Ultra Scientist Vol. 27(3)A, 223-228 (2015).

New Class of n-ary Semilattices And Their Properties

M. REEHANA PARVEEN[1] and P. SEKAR[2]

[1]Dept of Mathematics, Justice Baheer Ahmed Sayeed College, Chennai (India)
[2]Principal, Kandasamy Naidu College, Chennai (India)

(Acceptance Date 23rd November, 2015)

Abstract

In this paper for the first time we define the new notion of n-ary semilattices under the intersection operation. Several new notions likes pseudo buds and uniform t-length n-ary semilattices are introduced and their properties analysed.

Key words: n-ary semilattice, uniform t-length n-ary semilattice, pseudo buds.

1. Introduction

In this paper authors for the first time introduce the notion of n-ary semi lattices and some interesting concepts like pseudo buds and uniform t-length n-ary semilattices associated with them. They are defined and described. This paper has four sections. Section one is introductory in nature. For definition of semilattices refer[1,2]. For concept of trees please refer[3]. Section two defines the new notion of n-ary semilattices under the binary operation (intersection). Here the notion of pseudo buds and uniform t-length semilattices are defined, described and developed and their properties analysed In section three the applications of this new concept is briefly given. Final section gives the conclusions based on this research work.

2. New class of n-ary semilattices under the binary operation $\cap$:

In this section the new notion of n-ary semilattices using intersection, '$\cap$' operation is defined and their properties analysed. The n-ary semilattices under $\cap$ have the least element which is unique and has several maximal elements. The notion of pseudo buds are defined for n-ary semilattices under $\cap$ operation.

Definition 2.1: Let $\{S, \cap\}$ be a semilattice under the intersection operation $\{S, \cap\}$ is defined as a n-ary semilattice (n a positive integer $1 \leq n < \infty$) if the following conditions are satisfied.

i) Every vertex of the semilattice has exactly n edges or lines emerging out of it in the upward direction and the operation on S is $\cap$.

ii) There are many maximal elements but *one and only one least element.*

iii) When n=1 the n-ary semilattice is defined as the 1-ary semilattice under $\cap$.

When n = 2 the n-ary semilattice is defined as the binary semilattice and so on.

Here all n-ary semilattices $\{S, \cap\}$ are taken only of finite order. That is number of vertices and edges based on $\cap$ are only finite in number.

This situation is illustrated by the following examples.

Example 2.1:

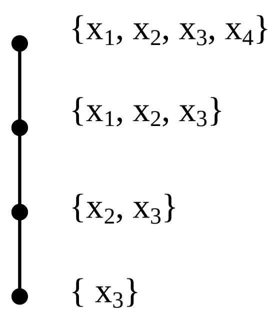

Figure 2.1

The above Figure 2.1 gives a 1-ary semilattice where S = $\{\{x_3\}, \{x_2, x_3\}\ \{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}, \cap\}$ under the intersection operation and the partial order is the containment relation.

$\{x_3\} \subseteq \{x_2, x_3\} \subseteq \{x_1, x_2, x_3\} \subseteq \{x_1, x_2, x_3, x_4\}$; $\{x_3\} \cap \{x_2, x_3\} = \{x_3\}$ and so on. The least element is $\{x_3\}$.

The maximal element is unique and is $\{x_1, x_2, x_3, x_4\}$.

In view of this the following result is proved.

Proposition 2.1: The one-ary semilattice $\{S, \cap\}$ has a unique maximal and a unique minimal element.

Proof: Follows from the very definition that the semilattices are one-ary so only one edge emerges from the vertex in the upward direction.

Example 2.2: Let $\{S, \cap\}$ be the 3-ary semilattice given by Figure 2.2.

Clearly c_1, c_2 and c_3 are the maximal elements of the 3-ary semilattice; $\{S, \cap\}$. 0 is the least element.

It is interesting to note that this has several chains of varying lengths. There are 3 chains of length three. 3 chains of length two and 2 chains of length 1 with 0 as least element.

Example 2.3: Let $\{S, \cap\}$ be the 4-ary semilattice which is given by Figure 2.3.

Clearly the 4-ary semilattice has 0 to be the least element and $\{c_1, c_2, c_3, \ldots, c_8\}$ are the 8 maximal elements of $\{S, \cap\}$. There are 8 number of chain lattices of order three.

There are 6 number of chain lattices of order two all of which have zero as the least element and has one maximal element. Now the definition of length of the n-ary semilattice $(S, \cap)$ is given in the following.

Definition 2.2: Let $\{S, \cap\}$ be the n-ary semilattice of finite order. The length *of S denoted by $l(S) = max \{|C| - 1\}$ where C denotes the maximal chain of S.*

Examples of length of n-ary semilattices $\{S, \cap\}$ is described by the following examples.

Example 2.4: Let $\{S, \cap\}$ be the 5-ary semilattice given by the Figure 2.4.

The length of the 5-ary semilattice $(S, \cap)$ is four.

Example 2.5: Consider the 2-ary semilattice $(S, \cap)$ given by Figure 2.5.

It is clear $(S, \cap)$ has 0 to be the least element and $c_1, c_2, c_3, c_4, .., c_{15}, c_{16}$ are 16 distinct maximal elements of $(S, \cap)$. Clearly the binary semilattice has the length of the semilattice to be four.

All chains of maximal length are only four and are equal.

These n-ary semilattices which has all chains connecting the least element 0 and the maximal element to be the chains of same length and they are defined as the uniform t-length n-ary semilattices.

The above binary semilattice is a uniform 4-length binary semilattice.

Consider the following example.

Example 2.6: Figure 2.6 gives a trinary-semilattice. $(S, \cap)$ is a uniform 2-length semilattice.

In view of this property the following definition is mandatory.

Definition 2.3: Let $(S, \cap)$ be a uniform t-length n-ary semilattice. The maximal elements of $(S, \cap)$ are defined as the pseudo buds of $(S, \cap)$.

Theorem 2.1: The pseudo buds of $(S, \cap)$ can generate $(S, \cap)$ in a very special way.

Proof: Let $(S, \cap)$ be the n-ary semilattice which is uniform t-length.

Suppose $c_1, c_2, \ldots, c_m$ be the pseudo buds of $(S, \cap)$.

Let d_1 be the intersection of $c_1, c_2, \ldots, c_n$, d_2 be the intersection of $c_{n+1}, \ldots, c_{2n}$ and so on d_r be the intersection of $c_{ln, ln+1, \ldots, m}$ where $m = c_{(l+1)n}$.

Now let intersection of $d_1, d_2, \ldots, d_n$ be b_1, $d_{n+1}, d_{n+2}, \ldots, d_{2n}$ be b_2 and so on. Next let intersection of $b_1, \ldots, b_n$ be a_1 and so on.

When the last but one set of vertices say are $x_1, x_2, \ldots, x_n$, then their intersection is 0, the least element of $(S, \cap)$. Hence the theorem.

This is represented by the following example.

Example 2.7: Let $(S, \cap)$ be the 2-ary semilattice given by Figure 2.7 which is maximum 4-length.

$$c_1 \cap c_2 = d_1, \quad c_3 \cap c_4 = d_2,$$
$$c_6 \cap c_5 = d_3,$$
$$c_7 \cap c_8 = d_4, \ \ldots, c_{15} \cap c_{16} = d_8$$

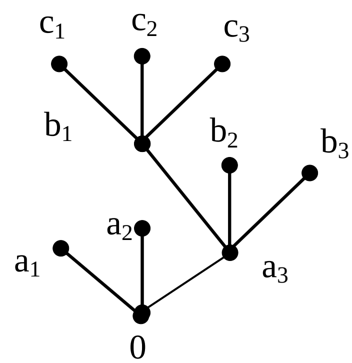

Figure 2.2

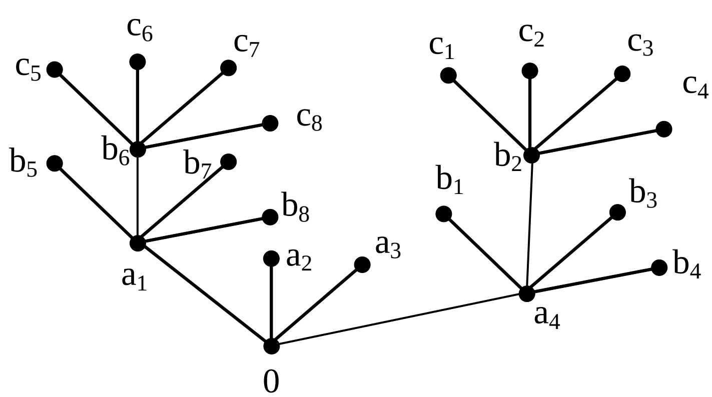

Figure 2.3

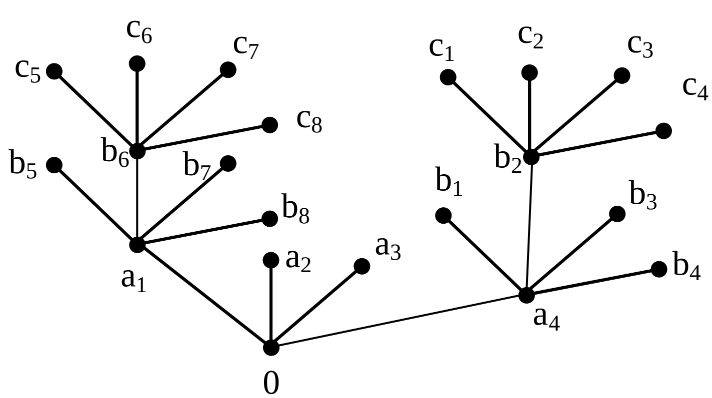

Figure 2.4

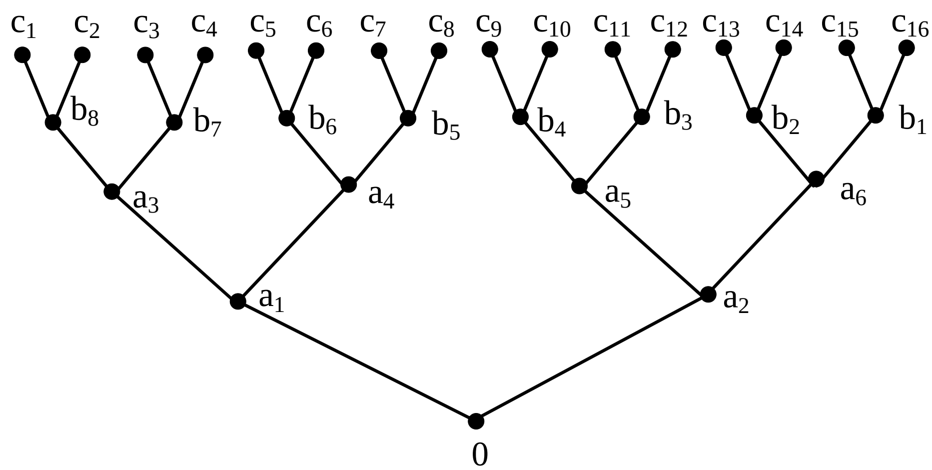

Figure 2.5

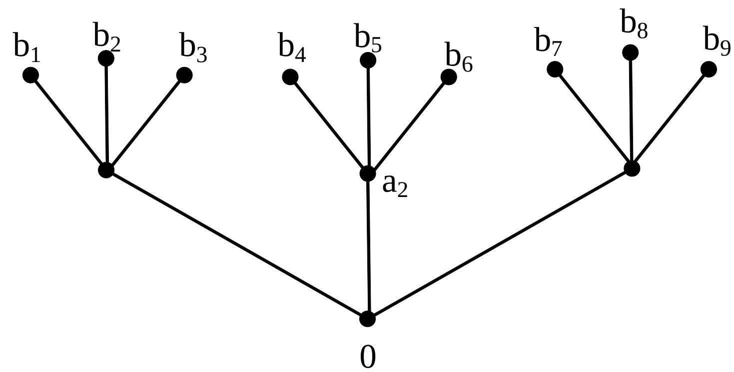

Figure 2.6

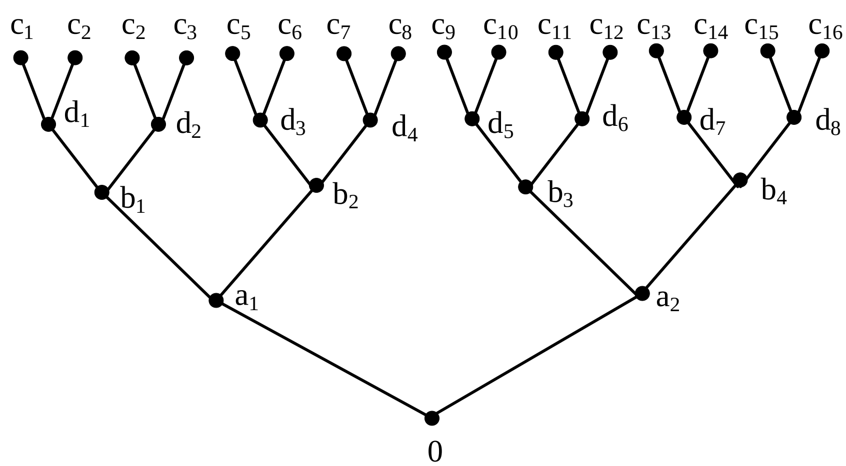

Figure 2.7

$d_1 \cap d_2 = b_1$, $d_3 \cap d_4 = b_2$,
$d_5 \cap d_6 = b_3$, $d_7 \cap d_8 = b_4$.
$b_1 \cap b_2\ a_1$, $b_3 \cap b_4 = a_2$
and $a_1 \cap a_2 = 0$.

This is the way this binary semilattice is four length.

3. Applications of n-ary semilattices (S, ∩) :

In this section a brief possible applications of the n-ary semilattices (S, ∩) is given.

Clearly by the very definition of the new concept of (S, ∩) the n-ary semilattices given in the figures one sees all these n-ary semilattices are n-ary trees.

One knows very well the applications of trees to various fields in IT, computer science and other fields. Specially the n-ary semilattices have application to sorting and determination of data mining.

Thus these n-ary semilattices (S, ∩) will certainly find applications as there are only trees and wherever trees find their applications these n-ary semilattices can be applied.

4. Conclusions

In this paper for the first time the new concept of n-ary semilattice (S, ∩) is defined, developed and described. Several interesting concepts and new notions of pseudo buds and uniform t-length semilattices are introduced and their properties analysed.

They are powerful structures as they are trees. Further these can be naturally constructed using subsets of a power set of a large set. Hence the existence of these new structures is guaranteed.

References

1. Birkhoff G, Lattice Theory, 2nd Edition, Amer, Math. Soc, (1948).
2. Gratzer G., General Lattice Theory, Birkhauser VerlageBasel and Stullgart, (1978).
3. Cormen, T.H., *et al.*, Introduction to Algorithms, McGraw Hills, (2001).