise fuzzy $\mathfrak{C}$ - irresolute if f and g are pairwise fuzzy $\mathfrak{C}$ - irresolute.

(ii) pairwise fuzzy $\mathfrak{C}$ - semi continuous if f is pairwise fuzzy $\mathfrak{C}$ - irresolute and g is pairwise fuzzy $\mathfrak{C}$ - semi continuous.

Proof :

(a) Let λ be a fuzzy $\mathfrak{C}$ - (δ_i, δ_j) - semi open in Z. Since g is pairwise fuzzy $\mathfrak{C}$ - irresolute, $g^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - $(\sigma_{i,j})$ - semi open in Y. Since f is pairwise fuzzy $\mathfrak{C}$ - irresolute, $f^{-1}(g^{-1}(\lambda))$ is fuzzy $\mathfrak{C}$ -(τ_i,τ_j)- semi open in X. That is $(g \circ f)^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open in X. Therefore by Definition5.1, g o f is pairwise fuzzy $\mathfrak{C}$ - irresolute.

(b) Let λ be a fuzzy δ_i - open in Z. Since g is pairwise fuzzy $\mathfrak{C}$ - semi continuous, $g^{-1}(\lambda)$ is fuzzy $\mathfrak{C}$ - (σ_i, σ_j) - semi open. Since f is pairwise fuzzy $\mathfrak{C}$ - irresolute, $f^{-1}(g^{-1}(\lambda))$ is pairwise fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. That is $(g \circ f)^{-1}(\lambda)$ is pairwise fuzzy $\mathfrak{C}$ - (τ_i, τ_j) - semi open. Therefore g o f is pairwise fuzzy $\mathfrak{C}$ - semi continuous.

References

1. C. Alegre, J. Ferrer and V. Gregori, On pairwise Baire bitopological spaces, *Publ. Maths Debrecen*, *55*, 3-15 (1999).
2. K. Bageerathi, G. Sutha and P. Thangavelu, A generalization of fuzzy closed sets. *International Journal of fuzzy systems and rough systems, 4(1)*, 1-5 (2011).
3. C. L. Chang, Fuzzy topological spaces, *J. Math. Anal. Apspl., 24,* 182-190 (1968).
4. George J. Klir and Bo Yuan, *Fuzzy Sets and Fuzzy Logi*$\mathfrak{C}$ Theory and Applications, Prentice – Hall, Inc., (2005).
5. A. Kandil, *Biproximaties and fuzzy*

bitopological spaces, Simon steven 63 (1989).

6. S.S. Thakur, R. Malviya *Pairwise Fuzzy Irresolute mappings Mathematica Bohemica*, Vol. *121*, No. 3, 273-280 (1996).
7. S.S. Thakur, R. MalviyaSemi open sets and semi continuity in fuzzy bitopological spaces, *Fuzzy sets and systems 79*, 251-256 (1996).
8. T. Thangam, K. Bageerathi Generalization of Fuzzy Bi-closed sets and Pair wise Baire Spaces Proceedings of the national Seminar On Advances in Fuzzy Algebra, Fuzzy Topology and Fuzzy Graphs (AFATG2015) A.P.C. Mahalaxmi College, Thoothukudi, Jan 22-23, (2015). **(To appear).**
9. T. Thangam, K. Bageerathi Generalization of Fuzzy semi boundary in Fuzzy Bitopological Spaces. **(To appear).**
10. L. A. Zadeh, Fuzzy sets, *Information and control*, *8*, 338 - 353 (1965).