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A New Type of Closed Sets in Bitopological Spaces

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Abstract

In this paper, we introduce $\overline{g}$-closed sets[19] in bitopological spaces. Properties of these sets are investigated and we introduce the two new bitopoligical spaces (i, j)-$\overline{T}$ and (i, j)-$\overline{T}^*$ spaces as applications. Further we introduce and study $\overline{g}$-continuity[19] in bitopological spaces.

Key words : (i, j)-$\overline{g}$-closed sets; (i, j)-$\overline{T}$ spaces; (i, j)-$\overline{T}^*$ spaces; $\overline{D}$ (i, j)-continuity.

1. Introduction

A triple (X, τ_1, τ_2) where X is non empty set and τ_1, τ_2 are two topologies on X is called a bitopological space. Kelly[3] in 1985 initiated and study of such space. Fukutaka[6] in 1985 introduced the concept of g-closed sets[2] in bitopological spaces and after that many authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Recently Shailendra and Manoj[19] introduced and studied the concepts of $\overline{g}$-closed sets and $\overline{g}$-continuity in topological spaces. $\overline{g}$-closed sets lies between closed sets and g-closed sets.

In this paper we introduce the concepts of $\overline{g}$-closed sets[19], $\overline{T}$-spaces[19], $\overline{T}^*$-spaces[19] and $\overline{g}$-continuity[19] for bitopological spaces and investigate some of their properties.

2. *Preliminaries:*

If A is a subset of X with a topologt τ, then the closure of A is denoted by τ-cl(A) or cl(A), the interior of A is denoted by τ-int(A) or int(A) and the complement of A is denoted by A^C.

We recall the following definitions:

Definition 2.01 : (i) A subset A of a topological space (X, τ) is called semi-open[1]. (resp. regular open[4], pre-open[2]) if $A \subseteq cl(int(A))$ (resp. $A = int(Cl(A))$, $A \subseteq int(cl(A))$).

(ii) A subset A of a topological space (X, τ) is called g-closed set[2]. (resp. $\hat{g}$-closed set[14], $\hat{\hat{g}}$-closed set[17], $\bar{g}$-closed set[19]) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open (resp. semi-open, sg-open, $\hat{\hat{g}}$-open) in (X, τ).

Definition 2.02 : The intersection of all pre-closed sets containing A is called the pre-closure of A and it is denoted by pcl(A).

Throughout this paper, X and Y represent non-empty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed unless otherwise explicitly mentioned and integers i, j, k $\in \{1, 2\}$. For a subset A of X, τ_i -cl(A) (resp. τ_i -int(A) denote the closure (resp. interior) of A w.r.t. topology τ_i. We denote the family of all $\hat{\hat{g}}$-open subsets of X w.r.t. topology τ_i by $\hat{\hat{G}}O(X, \tau_i)$ and the family of all τ_j-closed sets is denoted by F_j. By (i, j) we mean the pair of topologies (τ_i, τ_j).

Definition 2.03 : A subset A of a bitopolocal space (X, τ_1, τ_2) is called :

(i) (i, j)-g-closed[6] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U $\in \tau_i$.

(ii) (i, j)-rg-closed[8] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in τ_i.

(iii) (i, j)-gpr-closed[12] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in τ_i.

(iv) (i, j)-Wg-closed[10] if τ_j-cl(τ_i-int(A)) $\subseteq$ U whenever A $\subseteq$ U and U $\in \tau_i$.

(v) (i, j)-ω-closed[12] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in τ_i.

(vi) (i, j)-g*-closed[15] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is generalized-open in τ_i.

(vii) (i, j)-*g-closed[18] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$-open in τ_i.

The family of all (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-Wg-closed, (i, j)-ω-closed, (i, j)-g*-closed and (i, j)-*g-closed) subsets of a bitopological space (X, τ_1, τ_2) is denoted by D(i, j) (resp. Dr(i, j), ζ(i, j), W(i, j), C(i, j), D*(i, j) and *D(i, j)).

Definition 2.04 : (i) A bitopological space (X, τ_1, τ_2) is said to be (i, j)-$T_{1/2}$[6] (resp. (i, j)-$T^*_{1/2}$[15], (i, j)-$^*T_{1/2}$[15], (i, j)*-g$T^*_{1/2}$[18] and (i, j)-*g*$T_{1/2}$[18]) if every (i, j)-g-closed (resp. (i, j)-g*-closed, (i, j)-g-closed, (i, j)-*g-closed, (i, j)-g-closed) sets is τ_j--closed (resp. τ_j-closed, (i, j)-g*-closed, τ_j-closed, (i, j)-*g-closed).

(ii) A bitopological space (X, τ_1, τ_2) is said to be strongly pairewise-$T_{1/2}$[6] (resp. strongly pairwise $T^*_{1/2}$[15], strongly pairwise *g$T^*_{1/2}$[18]) space if it is (1, 2)-$T_{1/2}$ and (2, 1)-$T_{1/2}$ (resp. (1, 2)-$T^*_{1/2}$ and (2, 1)-$T^*_{1/2}$, (1, 2)*-g$T^*_{1/2}$ and (2, 1)*-g$T^*_{1/2}$) space.

Definition 2.05 : A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called :

(i) τ_j-σ_k-continuous[7] if $f^{-1}(U) \in \tau_j$ for every U $\in \sigma_k$.

(ii) D (i, j)-σ_k-continuous[7] (resp.Dr(i, j)-σ_k-continuous[8], ζ(i, j)-σ_k-continuous[12], W(i, j)-σ_k-continuous[10], C(i, j)-σ_k-continuous[12], D*(i, j)-σ_k-continuous[15] and *D(i, j)-σ_k-continuous[18]) if the inverse image of every σ_k-closed set is (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-Wg-closed, (i, j)-ω-closed (i, j)-g*-closed and (i, j)-*g-closed) set in (X, τ_1, τ_2).

Definiiton 2.06 : A topological space (X, τ) is called $\overline{T}$-space[19] if every $\overline{g}$-closed set is closed.

3. (i, j)-$\overline{g}$-Closed sets:

In this section we introduce the concepts of $\overline{g}$-closed sets in bitopological spaces.

Definition 3.01 : A subset A of a bitopological space (X, τ_1, τ_2) is said to be an (i,j)-$\overline{g}$-closed set if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U $\in \hat{\hat{G}}O(X, \tau_i)$.

Family of all (i, j)-$\overline{g}$-closed sets in (X, τ_1, τ_2) is denoted by $\overline{D}$(i, j).

Remark 3.02 : By setting $\tau_1 = \tau_2$ in definition (3.01), (i, j)-$\overline{g}$-closed set is $\overline{g}$-closed set.

Proposition 3.03 : If A is τ_j-closed subset of (X, τ_1, τ_2), then A is (i, j)-$\overline{g}$-closed.

The converse of the above proposition is not necessarily true as seen from following example.

Example 3.04 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b} , {a, b}, X} and τ_2 = {ϕ, {c}, {a, c}, X} then the subset {a, c} is (1, 2)-$\overline{g}$-closed but not τ_2-closed.

Proposition 3.05 : In a bitopological space every (i, j)-$\overline{g}$-closed set is (i, j)-g-closed but not conversely.

Proposition 3.06 : In a bitopological space every (i, j)-$\overline{g}$-closed set is (i) (i, j)-rg-closed (ii) (i, j)-gpr-closed (iii) (i, j)-Wg-closed.

The following examples show that the reverse implications of the above proposition are not true.

Example 3.07 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, {c}, {a, c}, X} and τ_2 = {ϕ, {a}, X} then the subset {a, c} is (1, 2)-rg-closed but not (1, 2)-$\overline{g}$-closed.

Example 3.08 : Let X = {a, b, c}, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X} and τ_2 = {ϕ, {a, b}, X} then the subset {a} is (1, 2)-gpr-closed but not (1, 2)-$\overline{g}$-closed.

Example 3.09 : In example (3.08) {b, c} is (1, 2)-Wg-closed but not (1, 2)-$\overline{g}$-closed.

Remark 3.10 : The following example shows that (i, j)-$\overline{g}$-closed set is not (i, j)-ω-closed set.

Example 3.11 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X} and τ_2 = {ϕ, {a}, {b, c}, X} then the subset {a, c} is (1, 2)-$\overline{g}$-closed but not (1, 2)-ω-closed.

Remark 3.12 : It can be shown that

(i, j)-ω-closed set is not (i, j)-$\overline{g}$-closed set.

Proposition 3.13 : In a bitopological space every (i, j)-g*-closed set is (i, j)-$\overline{g}$-closed set.

The following example shows that the reverse implication of the above proposition is not true.

Example 3.14 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {b}, {a, c}, X} then the subset {a, b} is (2, 1)-$\overline{g}$-closed but not (2, 1)-g*-closed.

Proposition 3.15 : In a bitopological space every (i, j)-*g-closed set is (i, j)-$\overline{g}$-closed but not conversely.

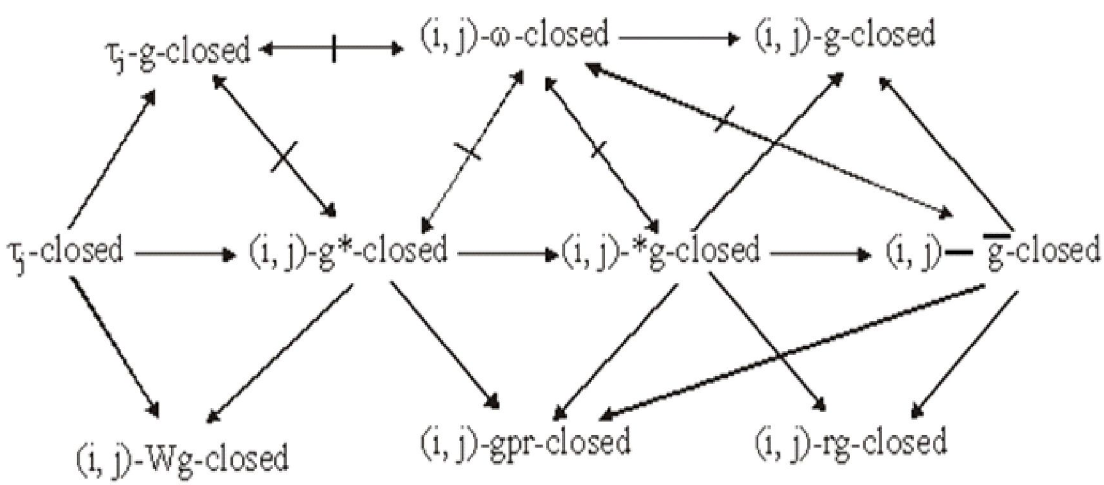
Example 3.16 : In example (3.14), the subset {a, b} is (2, 1)-$\overline{g}$-closed but not (2, 1)*g-closed.

Remark 3.17 : The following examples show that (i, j)-$\overline{g}$-closed sets and τ_j-g-closed sets are independent.

Example 3.18 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {a}, {c}, {a, c}, X} then the subset {b, c} is (1, 2)-$\overline{g}$-closed but not τ_2-g-closed.

Example 3.19 : In example (3.07), the subset {a, c} is τ_2-g-closed but not (1, 2)-$\overline{g}$-closed.

The following diagram summarizes the above discussions.



Where A → B (resp. A ↮ B) represents A implies B but not conversely (resp. A and B are independent).

Diagram (3.20)

Proposition 3.21 : If A, B $\in \overline{D}(i, j)$, then $A \cup B \in \overline{D}(i, j)$.

Remark 3.22 : The following example shows that the intersection of two (i, j)-$\overline{g}$-closed sets need not be (i, j)-$\overline{g}$-closed set.

Example 3.23 : In example (3.14), {a, b} and {a, c} are (1, 2)-$\overline{g}$-closed sets but their intersection {a} is not a (1, 2)-$\overline{g}$-closed set.

Remark 3.24 : (1, 2) is necessarily not equal to (2, 1), For in example (3.14), $(1, 2) \neq (2, 1)$.

Proposition 3.25 : If $\tau_1 \subseteq \tau_2$ in (X, τ_1, τ_2) then $\overline{D}(2, 1) \subseteq \overline{D}(1, 2)$.

The following example shows that the reverse implication of the above proposition is not true.

Example 3.26 : Let X = {a, b, c}, $\tau_1 = \{\phi, \{b\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$ then $\overline{D}(2, 1) \subseteq \overline{D}(1, 2)$ but τ_1 is not contain in τ_2.

Proposition 3.27 : For each element x of (X, τ_1, τ_2), {x} is τ_i-$\hat{\hat{g}}$-closed or $\{x\}^C$ is (i, j)-$\overline{g}$-closed.

Propositon 3.28 : If A is (i, j)-$\overline{g}$-closed then τ_j-cl(A) - A contains no non-empty τ_i-$\hat{\hat{g}}$-closed set.

Proof : Let A be an (i, j)-$\overline{g}$-closed set and B be τ_i-$\hat{\hat{g}}$-closed set such that $B \subseteq \tau_j\text{-cl}(A) = A$. Since $A \in \overline{D}(i, j)$, we have $\tau_j\text{-cl}(A) \subseteq B^C$. Thus $B \subseteq \tau_j\text{-cl}(A) \cap (\tau_j\text{-cl}(A))^C = \phi$.

The converse of the above proposition is not true as it can be seen from the following example.

Example 3.29 : Let X = {a, b, c}, $\tau_1 = \{\phi, \{a\}, \{b, c\}, X\}$ and $\tau_2 = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$. If A = {b} then τ_1-cl(A) = A = {c} does not contain any non-empty τ_2-$\hat{\hat{g}}$-closed set. But A is not a (2, 1)-$\overline{g}$-closed.

Proposition 3.30 : If A is (i, j)-$\overline{g}$-closed set in (X, τ_1, τ_2) then A is τ_j-closed iff τ_j-cl(A) = A is τ_i-$\hat{\hat{g}}$-closed.

Proposition 3.31 : If A is an (i, j)-$\overline{g}$-closed set of (X, τ_1, τ_2) such that $A \subseteq B \subseteq \tau_j$-cl(A) then B is also an (i, j)-$\overline{g}$-closed set of (X, τ_1, τ_2).

Proposition 3.32 : Let $A \subseteq Y \subseteq X$ and A is (i, j)-$\overline{g}$-closed in X. Then A is (i, j)-$\overline{g}$-closed related to Y.

Theorem 3.33 : In a bitopological space (X, τ_1, τ_2), $\hat{\hat{G}}O(X, \tau_i) \subseteq F_j$ (Family of all closed sets in τ_j) iff every subset of X is an (i, j)-$\overline{g}$-closed set.

Proof : Suppose that $\hat{\hat{G}}O(X, \tau_i) \subseteq$

F_j. Let A be a subset of X such that $A \subseteq U$ where $U \in (X, \tau_i)$ then τ_j-cl(A) $\subseteq \tau_j$-cl(U) =U and hence A is (i, j)-$\overline{g}$-closed.

Conversely let every subset of X is (i, j)-$\overline{g}$-closed. Let $U \in \hat{\hat{G}}O(X, \tau_i)$. Since U is (i, j)-$\overline{g}$-closed, we have τ_j-cl(U) $\subseteq$ U. So $U \in F_j$ and hence $\hat{\hat{G}}O(X, \tau_i) \subseteq F_j$.

4. (i, j)- spaces and (i, j)-*spaces

In this section we introduce the following definitions:

Definition 4.01 : A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-$\overline{T}$ space if every (i, j)-$\overline{g}$-closed set is τ_j-closed.

Definition 4.02 : A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-$\overline{T}$* space if every (i, j)-g-closed set is (i, j)-$\overline{g}$-closed set.

Proposition 4.03 : If (X, τ_1, τ_2) is (i, j)-$T_{1/2}$ space then it is (i, j)-$\overline{T}$ space but converse is not necessarily true.

Proposition 4.04 : If (X, τ_1, τ_2) is (i, j)-$\overline{T}$ space then it is (i, j)-*g$T^*_{1/2}$ space but not conversely.

Example 4. 05 : Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ , {b}, {a, c}, X}. Then (X, τ_1, τ_2) is (2, 1)-*g$T^*_{1/2}$ space but not (2, 1)-$\overline{T}$ space.

Theorem 4.06 : A bitopological space (X, τ_1, τ_2) is an (i, j)-$\overline{T}$ space iff {x} is τ_j-open or τ_i-$\hat{\hat{g}}$-closed for each $x \in X$.

Proof : Suppose that {x} is not τ_i-$\hat{\hat{g}}$-closed then $\{x\}^C$ is (i, j)-$\overline{g}$-closed by proposition (3.27). Since (X, τ_1, τ_2) is an (i, j)-$\overline{T}$ space, $\{x\}^C$ is τ_j-closed *i.e.* $\{x\}^C$ is τ_j-open.

Conversely let F be an (i, j)-$\overline{g}$-closed set, By assumption {x} is τ_j-open or τ_i-$\hat{\hat{g}}$-closed for any $x \in \tau_j$-cl(F). Now consider two cases.

Case-I : Suppose {x} is τ_j-open. Since $\{x\} \cap F \neq \phi$ we have $x \in F$.

Case-II : Suppose {x} is τ_j-$\hat{\hat{g}}$-closed. If $x \notin F$ then $\{x\} \subseteq \tau_j$-cl(F)–F, which is a contradiction to proposition (3.30) so $x \in F$.

Thus in both cases we conclude that F is τ_j-closed. Hence (X, τ_1, τ_2) is (i, j)-$\overline{T}$ space.

Remark 4.07 : (X, τ_1, τ_2) is not generally (1, 2)-$\overline{T}$ space even if both (X, τ_1) and (X, τ_2) are $\overline{T}$-spaces. It can be shown by the following example.

Example 4. 08 : Let X = {a, b, c}, τ_1= {ϕ, {a}, {c}, {a, c}, X} and τ_2 = {ϕ, {a}, {b}, {a, b}, X}. Then (X, τ_1) and (X, τ_2) are $\overline{T}$-space but (X, τ_1, τ_2) is not (1, 2)-$\overline{T}$ space.

Definition 4.09 : A bitopological

space (X, τ_1, τ_2) is said to be strongly pairwise $\overline{T}$-space if it is both $(1, 2)$-$\overline{T}$ space and $(2, 1)$-$\overline{T}$ space.

Proposition 4.10 : If (X, τ_1, τ_2) is strongly pairwise $T_{1/2}$-space then it is strongly pairwise $\overline{T}$-space.

Proposition 4.11 : If (X, τ_1, τ_2) is strongly pairwise $\overline{T}$-space then it is strongly pairwise $*gT*_{1/2}$-space.

Proposition 4.12 : Every (i, j)-$\overline{T}$ space is (i, j)-$T*_{1/2}$ space but not conversely.

Example 4.13 : In example (4.05), (X, τ_1, τ_2) is (1, 2)-$T*_{1/2}$ space but not (1, 2)-$\overline{T}$ space.

Proposition 4.14 : Every (i, j)-$T_{1/2}$ space is (i, j)-$\overline{T}$ * space but not conversely.

Example 4.15 : Let $X=\{a, b, c\}$, $\tau_1= \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi , \{a\}, \{c\}, \{a, c\}, X\}$. Then (X, τ_1, τ_2) is (2, 1)-$\overline{T}$ *space but not (2, 1)-$T_{1/2}$ space.

Proposition 4.16 : Every (i, j)-$*T_{1/2}$ space is (i, j)-$\overline{T}$ * space but not conversely.

Example 4.17 : In example (3.11), (X, τ_1, τ_2) is (2, 1)-$\overline{T}$ *space but not (2, 1)-$*T_{1/2}$ space.

Proposition 4.18 : Every (i, j)-$*g*T_{1/2}$ space is (i, j)-$\overline{T}$ * space but not conversely.

Example 4.19 : In example (3.11), (X, τ_1, τ_2) is (2, 1)-$\overline{T}$ *space but not (2, 1)-$*g*T_{1/2}$ space.

Remark 4.20 : (i, j)-$\overline{T}$ * space is not necessarily (i, j)-$\overline{T}$ space as it can be seen from the following example.

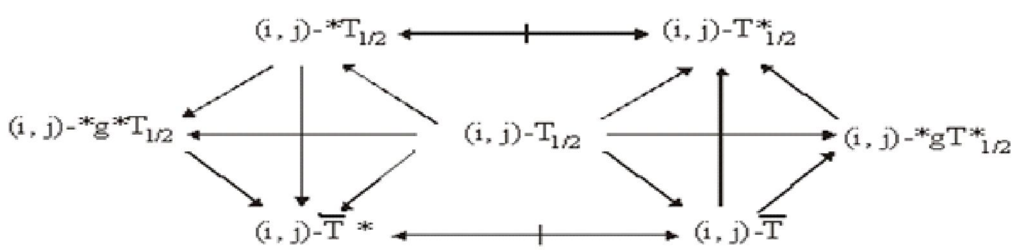
Example 4.21 : In example (3.08), (X, τ_1, τ_2) is (1, 2)-$\overline{T}$ *space but not (1, 2)-$\overline{T}$ space.

Remark 4.22 : In a bitopological space (X, τ_1, τ_2), (i, j)-$\overline{T}$ space is not necessarily (i, j)-$\overline{T}$ *space.

Theorem 4.23 : A bitopological space (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space iff it is both (i, j)-$\overline{T}$ space and (i, j)-$\overline{T}$ * space.

Propositioin 4.24 : If (X, τ_1, τ_2) strongly pairwise $\overline{T}$-space then it is strongly pairwise $T*_{1/2}$-space.

The following diagram summarizes the above discussions.



5. $\bar{g}$ -Continuous maps and $\bar{g}$ -irresolute maps :

In this section we introduce the following definitions.

Definition 5.01 : A map. f : (X, τ_1, τ_2)$\rightarrow$(Y, σ_1, σ_2) is called (i, j)- $\bar{g}$ -σ_k-continuous if the inverse image of every σ_k-closed set is an (i, j)- $\bar{g}$ -closed set. It is denoted by $\bar{D}$ (i, j)-σ_k-continuous.

Proposition 5.02 : If f : (X, τ_1, τ_2)$\rightarrow$ (Y, σ_1, σ_2) is τ_j-σ_k-continuous then it is $\bar{D}$ (i, j)-σ_k-continuous but not conversely.

Example 5.03 : Let X={a, b, c}, τ_1= {ϕ, {a}, X}, τ_2 = (ϕ, {b}, {a, c}, X } and Y = {p, q}, σ_1 = {ϕ, {p}, Y}, σ_2 = {ϕ, {q}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by f(c) = p, f(a) = q, f(b) = q then f is $\bar{D}$ (1, 2)-σ_2-continuous but not τ_2–σ_2-continuous.

Proposition 5.04 : Every $\bar{D}$ (i, j)-σ_k-continuous map is D(i, j)-σ_k-continuous map.

Proposition 5.05 : If f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is $\bar{D}$ (i, j)-σ_k-continuous then it is (i) Dr(i, j)-σ_k-continuous (ii) ζ(i, j)-σ_k-continuous (iii) W(i, j)-σ_k-continous.

However the reverse implications of the above proposition are not true in general as seen from the following examples.

Example 5.06 : Let X = Y ={a, b, c}, τ_1 = {ϕ, {a}, {a, c}, X}, τ_1 = (ϕ, {a, b}, X} and σ_1 = {ϕ, {b}, {a, b}, Y}, σ_2 = {ϕ, {a}, {a, b}, {a, c}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by f(a) = b, f(b) = c, f(c) = a then f is Dr(2, 1)-σ_2-continuous but not $\bar{D}$ (2, 1)-σ_2-continuous.

Example 5.07 : Let X = Y ={a, b, c}, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X}, τ_2 = (ϕ, {a, b}, X } and σ_1 = {ϕ, {a}, {b}, {a, b}, Y}, σ_2 = {ϕ, {b}, {c}, {b, c}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by f(a) = c, f(b) = a, f(c) = b then f is ζ(1, 2)-σ_1-continuous but not $\bar{D}$ (1, 2)-σ_1-continuous.

Example 5.08 : Let X = Y ={a, b, c}, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X}, τ_2 = (ϕ, {a, b}, X } and σ_1 = {ϕ, {a}, {c}, {a, c}, Y}, σ_2 = {ϕ,{c}, {b, c}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by identity mapping then f is W(1, 2)-σ_2-continuous but not $\bar{D}$ (1, 2)-σ_2-continuous.

Proposition 5.09 : If f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is D*(i, j)-σ_k-continuous then it is $\bar{D}$ (i, j)-σ_k-continuous.

However the reverse implication of the above proposition is not true in general as seen from the following example.

Example 5.10 : Let X = Y ={a, b, c}, τ_1 = {ϕ, {a}, {c}, {a, c}, X}, τ_2 = (ϕ, {a}, X} and σ_1 = {ϕ, {a}, {a, b}, {a, c},Y}, σ_2 = {ϕ, {a}, {a, c}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by f(a) = a, f(b) = c, f(c) = b then f is $\bar{D}$ (2, 1)-σ_1-continuous but not D*(2, 1)-σ_1-continuous.

Proposition 5.11 : If f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is *D(i, j)-σ_k-continuous then it is $\bar{D}$ (i, j)-σ_k-continuous.

However the reverse implication of the above proposition is not true in general as seen from the following example.

Example 5.12 : Let X = Y ={a, b, c}, τ_1={ϕ, {a}, {b}, {a, b}, X}, τ_2=(ϕ, {a}, {b, c}, X } and σ_1 = {ϕ, {b}, {b, c},Y}, σ_2 = {ϕ, {a}, {a, b}, {a, c}, Y}. Define f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by identity mapping then f is $\overline{D}$(2, 1)-σ_2-continuous but not *D(2, 1)-σ_2-continuous.

Remark 5.13 : $\overline{D}$(i, j)-σ_k-continuity is not necessarily C(i, j)-σ_k- continuity as seen from the following example.

Example 5.14 : Let X = Y ={a, b, c}, τ_1 = {ϕ, {a}, {b, c}, X}, τ_2 = (ϕ, {a}, {a, b}, X } and σ_1 = {ϕ, {b}, {c}, {b, c},Y}, σ_2 = {ϕ, {b}, {a, b}, {b, c}, Y}. Define f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) by f(a) = c, f(b) = b, f(c) = a then f is $\overline{D}$(2, 1)-σ_2-continuous but not C(2, 1)-σ_2-continuous.

Remark 5.15 : C(i, j)- σ_k-continuity is not necessarily $\overline{D}$(i, j)-σ_k-continuity.

The following diagram summarizes the above discussions.

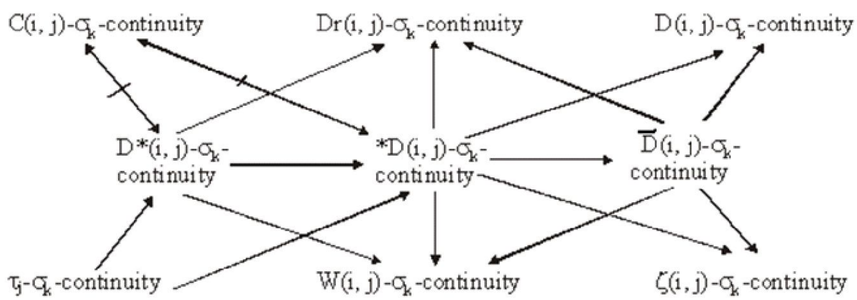


Diagram (5.16)

Theorem 5.17 : Let f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) be a map:

(i) if (X, τ_1, τ_2) is an (i, j)-$T_{1/2}$ space then f is D(i, j)-σ_k-continuous iff it is $\overline{D}$(i, j)-σ_k-continuous.

(ii) If (X, τ_1, τ_2) is an (i, j)-T_u-space then f is τ_j-σ_k-continuous iff it is $\overline{D}$(i, j)-σ_k-continuous.

Proof : (i) : Let f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) is D(i, j)-σ_k-continuous then inverse image of every σ_k-closed set in Y is (i, j)-g-closed in X. Since every (i, j)-$\overline{g}$-closed set is (i, j)-g-closed so inverse image of every σ_k-closed set in Y is (i, j)-$\overline{g}$-closed *i.e.* f is $\overline{D}$(i, j)-σ_k-continuous.

Conversely f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) is $\overline{D}$(i, j)-σ_k-continuous then by proposition (5.04 (i)), f is D(i, j)-σ_k-continuous.

(ii) Same as (i).

Definition 5.18 : A map. f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) is called $\overline{D}$(i, j)-σ_k-irresolute if the inverse image of every σ_k-$\overline{g}$-closed set in Y is (i, j)-$\overline{g}$-closed set in X.

Proposition 5.19 : If f : (X, τ_1, τ_2)

$\rightarrow$ (Y, σ_1, σ_2) is $\overline{D}$ (i, j)-σ_k-irresolute then it is $\overline{D}$ (i, j)-σ_k-continuous but not conversely.

Example 5.20 : Let X = Y = {a, b, c}, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = (\phi, \{a\}, \{b, c\}, X\}$ and $\sigma_1 = \{\phi, \{a\}, Y\}$, $\sigma_2 = \{\phi, \{b\}, \{b, c\}, Y\}$. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by identity mapping then f is $\overline{D}$ (1, 2)-σ_1-continuous but not $\overline{D}$ (1, 2)-σ_1-irresolute.

Proposition 5.21 : If f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is $\overline{D}$ (i, j)-σ_k-continuous and (Y, σ_k) is $\overline{T}$ -space then f is $\overline{D}$ (i, j)-σ_k-irresolute.

Proof : Let f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is $\overline{T}$ (i, j)-σ_k-continuous then inverse image of every σ_k-closed set in Y is (i, j)- $\overline{g}$ -closed inX. Since (Y, σ_k) is $\overline{T}$ -space so every σ_k-$\overline{g}$ -closed set is σ_k-closed *i.e.* inverse image of every σ_k- $\overline{g}$ -closed set is (i, j)- $\overline{g}$ -closed so f is (i, j)- $\overline{g}$ -σ_k-irresolute.

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