Ultra Scientist Vol. 27(3)A, 263-266 (2015).

On Quasi Umbilical Submanifold of CO-Dimension -2 of Almost Hyperbolic Manfold

B.B. PANDEY and S.B. PANDEY

Department of Mathematics, Kumaun University
Soban Singh Jeena campus, Almora (Uttarakhand) (India)

(Acceptance Date 27th December, 2015)

Abstract

Hypersurfaces immersed in an almost hyperbolic Hermitian manifold studied by Dube and Mishra[3]. Almost hyperbolic hermite manifold have been studied by Dube[2]. Hypersurfaces of almost hyperbolic Hermitian manifold I,II ,have been studies by Bhatt and Dube[1]. The purpose of the present paper is to study the Quasi – Umbilical submanifold of co-dimension -2 of an almost hyperbolic manifold. We have obtained the condition for this submanifold, to be W –Quasi umbilic.

Introduction

Let V_{2n+2} be C^∞ - manifold and there exist a vector valued linear function F of differentiability class C^∞ - satisfying.

(1.1) $F^2 X=X$.

Then V_{2n-1} is said to be an almost hyperbolic manifold and F is said to given an almost hyperbolic structure to V_{2n+2} let the almost hyperbolic manifold V_{2n+2} be endowed with Riemannian metric g satisfying[4].

(1.2) $G(FX ,FY)=G(X,Y)$

Then V_{2n+2} is called almost hyperbolic Riemannian manifold. Let D be the Riemannian in almost hyperbolic Riemannian manifold V_{2n+2} if,

(1.3) $(D_x F) Y =0$

V_{2n+2} is called almost hyperbolic manifold. V_{2n+2} be a differentiable submanifold of Co-dimension -2 of an almost hyperbolic manifold V_{2n+2},such that.

(1.4) $FBX = B f X + u \otimes P + v \otimes Q$

(1.5) $FP=BU-\lambda q$

(1.6) $Fq=bv-\lambda p$

Where P Q are two unit normal vector fields to V_{2n} f is the fields of type (1,1), U,V are vector fields u, v,1-forms and λ is a C^∞ function. Operating equation (1.4)(1.5)(1.6) by F and using equation (1.1)(1.2)(1.4)(1.5)(1.6) and using tangential and normal part, we have

(1.7)
(a) $f^2X=X+u(X)U+v(X)V$,
(b) $U(fX)=\lambda v(X)$, $v(fX)=\lambda u(X)$
(c) $fU=-\lambda V$, $fV=\lambda U$,
(d) $u(U)=-(1+\lambda^2)$, $u(V)=0$
(e) $v(V)=-(1+\lambda^2)$, $v(U)=0$.

Thus we get the almost hyperbolic contact $\{f,g,\eta,\xi\}$ structure[4]. Let g be the induced Riemannian metric in V_{2n} defined by

(1.8) (a) $g(X,Y)=G(BX,BY)$
B being differential map.
(1.8) (b) $G(BX,P)=0=G(BX,Q)$
(1.8) (c) $G(P,P)=1=G(Q,Q)$.

Then using equation (1.4)and (1.8)in equation (1.2) we get

(1.9) $g(fX,fY)=-g(X,Y)-u(X)\,u(Y)-v(X)\,v(Y)$
(1.10) $g(U,X)=u(X)$, $g(V,X)=v(x)$

Thus the submanifold V_n of an almost product and decomposable manifold. Let D be an affine connection is V_{2n} induced by the Riemannian connexion of almost hyperbolic manifold V_{2n+2} then Gauss and Weingarten's equation are given by

(1.11) $D_{BX}BY=BD_XY+h(X,Y)\,P+K(X,Y)Q$,
(1.12) $D_{BX}P=-BHX+1\,()X)\,Q$
(1.13) $D_{BX}Q=-BKX+I(X)Q$

Where h and K are second fundamental forms and 1 is the third fundamental form defined by

(1.14) $g(HX,Y)=h(X,Y)$, $g(KX,Y)=K(X,Y)$
H and K being the tensors of type (1,1) differentiating equation (1.4)(1.5) and (1.6) and using equations (1.1)(1.6) (1.11) (1.12) and (1.13) we get
(1.15) $(D_xf)Y=h(X,Y)U+k(X,Y)V-u(Y)HX-v(Y)KX$,
(1.16) $(D_Xu)Y=-h(X,fY)-\lambda K(X,Y)-v(Y)l(X)$,
(1.17) $(D_Xv)Y=-\lambda h(X,Y)-K(X,fY)-v(Y)l(X)$,
(1.18) $(D_XU)=-fHX-\lambda KX+l(X)V$,
(1.19) $(D_XV)=-fKX-\lambda HX+l(X)V$,
(1.20) $h(X,U)=u(H,X)$,
(1.21) $K(X,V)=v(K,X)$.

2. Quasi Umbilical Submanifold :

Let in submanifold V_{2n}

(2.1)(a) $h(X,Y)=\alpha g(X,Y)+\beta w(X)w(Y)$
(2.1)(b) $K(X,Y)=\alpha'\,g(X,Y)+\beta'\,w(X)w(Y)$

Be satisfied in which α,β and α', β' are scaler functions and w is 1-form then V_{2n} is called Quasi-Umbilical submanifold of codimension-2 [7].

(2.2) $w(X)=q(w,X)$

Where W is the vector field then V_{2n} is called a W-Quasi submanifold if $\alpha=0=\alpha'$, $\beta\neq 0$, $\beta'\neq 0$ and l=0, then w-Quasi Umbilical submanifold is called para cylindrical submanifold[5,6].

Theorem (2.1): If the submanifold V_{2n} of Co-dimension-2 of an almost hyperbolic manifold V_{2n+2} is w-Quasi umbilical thus we have

(2.3)(a) $(D_Xf)Y=\alpha g(X,Y)U+\beta w(X)w(Y)U+\alpha' g(X,Y)V+\beta' w(X)w(Y)Vu(Y)$
$\{\alpha(X)+\beta w(X)w\}-v(Y)\{\alpha'(X)+\beta' w(X)W\}$
(2.3)(b) $(D_Xu)Y=-\alpha g(X,fY)-\beta w(X)w(fY)-\lambda\alpha'\,g(X,Y)-\beta' w(X)w(Y)-V(Y)l(X)$.
(2.3)(c) $(D_Xv)Y=-\lambda\alpha g(X,Y)-\lambda\beta w(X)w(Y)-\alpha' g(X,fY)-\beta' w(X)w(fY)-u(Y)l(X)$.
(2.3)(d) $(D_XU)=-\alpha fX-\beta w(X)fW-\alpha'\lambda X-\beta' w(X)\lambda w+l(X)V$.
(2.3)(e) $(D_XV)=-\alpha' fX-\beta' w(X)fW-\alpha\lambda W-\beta w(X)\lambda w+l(X)U$.
(2.3)(f) $h(X,U)=\alpha u(X)+\beta w(X)u(W)$.
(2.3)(g) $K(X,V)=\alpha' \nu(X)+\beta' w(X)V(W)$.

Proof: In view of equation (2.1) (a) (b) and equation(1.15)(1.16) (1.17)(1.18) (1.19)(1.20)(1.21),we get the result of equation (2.3)(a)(b)(c)(d)(e)(f)(g), we get

Cor (2.1) if $\alpha\neq 0$, $\alpha'\neq 0$, $\beta=\beta'=0$ and l=0 then 1-form u,v are projectively killing in V_{2n} *i.e.*

(2.4)(a) $(D_X u)(Y)+(D_Y u)(X)=-2\lambda\alpha' g(X,Y)$

(2.4)(b) $(D_X v)(Y)+(D_Y v)(X)=-2\lambda\alpha\, g(X,Y)$

Proof: by using the equation (2.3)(b) and (2.3)(c) we get equation(2.4)(a)(2.4)(b),

Cor(2.2): if $\alpha=\alpha' = 0$, $\beta\neq 0$, $\beta'\neq 0$ and l=0 be satisfied the submanifold V_{2n} of codimension-2 (para cylinderical)

(2.5)(a) $(D_X f)Y=(\beta U+\beta' V)w(X)w(Y)-\beta w(X)u(Y)W-\beta' w(X)v(Y)W.$

(2.5)(b) $(D_X u)Y=-\beta w(X)w(fY)-\beta' w(X)w(Y).$

(2.5)(c) $(D_X V)Y=-\lambda\beta w(X)w(Y)-\beta' w(X)w(fY).$

Proof: by using the condition in equation (2.3)(a),(2.3)(b),(2.3)(c). we get (2.5)(a), (2.5)(b),(2.5)(c).

Theorem(2.2): In a paracylindrical Hyperbolic submanifold V_{2n} of Co-dimension-2 we have

(2.6)(a) $(D_X u)Y-(D_Y u)X=\beta[w(fX)w(y)-w(X)w(fX)]$

(2.6)(b) $(D_X v)Y-(D_Y v)X=\beta' [w(fX)w(y)-w(X)w(fX)]$

Proof: From equation (2.5)(b), we have

$(D_X u)Y=-\beta w(X)w(fX)-\beta' w(Y)$

$(D_Y u)X=-\beta w(Y)w(fX)-\beta' w(Y)w(X)$

By subtraction the above equation we get the equation (2.6)(a), similarly from equation (2.5)(c), we get equation (2.6)(b).

Theorem(2.3): The Nijenhuis tensor of w-Quasi umbilical submanifold V_{2n} of co-dimensionan-2 of an almost hyperbolic manifold with hyperbolic (f,g,u,v,λ) structure.

(2.7) $N(X,Y) = 2\alpha g(x,fY)U+2\alpha' g(fY,X)V +\beta[\{u(Y)w(X)-u(X)w(Y)\}fW+ \{u(X)w(fY)-u(Y)w(fX)\}W]+\beta'[\{v(Y)w(X)-v(X)w(Y)\}fW+\{v(X)w(fY)- v(Y)w(fX)\}W]+\{w(fX)w(Y)-w(fY)w(X)\}(\beta U+\beta' V)$

Proof: If N be the Nijenhuis tensor Corresponding to the tensor field f in V_{2n}

(2.8) $N(X,Y)=(D_{fX}f)(Y)-(D_{fY}f)(X)-f(D_X f)Y+f(D_Y f)(X)$

From equation (2.3)(a)and (2.8), we get equation (2.7).

Theorem (2.4): The Nijenhuis tensor Corresponding to the tensor field fin $V_{2n}$of co-dimension-2 of a hyperbolic almost manifold V_{2n+2} is given by

(2.9) $N(X,Y)=\beta[\{u(Y)w(X)-u(X)w(Y)\}fW +\{u(X)w(fY)-u(Y)w(fX)\}W]+\beta'[\{v(Y)w(X)-v(X)w(Y)\}fW+\{v((X)w(fY)-v(Y)w()fX)\}W] + \{w(fX)w(Y)-w(fY)w(X)\}(\beta' U+).$

Proof: By putting $\alpha=\alpha'=0$ and $\beta\neq 0$, $\beta' \neq 0$ in equation (2.7), we get equation (2.9).

References

1. Bhatt, L. and Dube, K.K., Hypersurface of hyperbolic Hermitian manifold-I,II (comm.)

2. Dube, K.K., Almost hyperbolic Hermitian manifold,Annalele Univ.din, Tim Seria stiinle *Math. Vol xi fase.* (1973,47-54).
3. Dube, K.K., Mishra, M.M., hypersurface immersed in an almost hyperbolic Hermatian manifold, camptes *Rendus,Tome* 34, q-10, 1343-1385 (1981).
4. Dube, K.K. and Upadhyay, M.D $\therefore$ Almost hyperbolic contact $\{f,g,\eta,\xi\}$ Structure Acta Mathematica, *Academaiae Scientiarum Hungarical, tomus 28* (H-1053), 13.
5. Dube, K.K., Niwas, R. $\therefore$ Almost r-contact hyperbolic structure in a product manifold *Demonstratio Mathematica* Vol *XI*, no-4, 887-897, (1978).
6. Goldberg, S.I $\therefore$ Totally Geodesic Hypersurface of Kahler manifold, *Pacific Math., 27*, 275-281 (1963).
7. Pal, R.B. and Mishra, R.S $\therefore$ Hypersurfaces of almost hyperbolic hermite manifold *Indian J. pure Appl. Math., 11(5)* 628-632 (1980).