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Role of Fuzzy Stacks on Extended Topology

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Abstract

The paper is addressed to the introduction of some fuzzy stacks as to how they play an important role on extended fuzzy topology and its results.

1. Introduction

The extended fuzzy topology introduced by Gahler and Abd-Allah[1] is defined as interior operator for fuzzy sets which is a completely distributive lattice with $\tilde{0}$ and $\tilde{1}$ as the least and the largest element. It is an isotone kernel operator int : $L^X \to L^X$

In the classical case of $L = \{0,1\}$, the idea of extended fuzzy topology coincides with that of extended topology by Hammer[2] In this paper, we mention some results on fuzzy stacks given by W. Gahler and Allah[3]. The new results on pre-image of fuzzy stacks and on fuzzy stack products follow.

For each extended fuzzy topological space (X, int.), the mapping which assigns to each point $x \in X$, the fuzzy neighborhood stack at x is called fuzzy stack pre topology of this space. Thus fuzzy stacks play an important role on various extended fuzzy topologies in their categories and sub categories.

2. Preliminaries :

Let L be a completely distributive lattice with different least and largest elements $\tilde{0}$ and $\tilde{1}$ respectively which obeys infinite distributive laws

$$\bigvee_{i\in I} (\propto_i \wedge \propto) = (\bigvee_{i\in I} \propto_i) \wedge \propto$$

and let $L_0 = L\sim\{0\}$

If X is any set, the mappings $f : X \to L$ are L fuzzy subsets of X and L^X is the set of all fuzzy subsets where $f \le g$ means $f(x) \le g(x)$ $\forall x \in X$, $f, g \in L^X$

If any set X is fixed, then for each $\alpha \in I$, $\overline{\alpha}$ will denote the constant mapping of X into L

Def (2.1) Fuzzy stacks : A fuzzy stacks on a non empty set X is a mapping u : $L^X \rightarrow L$ satisfying the conditions :

(i) For each non empty finite set of fuzzy sets $f_1, f_2 \ldots\ldots f_n \in L^X$, we hav $u(f_1) \wedge u(f_2) \wedge \ldots\ldots\ldots \wedge u(f_n) \leq \sup (f_1 \wedge f_2 \wedge \ldots\ldots \wedge f_n)$

(ii) u is isotone

In case of $L = \{0,1\}$, L^X with the power set p (x) is called a stack instead of fuzzy stack satisfying the conditions

(a) For each non empty finite subsets $\{M_1, M_2 \ldots\ldots M_K\}$ of u, $\bigcap_{i \in I} M_i \neq$? and

(b) $M \in u$, $M \subseteq N$ implies $N \in u$

special fuzzy stacks are fuzzy filters which is a mapping u : $L^X \rightarrow L$ satisfying

F_1 : $u(\bar{\alpha}) \leq \alpha$ holds for all $\alpha \in L$, $u(\tilde{1}) = 1$

F_2 : $u(f \wedge g) = u(f) \wedge u(g)$ for all $f, g \in L^X$

A fuzzy stack is called homogeneous if $u(\bar{\alpha}) = \alpha$ for all $\alpha \in L$ and it is said to have a cutting property if

$$u(f \wedge \bar{\alpha}) = u(f) \wedge u(\bar{\alpha}) \text{ for all } \alpha \in L, f \in L^X$$

Proposition (2.2) :

For each fuzzy stack u on X, the mapping : $L^X \rightarrow L$ defined by

$$\mu(f) = \begin{cases} V(u(f_1) \wedge u(f_2) \wedge \ldots\ldots \wedge u(f_n)) & \text{if } f \neq \tilde{1} \\ 1, & \text{otherwise} \end{cases}$$

is the coarsest fuzzy filter on X which is finer than u. A fuzzy stack u is called homogeneous[1] if $u(\bar{\alpha}) = \alpha$ for all $\alpha \in L$

Proposition (2.3) :

For each fuzzy stack u on X, the mapping $\bar{u} : L^X \rightarrow L$

defied by

$$\bar{u}(f) = V(u(g) \wedge u(\bar{\alpha}_1) \wedge u\,\bar{\alpha}_n))$$
$$g \wedge \bar{\alpha}_1 \wedge \ldots\ldots\ldots \wedge \bar{\alpha}_n \leq f \text{ for all } f \in L^X$$

is the coarsest fuzzy stack on X finer than u which has the culting property[3]

In case of $L = \{0,1\}$, the notion of clting property is trivial.

Definition (2.4): (Valued fuzzy stack bases)

There are two important notions of fuzzy stack bases A family $(B_\alpha)_{\alpha \in L}$ of subsets of L^X is called a valued fuzzy stack[1] if it satisfies the condition

For each nonempty finite subsets $\{\alpha_1, \alpha_2, \ldots\ldots\ldots \alpha_n\}$ of L_0 and all mappings $f_1 \in B_{\alpha 1}, \ldots\ldots\ldots f_n \in B_{\alpha n}$, we have $\alpha_1 \wedge \ldots\ldots\ldots \wedge \alpha_n \leq (f_1 \wedge \ldots\ldots\ldots \wedge f_n)$

Proposition (2.5) :

Each valued fuzzy stack base $(B_\alpha)_{\alpha \in L}$ defines a fuzzy stack u by taking $u(f) = \vee_{\alpha \in B_{\alpha j}} \alpha$ for all $\alpha \in L^X$

and conversely

$(\alpha, pr(\mu)\ \alpha \in L_0$ is a family of prestacks on X and is called the large valued fuzzy stack base of μ when by a pre-stack on X, we mean a subset of L^X (say K) such that[3]

(i) $f_1, f_2 \ldots\ldots\ldots f_n \in K \Rightarrow 0 < \vee (f_1 \wedge f_2 \wedge \ldots \ldots\ldots \wedge f_n)$ and

(ii) $f \in k$ and $f \leq g \Rightarrow g \in k$

Definition (2.6) :

Superior fuzzy stack bases :-

A subset B of L^X is called a superior fuzzy stack base if the following conditions hold

S_1 : $\bar{\alpha} \in \beta$ for every $\alpha \in L$

S_2 : for each non empty finite subset $\{f_1, f_2 \ldots \ldots f_n\}$ of β, we have

$$(\vee f_1) \wedge (\vee f_2) \wedge \ldots\ldots (\vee f_n) = \vee (f_1 \wedge f_2 \ldots\ldots \wedge f_n)$$

Preposition (2.7) :

Each superior fuzzy stack base B defines a homogeneous fuzzy stack u on X by taking

$$u(f) = \bigvee_{\substack{g \in B \\ g \leq f}} [Vg] \quad \text{for all } f \in L^X$$

For each homogeneous fuzzy stack base $u = \{f \in L^X, u(f) = \vee f\}$ is a superior fuzzy stack base. For the fuzzy stack u^b generated by base[3] u, we have $u \leq u^b$ and base u= base u^b

Preposition (2.8) :

Let u be a homogeneous fuzzy stack on X with the culting property and B be a superior fuzzy stack base of u Then

$(B_\alpha)_{\alpha \in L}$ with $B_\alpha = \{g \in B, \alpha \leq vg\}$ is valued fuzzy stack base of u.

Preposition (2.9) :

Let u be a homogeneous fuzzy stack on X with the culting property and $(B_\alpha)_{\alpha \in L}$ be a valued fuzzy stack of u such that

(i) $\bar{\alpha} \in \beta_\alpha$ for all $\alpha \in L$

(ii) $f \wedge \bar{\alpha} \in \beta_\alpha$ for all $\alpha \in L_0$ and $f \in \beta_\alpha$

(iii) $0 < \alpha \leq \beta \Rightarrow \beta_\alpha \supseteq B_\beta$ for all $\alpha, \beta \in L_0$

Then, $B = \{\tilde{0}\} \cup \{f \in L^X : f \neq \tilde{0}, f \in \beta_\alpha\}$ is a superior fuzzy stack base of u.

3. Pre Image :

For the notion of pre image of a fuzzy stack, important results of fuzzy stack products will be given.

Let $f : X \to Y$ be a mapping. For each fuzzy stack λ on Y by the pre image of λ with respect to f we mean the coarsest fuzzy stack u on X for which $\lambda(f(u)) \leq 1$ holds provided if exists. The pre image of λ with respect to f will be denoted by $\overline{p_i}\ f(\lambda)$ if the pre image of λ with respect to f exists

Then

$p_i f(\overline{p_i} f(\lambda)) \leq 1$ Type equation here.

obviously $u \leq \overline{p_i}\ f(p_i f(\lambda))$

Recall that the pre image of a fuzzy filter on Y with respect to the mapping $f : X \to Y$ is the coarsest fuzzy filter u on X for which[4,5] $\overline{p_i}\ f(u) \leq 1$

Theorem (3.1):

Let $f : X \to Y$ be a mapping and λ be a fuzzy stack on Y. Then the following conditions are equivalent

(i) The pre image of λ with respect to f exists.

(ii) $\{u \in p_L X : P_L (f(u)) \leq 1\}$ is non empty

(iii) $\{u \in P_L \overline{X} : \overline{P}_L (f(u)) \leq 1\}$ is non empty

(iv) $\lambda (h_1) \wedge \lambda (h_2) \wedge \ldots \wedge \lambda (h_n) \leq \vee (h_i \text{ o } f)$

Where $(h_n \text{ o } f)$ holds for all finite subsets $(h_1, h_2 \ldots h_n)$ of L^Y

Proof : Since $p_L f$ preserves non empty suprimum of fuzzy stacks, the equivalent of (i) and (ii) follows

Because of the proposition (2.2), each fuzzy stack has a finite fuzzy filter. This implies that (ii) and (iii) are equivalent[6-7]

If the pre image $\overline{p}_L (f(\lambda))$ exists, then for each finite subset $\{h_1, h_2, \ldots h_n\}$ of L^Y $(h_1) \leq \overline{P}_L (\overline{f} (\lambda)) (h_1 \text{ o } f) \wedge \ldots \leq \vee [((h_1 \text{ o } f) \wedge \ldots]$ which shows that (iv) is fulfilled.

Theorem (3.2) :

Let $f : X \to Y$ be a mapping and λ be a fuzzy stack on Y for which the pre image $\overline{P}_L (f(\lambda))$ exists. For each valued fuzzy stack base $\{S_\alpha\}_{\alpha \in L0}$ of λ

The family $\{B_\alpha\}_{\alpha \in L0}$ defined by

$B_\alpha = \{h \text{ o } f : h \in S_\alpha\}$ for all $\alpha \in L_0$ is a valued fuzzy stack base of $\overline{P}_L f(\lambda)$

Moreover $\overline{P}_L (f(\lambda))\, g = \bigvee_{hof \leq g} \lambda(h)$ for all $g \in L^X$. If the mapping f is surjective Then

$$p_L \; f [\overline{P}_L (f(\lambda))] = \lambda \text{ for all } \lambda \in p_L Y$$

If is fuzzy filter, Then $\overline{p}_L f(\lambda)$is also so.

Proof : Let $\{S_\alpha\}_{\alpha \in L0}$ be a valued fuzzy stack of λ and let $\{B_\alpha\}_{\alpha \in L0}$ be defined by means of hof : $h \in \delta_\alpha$

$\lambda (hof) = \vee\alpha \geq \vee \alpha = \lambda$

Hence $p_L (f(\lambda)) \leq 1$ holds

If f is surjective, then for all $g \in L^Y$

$$p_L \; f (\overline{p}_L (f(II)) (g) = \vee \lambda(h) = \lambda(g)$$

If λ be a fuzzy filter, then for $g_1, g_2 \in L^X$, we get

$\vee \lambda (h_1) \wedge \vee \lambda (h_2) \leq \vee (h_1 \wedge h_2) = \vee (h)$

Hence the result.

4. Fuzzy stack products :

Let I be a non empty set and for each $i \in I$, let u_i be fuzzy stacks on X_i. The Coarsest fuzzy stack p on the Cartesian

product $X = \prod_{i \in I} X_i$ for which $p_L \, II_i (\lambda) \leq u_i$ holds for all $i \in I$, is

called the fuzzy stack products of $\{u_i\}_{i \in I,}$?$_i$ being the ith projection of X.

If for each $i \in I$, p_i is a fuzzy filter on X, then the coarsest fuzzy filter λ on the Cartesian product $\prod_{i \in I} X$ is called the fuzzy filter product of $\{u_i\}_{i \in I}$. According to the following proposition, all fuzzy stack products exist.

Proposition (4.1): If I be a non empty set and for each $i \in I$ u_i be a fuzzy stack on X, then the fuzzy stack products of $\{u_i\}_{i \in I}$ exist and we have

$$\prod_{i \in I} u_i = \wedge \, \bar{p}_L \, II_i \, (u_i)$$

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