Ultra Scientist Vol. 27(3)A, 267-273 (2015).

Generalization of Fibonacci Sequence in Case of Multiplicative Coupled Fibonacci Sequences

SANJAY HARNE[1] and MANJEET SINGH TEETH[2]

[1]Government College, Mandeleshwar, M.P. (India)
[2]M.B. Khalsa College, Indore, M.P. (India)

(Acceptance Date 27th December, 2015)

Abstract

In this paper we generalize the Fibonacci sequence in case multiplicative coupled Fibonacci sequence. We consider two infinite sequences $\{\alpha_i\}_{i=0}^{\infty}$ and $\{\beta_i\}_{i=0}^{\infty}$ which have given eight initial values and we derive new identities.

Key word : Fibonacci sequence.

Mathematics Subject Classification: 11B39

1. Introduction

The concept of multiplicative coupled Fibonacci sequences was first introduced by Atanassov[1] in 1995 and also discussed many curious properties and new direction of generalization of Fibonacci sequence. Atanassov was defined and studied about four different ways to generate coupled Fibonacci sequences and called them 2-Fibonacci sequences of second order. Let $\{\alpha_i\}_{i=0}^{\infty}$ and $\{\beta_i\}_{i=0}^{\infty}$ be two infinite sequences and let a, b, c and d be arbitrary four arbitrary real numbers with initial values $\alpha_0 = a, \beta_0 = b, \alpha_1 = c, \beta_1 = d,$ $\alpha_2 = e, \beta_2 = f$.

Then four different schemes of multiplicative coupled Fibonacci sequences of second order as follows:

First Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d$$
$$\alpha_{n+2} = \beta_{n+1}\ .\ \beta_n,\quad n \geq 0$$
$$\beta_{n+2} = \alpha_{n+1}\ .\ \alpha_n\ ,\ n \geq 0.$$

Second Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d$$
$$\alpha_{n+2} = \alpha_{n+1}\ .\ \beta_n,\quad n \geq 0$$
$$\beta_{n+2} = \beta_{n+1}\ .\ \alpha_n\ ,\ n \geq 0.$$

Third Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d$$
$$\alpha_{n+2} = \beta_{n+1}\ .\ \alpha_n,\ \ n \geq 0$$
$$\beta_{n+2} = \alpha_{n+1}\ .\ \beta_n\ ,\ n \geq 0.$$

Fourth Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d$$
$$\alpha_{n+2} = \alpha_{n+1}\ .\ \alpha_n,\ \ n \geq 0$$
$$\beta_{n+2} = \beta_{n+1}\ .\ \beta_n\ ,\ n \geq 0.$$

Many work has been done on the multiplicative coupled Fibonacci sequences. Rathore *et al.*[3] studied fundamental properties of the multiplicative coupled Fibonacci sequences. Harne[2] consider two infinite sequences $\{\alpha_i\}_{i=0}^{\infty}$ and $\{\beta_i\}_{i=0}^{\infty}$ and six arbitrary real numbers $a,\ b,\ c,\ d,\ e$ and f be given. The eight different multiplicative schemes for multiplicative coupled Fibonacci sequences are as follows:

First Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d,\alpha_2 = e,\beta_2 = f$$
$$\alpha_{n+3} = \beta_{n+2}\ .\ \beta_{n+1}.\ \beta_n\ ,\ \ n \geq 0$$
$$\beta_{n+3} = \alpha_{n+2}\ .\ \alpha_{n+1}\ .\alpha_n\ ,\ n \geq 0.$$

Second Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d,\alpha_2 = e,\beta_2 = f$$
$$\alpha_{n+3} = \alpha_{n+2}\ .\ \alpha_{n+1}.\ \alpha_n\ ,\ n \geq 0$$
$$\beta_{n+3} = \beta_{n+2}\ .\ \beta_{n+1}\ .\beta_n\ ,\ n \geq 0.$$

Third Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d,\alpha_2 = e,\beta_2 = f$$
$$\alpha_{n+3} = \beta_{n+2}\ .\ \beta_{n+1}.\ \alpha_n\ ,\ n \geq 0$$
$$\beta_{n+3} = \alpha_{n+2}\ .\ \alpha_{n+1}\ .\beta_n\ ,\ n \geq 0.$$

Fourth Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d,\alpha_2 = e,\beta_2 = f$$
$$\alpha_{n+3} = \alpha_{n+2}\ .\ \beta_{n+1}.\ \beta_n\ ,\ n \geq 0$$
$$\beta_{n+3} = \beta_{n+2}\ .\ \alpha_{n+1}\ .\alpha_n\ ,\ n \geq 0.$$

Fifth Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d,\alpha_2 = e,\beta_2 = f$$
$$\alpha_{n+3} = \beta_{n+2}\ .\ \alpha_{n+1}.\ \alpha_n\ ,\ n \geq 0$$
$$\beta_{n+3} = \alpha_{n+2}\ .\ \beta_{n+1}\ .\beta_n\ ,\ n \geq 0.$$

Sixth Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d, \alpha_2 = e, \beta_2 = f$$
$$\alpha_{n+3} = \alpha_{n+2}\ .\ \beta_{n+1}.\ \alpha_n\ ,\ n \geq 0$$
$$\beta_{n+3} = \beta_{n+2}\ .\ \alpha_{n+1}\ .\beta_n\ ,\ n \geq 0.$$

Seventh Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d, \alpha_2 = e, \beta_2 = f$$
$$\alpha_{n+3} = \beta_{n+2}\ .\ \alpha_{n+1}.\ \beta_n\ ,\ n \geq 0$$
$$\beta_{n+3} = \alpha_{n+2}\ .\ \beta_{n+1}\ .\alpha_n\ ,\ n \geq 0.$$

Eight Scheme

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d, \alpha_2 = e, \beta_2 = f$$
$$\beta_{n+3} = \beta_{n+2}\ .\ \beta_{n+1}\ .\alpha_n\ ,\ n \geq 0$$
$$\alpha_{n+3} = \alpha_{n+2}\ .\ \alpha_{n+1}\ .\beta_n\ ,\ n \geq 0.$$

2. Multiplicative Coupled Fibonacci Sequences of Forth Order :

We consider two infinite sequences $\{\alpha_i\}_{i=0}^{\infty}$ and $\{\beta_i\}_{i=0}^{\infty}$ Which have given eight initial values *p,q,r,s,t,u,v, and w* (which are real numbers). The sixteen different multiplicative schemes for multiplicative coupled Fibonacci sequences are as follows:

First Scheme

$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \beta_{n+3}\beta_{n+2}\beta_{n+1}\beta_n,\ \ n \geq 0 \quad (2.1)$$
$$\beta_{n+4} = \alpha_{n+3}\alpha_{n+2}\alpha_{n+1}\alpha_n,\ \ n \geq 0$$

Second Scheme

$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \alpha_{n+3}\alpha_{n+2}\alpha_{n+1}\alpha_n,\ \ n \geq 0 \quad (2.2)$$
$$\beta_{n+4} = \beta_{n+3}\beta_{n+2}\beta_{n+1}\beta_n,\ \ n \geq 0$$

Third Scheme

$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \alpha_{n+3}\alpha_{n+2}\alpha_{n+1}\beta_n,\ \ n \geq 0$$
$$\beta_{n+4} = \beta_{n+3}\beta_{n+2}\beta_{n+1}\alpha_n,\ \ n \geq 0$$

(2.3)

Fourth Scheme

$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \alpha_{n+3}\alpha_{n+2}\beta_{n+1}\alpha_n,\ \ n \geq 0$$
$$\beta_{n+4} = \beta_{n+3}\beta_{n+2}\alpha_{n+1}\beta_n,\ \ n \geq 0$$

(2.4)

Fifth Scheme

$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \alpha_{n+3}\beta_{n+2}\alpha_{n+1}\alpha_n,\ \ n \geq 0$$
$$\beta_{n+4} = \beta_{n+3}\alpha_{n+2}\beta_{n+1}\beta_n,\ \ n \geq 0$$

(2.5)

Sixth Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \beta_{n+3}\alpha_{n+2}\alpha_{n+1}\alpha_n,\ n \geq 0$$
$$\beta_{n+4} = \alpha_{n+3}\beta_{n+2}\beta_{n+1}\beta_n,\ n \geq 0 \tag{2.6}$$

Seventh Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \alpha_{n+3}\alpha_{n+2}\beta_{n+1}\beta_n,\ n \geq 0$$
$$\beta_{n+4} = \beta_{n+3}\beta_{n+2}\alpha_{n+1}\alpha_n,\ n \geq 0 \tag{2.7}$$

Eight Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \alpha_{n+3}\beta_{n+2}\alpha_{n+1}\beta_n,\ n \geq 0$$
$$\beta_{n+4} = \beta_{n+3}\alpha_{n+2}\beta_{n+1}\alpha_n,\ n \geq 0 \tag{2.8}$$

Nineth Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \beta_{n+3}\alpha_{n+2}\alpha_{n+1}\beta_n,\ n \geq 0$$
$$\beta_{n+4} = \alpha_{n+3}\beta_{n+2}\beta_{n+1}\alpha_n,\ n \geq 0 \tag{2.9}$$

Tenth Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\beta_{n+4} = \alpha_{n+3}\alpha_{n+2}\alpha_{n+1}\beta_n,\ n \geq 0$$
$$\alpha_{n+4} = \beta_{n+3}\beta_{n+2}\beta_{n+1}\alpha_n,\ n \geq 0 \tag{2.10}$$

Eleventh Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \alpha_{n+3}\beta_{n+2}\beta_{n+1}\beta_n,\ n \geq 0$$
$$\beta_{n+4} = \beta_{n+3}\alpha_{n+2}\alpha_{n+1}\alpha_n,\ n \geq 0 \tag{2.11}$$

Twelfth Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \beta_{n+3}\alpha_{n+2}\beta_{n+1}\beta_n,\ n \geq 0$$
$$\beta_{n+4} = \alpha_{n+3}\beta_{n+2}\alpha_{n+1}\alpha_n,\ n \geq 0 \tag{2.12}$$

Thirteenth Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\beta_{n+4} = \alpha_{n+3}\alpha_{n+2}\beta_{n+1}\beta_n,\ n \geq 0$$
$$\alpha_{n+4} = \beta_{n+3}\beta_{n+2}\alpha_{n+1}\alpha_n,\ n \geq 0 \tag{2.13}$$

Fourteenth Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \alpha_{n+3}\beta_{n+2}\beta_{n+1}\beta_n,\ n \geq 0$$
$$\beta_{n+4} = \beta_{n+3}\alpha_{n+2}\alpha_{n+1}\alpha_n,\ n \geq 0 \tag{2.14}$$

Fifteenth Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\beta_{n+4} = \beta_{n+3}\beta_{n+2}\beta_{n+1}\beta_n,\ n \geq 0$$
$$\alpha_{n+4} = \alpha_{n+3}\alpha_{n+2}\alpha_{n+1}\alpha_n,\ n \geq 0 \tag{2.15}$$

Sixteenth Scheme
$$\alpha_0 = p\ \ \alpha_1 = q\ \ \alpha_2 = r\ \alpha_3 = s$$
$$\beta_0 = t\ \ \beta_1 = u\ \ \beta_2 = v\ \ \beta_3 = w$$
$$\alpha_{n+4} = \beta_{n+3}\alpha_{n+2}\beta_{n+1}\beta_n,\ n \geq 0$$
$$\beta_{n+4} = \alpha_{n+3}\beta_{n+2}\alpha_{n+1}\alpha_n,\ n \geq 0 \tag{2.16}$$

The first few terms of the sequences defined in (2.1) are as under.

n	α_n	β_n
0	p	t
1	q	u
2	r	v
3	s	w
4	tuvw	pqrs
5	pqrsuvw	qrstuvw
6	$pq^2r^2s^2tuv^2w^2$	$pqr^2s^2tu^2v^2w^2$
7	$p^2q^3r^4s^4t^2u^3v^3w^4$	$p^2q^3r^3s^4t^2u^3v^4w^4$
8	$p^4q^6r^7s^8t^4u^6v^7w^7$	$p^4q^6r^7s^7t^4u^6v^7w^8$
9	$p^7q^{11}r^{13}s^{14}t^8u^{12}v^{14}w^{15}$	$p^8q^{12}r^{14}s^{15}t^7u^{11}v^{13}w^{14}$

3. Some Properties of First Schemes and Results:

Theorem 3.1: For every integer $n \geq 0$,

$$\beta_0.\alpha_{n+5} = \alpha_0\beta_{n+5}$$

Proof : To prove this, we used mathematical induction method.

If n=0, then $\beta_0.\alpha_5 = \beta_0.\beta_4\beta_3.\beta_2.\beta_1$ (By 2.1))

$$= \beta_0.\alpha_3.\alpha_2.\alpha_1.\alpha_0.\beta_3.\beta_2\beta_1$$

$$= \alpha_3.\alpha_2.\alpha_1.\alpha_0.\beta_3.\beta_2\beta_1.\beta_0$$

$$= \alpha_3.\alpha_2.\alpha_1.\alpha_0.\alpha_4 \text{ (By 2.1))}$$

$$= \alpha_0.\alpha_4.\alpha_3.\alpha_2.\alpha_1$$

$$= \alpha_0.\beta_5$$

Thus the result is true for n = 0. Let us assume that the result is true for some integer $n \geq 1$, then

$$\beta_0.\alpha_{n+6} = \beta_0.(\beta_{n+5}.\beta_{n+4}.\beta_{n+3}.\beta_{n+2})$$

$$= \beta_0.(\alpha_{n+4}.\alpha_{n+3}.\alpha_{n+2}.\alpha_{n+1})\beta_{n+4}.\beta_{n+3}.\beta_{n+2}$$

$$= (\beta_0.\alpha_{n+1})\alpha_{n+4}.\alpha_{n+3}.\alpha_{n+2}.\beta_{n+4}.\beta_{n+3}.\beta_{n+2}$$

$$= (\alpha_0.\beta_{n+1})\alpha_{n+4}.\alpha_{n+3}.\alpha_{n+2}.\beta_{n+4}.\beta_{n+3}.\beta_{n+2}$$

$$= \alpha_0.\beta_{n+1}\alpha_{n+4}.\alpha_{n+3}.\alpha_{n+2}.\beta_{n+4}.\beta_{n+3}.\beta_{n+2}$$

$$= \alpha_0.\beta_{n+4}.\beta_{n+3}.\beta_{n+2}\beta_{n+1}\alpha_{n+4}.\alpha_{n+3}.\alpha_{n+2}$$

$$= \alpha_0.\alpha_{n+5}.\alpha_{n+4}.\alpha_{n+3}.\alpha_{n+2}$$

$$= \alpha_0\beta_{n+6}$$

Hence the result is true for all integer $n \geq 0$.

Theorem 3.2: For every integer $n \geq 0$,

$$\beta_1\alpha_{n+6} = \alpha_1\beta_{n+6}$$

Proof. To prove this, we use mathematical induction method.

If $n = 0$, then $\beta_1\alpha_6 = \beta_1\beta_5\beta_4\beta_3\beta_2$

$$= \beta_1\beta_4\beta_3\beta_2(\alpha_4\alpha_3\alpha_2\alpha_1)$$

$$= (\beta_1\alpha_1)\alpha_4\alpha_3\alpha_2\beta_4\beta_3\beta_2$$

$$= (\alpha_1\beta_1)\alpha_4\alpha_3\alpha_2\beta_4\beta_3\beta_2$$

$$= \alpha_1\beta_4\beta_3\beta_2\beta_1\alpha_4\alpha_3\alpha_2$$

$$= \alpha_1\alpha_5\alpha_4\alpha_3\alpha_2$$

$$= \alpha_1\beta_6$$

Thus the result is true for $n = 0$. Let us assume that the result is true for some integer $n \geq 1$, then

$$\beta_1\alpha_{n+7} = \beta_1\beta_{n+6}\beta_{n+5}\beta_{n+4}\beta_{n+3}$$

$$= \beta_1(\alpha_{n+5}\alpha_{n+4}\alpha_{n+3}\alpha_{n+2})\beta_{n+5}\beta_{n+4}\beta_{n+3}$$

$$= (\beta_1\alpha_{n+2})(\alpha_{n+5}\alpha_{n+4}\alpha_{n+3})\beta_{n+5}\beta_{n+4}\beta_{n+3}$$

$$= (\alpha_1\beta_{n+2})(\alpha_{n+5}\alpha_{n+4}\alpha_{n+3})\beta_{n+5}\beta_{n+4}\beta_{n+3}\text{n+7}$$

$$= \alpha_1\beta_{n+5}\beta_{n+4}\beta_{n+3}\beta_{n+2}\alpha_{n+5}\alpha_{n+4}\alpha_{n+3}$$

$$= \alpha_1\alpha_{n+6}\alpha_{n+5}\alpha_{n+4}\alpha_{n+3}$$

$$= \alpha_1\beta_{n+7}$$

Theorem 3.3: For every integer $n \geq 0$,

$$\beta_2.\alpha_{n+7} \quad = \alpha_2\beta_{n+7}$$

Proof. To prove this, we use mathematical induction method.

If $n = 0$ then $\beta_2.\alpha_7 \quad = \beta_2.\beta_6\beta_5\beta_4\beta_3$

$$= \beta_2(\alpha_5\alpha_4\alpha_3\alpha_2)\beta_5\beta_4\beta_3$$

$$= (\beta_2\alpha_2)\alpha_5\alpha_4\alpha_3\beta_5\beta_4\beta_3$$

$$= (\alpha_2\beta_2)\alpha_5\alpha_4\alpha_3\beta_5\beta_4\beta_3$$

$$= \alpha_2(\beta_5\beta_4\beta_3\beta_2)\alpha_5\alpha_4\alpha_3$$

$$= (\alpha_6\alpha_5\alpha_4\alpha_3)\alpha_2$$

$$= \beta_7\alpha_2$$

Thus the result is true for $n = 0$. Let us assume that the result is true for some integer $n \geq 1$, then

$$\beta_2.\alpha_{n+8} \quad = \beta_2.(\beta_{n+7}\beta_{n+6}\beta_{n+5}\beta_{n+4})$$

$$= \beta_2.(\alpha_{n+6}\alpha_{n+5}\alpha_{n+4}\alpha_{n+3})\beta_{n+6}\beta_{n+5}\beta_{n+4}$$

$$= (\beta_2.\alpha_{n+3})(\alpha_{n+6}\alpha_{n+5}\alpha_{n+4})\beta_{n+6}\beta_{n+5}\beta_{n+4}$$

$$= (\alpha_2.\beta_{n+3})(\alpha_{n+6}\alpha_{n+5}\alpha_{n+4})\beta_{n+6}\beta_{n+5}\beta_{n+4}$$

$$= \alpha_2.\beta_{n+6}\beta_{n+5}\beta_{n+4}\beta_{n+3}(\alpha_{n+6}\alpha_{n+5}\alpha_{n+4})$$

$$= \alpha_2.\alpha_{n+7}\alpha_{n+6}\alpha_{n+5}\alpha_{n+4}$$

$$= \alpha_2.\beta_{n+7}$$

Hence the result is true for all integer $n \geq 1$.

Theorem 3.4: For every integer $n \geq 0$,

$$\beta_3.\alpha_{n+8} \quad = \alpha_3\beta_{n+8}$$

Proof: To prove this, we use mathematical induction method.

If $n = 0$, then $\beta_3.\alpha_8 \quad = \beta_3.\beta_7\beta_6\beta_5\beta_4$

$$= \beta_3.(\alpha_6\alpha_5\alpha_4\alpha_3)\beta_6\beta_5\beta_4$$

$$= (\beta_3\alpha_6\alpha_5\alpha_4)\beta_6\beta_5\beta_4\alpha_3$$

$$= (\alpha_6\alpha_5\alpha_4)\beta_6\beta_5\beta_4\beta_3\alpha_3$$

$$= \alpha_6\alpha_5\alpha_4\alpha_7\alpha_3$$

$$= \beta_7\alpha_3$$

Thus the result is true for $n = 0$. Let us assume that the result is true for some integer $n \geq 1$, then

$$\begin{aligned}
\beta_3.\alpha_{n+9} &= \beta_3.\beta_{n+8}\beta_{n+7}\beta_{n+6}\beta_{n+5} \\
&= \beta_3.(\alpha_{n+7}\alpha_{n+6}\alpha_{n+5}\alpha_{n+4})\beta_{n+7}\beta_{n+6}\beta_{n+5} \\
&= (\beta_3\alpha_{n+4}).(\alpha_{n+7}\alpha_{n+6}\alpha_{n+5})\beta_{n+7}\beta_{n+6}\beta_{n+5} \\
&= (\alpha_3\beta_{n+4}).(\alpha_{n+7}\alpha_{n+6}\alpha_{n+5})\beta_{n+7}\beta_{n+6}\beta_{n+5} \\
&= \alpha_3\beta_{n+7}\beta_{n+6}\beta_{n+5}\beta_{n+4}\alpha_{n+7}\alpha_{n+6}\alpha_{n+5} \\
&= \alpha_3\alpha_{n+8}\alpha_{n+7}\alpha_{n+6}\alpha_{n+5} \\
&= \beta_{n+9}\alpha_3
\end{aligned}$$

References

1. Atanassov, K.T., "Remark on a new direction for a generalization of the Fibonacci sequence", *The Fibonacci Quarterly, 33(3)*, 249-250 (1995).
2. Harne S., Badshah V.H., Sethiya S., Some Properties of the Generalization of the Multiplicative Coupled Fibonacci Sequences, *Applied Mathematical Sciences, 7(43)*, 2137-2142 (2013).
3. Rathore G.P.S., Jain, S. Sikhwal, O. Generalized Coupled Fibonacci Sequences, *Int. J. Contemp. Math Science, 59(8)*, (2012).