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Unsteady two-Dimensional Flow of an Electrically Conducting Visco-Elastic Fluid Along Vertical Porous Surface with Heat Source and Chemical Reaction

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Abstract

An unsteady two-dimensional natural convection flow of a visco-elastic (Walters B′ Model) incompressible, electrically conducting fluid along an infinite hot vertical porous surface with heat source and chemical reaction has been studied. The novelty of the present study is to investigate the effects of fluctuating bounding-surface-temperature as well as presence of a volumetric heat source on the flow and heat transfer phenomena. The governing equations of motion, energy and concentration are solved by successive perturbation technique. The expressions for the skin friction, Nusselt number and Sherwood number are also derived. The variations in the fluid velocity, temperature and concentration are shown graphically whereas numerical values of skin-friction, Nusselt number and Sherwood number are presented in a tabular form for various values of pertinent flow parameters. One of the significant contributions of the present study is that the lapse of time contributes to thickening of velocity boundary layer and thinning of thermal boundary layer and hence contributing to designing of MHD devices controlling fluid flow and heat transfer.

Key words: Electrically conducting fluid, Visco-elastic, heat source, chemical reaction, porous surface.

1. Introduction

When the forced flow through a parallel plate heat exchanger ceases, a free convection flow pattern is established since the fluid remains at an elevated temperature. If the fluid is a conductor of electricity, this flow can be influenced by an imposed magnetic field. The problem of an unsteady free convection flow along a vertical porous surface in the presence of an applied magnetic field is of general interest from the point of view of many applications, such as in space flight and nuclear fusion research.

Flows through porous media are frequently used in filtering of gases, liquids and drying of bulk materials. In electrochemical engineering, semi-permeable diaphragms, porous electrodes and permeable, are used to obtain improved current efficiencies. There is also large scale use of packed porous beds in chemical reactions, heat exchangers and separation of chemical components. In the field of agricultural engineering, porous media heat transfer plays an important role particularly in germination of seeds. Above all, man does a part of his breathing through his porous skin. More interestingly, application of magnetic field in the field of chemical engineering and agriculture attracts many researchers to work on this new field of interest. Kronenberg rightly stated that “If the agricultural industry knew the benefit of magnetically treated water, the total agricultural output could be approximately increased by 25 %”. When fluid passes through a properly focus magnetic field, the magnetic field breaks up some of the complexes that are carried in the water, freeing the captive mineral particulars, This process keeps minerals in the water rather than precipitated out. This is known as magnetic field conducting. Further, magnetically treated fuel has a tendency to attract oxygen molecules when mixed with air in a combustion cylinder. This results a more efficient and complete combustion of the fuel generating more power from the same amount of fuel. Research shows that dairy cows will drink more magnetically treated water than untreated and thus increases milk production appreciably. With beef cattle, feed conversion ratio has increased by 5 % approximately enhancing meat production about the same. A chemical reaction involves in breaking of bonds in the reactive substances and the formulation of bonds to form product species.

Walters[1] suggested a constitutive equation for elastico-viscous liquid with short memories (*i.e.*, short relaxation times).The problem of free convective magnetohydrodynamics (MHD) flow with heat and mass transfer has been a subject of interest of several researchers. Unsteady free convection with thermal and mass diffusion of an electrically conducting fluid past an infinite vertical plate subjected to uniformly accelerated motion and constant heat flux was studied by Acharya *et al.*[2], Sharma and Pareek[3] examined the unsteady flow and heat transfer in an elastico-viscous liquid along an infinite hot vertical porous moving plate with variable free steam and suction. Flow and heat transfer of an electrically conducting visco-elastic fluid between two horizontal squeezing/streatching plates have been studied by Rath *et al*. (2001). Alharbi*et al.*[5] have studied heat and mass transfer in MHD visco-elastic fluid flow through a porous medium over a stretching sheet with chemical reaction. Visco-elastic fluid flow past an infinite vertical porous plates in the presence of first order chemical reaction has been investigated by Damesh and Shannak[6]. Kumar and Sivaraj[7] have studied MHD mixed convective visco-elastic fluid flow in a permeable vertical channel with Dufour effect and chemical reaction parameter. Chen[8] worked on the analytic solution of MHD flow and heat transfer for two types of visco-elastic fluid over a stretching sheet with energy dissipation internal heat source and thermal diffusion.His study was restricted to a parabolic spatial distribution of plate temperature.

Unsteady free convection MHD flow and mass transfer through porous media of a second order fluid between two heated plates with source/sink has been studied by Dash *et al*.[9]. Rath *et al*.[10] have studied the three-dimensional free convection flow through porous medium in a vertical channel with heat source and chemical reaction. Barik *et al*.[11] have studied the chemical reaction effect on MHD oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and soret effect. Unsteady two-dimensional flow and heat transfer of an elastico-viscous liquid along an infinite hot vertical porous surface embedded in a porous medium has been studied by Sharma and Sharma[12]. Unsteady MHD flow of a visco-elastic fluid along vertical porous surface with chemical reaction has been investigated by Nayak *et al*.[13]. Their study suffers two limitations

i) The temperature of the bounding surface is considered constant which usually does not happen in practice.
ii) There is no additional volumetric heat source/sink in heat-equation which is the frequent occurrence in the process of exothermic and endothermic reactions.

Therefore, the novelty of the present study is to generalize the earlier study Nayak *et al*.[13] by letting the temperature of the bounding surface be a fluctuative one as well as considering the presence of the volumetric temperature dependent heat source. The flow within the semi-infinite region is due to the oscillatory motion of the free stream. The model has important applications in novel magneto biofluid and designing MHD devices requiring fluid flow control.

The main objective of the present study is to investigate the combined effects of magnetic field, heat source and chemical reaction on two-dimensional visco-elastic flow through porous media over a vertical surface.

2. Mathematical formulation of the problem:

An unsteady two dimensional MHD flow of visco-elastic conducting fluid (Walters B′ model)[1] past a vertical infinite surface through a porous medium with heat source and mass transfer has been considered. A uniform transverse magnetic field B_0 is applied normal to the main direction of the fluid flow. The x^*-axis is taken along the surface of the porous plate in upward direction *i.e.*, opposite to the direction of gravity and y^*-axis is taken normal to the surface.We have considered the density variation in the buoyancy terms and neglected the work against gravity along with viscous and joul heating. The interaction of the electromagnetic force with the free convection, both due to thermal buoyancy and mass buoyancy, affects the flow. With the above frame of reference and assumptions (the plate is infinite in extent and the flow is unsteady), the physical variables are functions of y and t only. Consequently the governing equations for visco-elastic flow with modified pressure gradients under usual boundary layer and Bousinesq approximations following Nayak *et al*.[13] are given by:

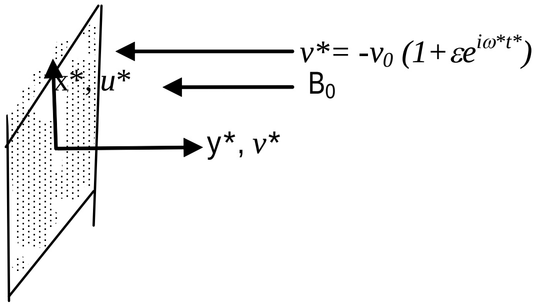

Figure. 1. Physical configuration of problem

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{2.1}$$

$$\rho\left(\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*}\right) = \rho\frac{\partial U^*}{\partial t^*} + \frac{\mu}{K_p^*}\left(U^* - u^*\right) + \mu\frac{\partial^2 u^*}{\partial y^{*2}} - K_0^*\left(\frac{\partial^3 u^*}{\partial t^*\partial y^{*2}} + v^*\frac{\partial^3 u^*}{\partial y^{*3}}\right) + \sigma B_0^2\left(U^* - u^*\right) + g\beta\rho\left(T^* - T_\infty\right) + g\beta^*\rho\left(C^* - C_\infty\right) \tag{2.2}$$

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + S^*(T^* - T_\infty) \tag{2.3}$$

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_c^*\left(C^* - C_\infty\right) \tag{2.4}$$

where u^*, v^*denote the components of velocity along x^* and y^* directions respectively, ρ the density of the fluid, g the acceleration due to gravity, v the Kinematic viscosity, μ the coefficient of dynamic viscosity, K_p^* the permeability parameter, K_0^* the elastic parameter, T^* the temperature of the fluid, k the thermal conductivity of fluid, C_p the specific heat of the fluid at constant pressure, t^* the time, C^* the concentration of fluid, K_c^* the chemical reaction parameter, D the mass diffusion coefficient, σ the electrical conductivity, S^* the heat source parameter, β and β^* are the volumetric coefficient of thermal and concentration expansion respectively.

The boundary conditions are :

$$y^*=0;\ u^*=0,\ T^* = T_w + \varepsilon\left(T_w - T_\infty\right)e^{i\omega t}, C^* = C_w$$

$$y^*\to\infty;\ u^*\to U^*(t^*),\ T^*\to T_\infty\ ,\ C^*\to C_\infty \tag{2.5}$$

where T_w and C_w are the constantsurface temperature and concentration respectively, U^* the free stream velocity, T_∞ the free stream temperature and C_∞ is the concentration of the fluid for away from the wall.

From equation of continuity (2.1), it is clear that the suction velocity normal to the plate is either a constant or a function of time. Hence, it is assumed to be of the form

$$v^*(t) = -v_0(1+ \quad \varepsilon e^{i\omega^* t^*}) \tag{2.6}$$

where ω^* the frequency of oscillation of suction,ε is the small suction parameter (real positive constant) *i.e.*, $0 < \varepsilon < 1$ and v_0 is a non-zero positive suction velocity. Here the negative sign indicates that the suction is towards the plate.

The equation (2.6) represents the flow subject to time dependent oscillatory suction with a mean v_0 frequency of oscillation ω and having a steady part and a transient part. Dash and Behera[14] have considered time dependent suction in their study. In a physical situation, We cannot ensure perfect insulation, in any experimental set up. There will always be some fluctuations in the temperature. The plate temperature is assumed to vary harmonically with time. It varies from $T_w \pm \varepsilon (T_w - T_\infty)$ as t varies from 0 to $2\pi/\omega$. Since the plate temperature varies, there may also occur some variations in suction at the plate. Here we assume that frequencies of suction velocity and temperature variations are same. Since the ε is small the variation of suction velocity and temperature at the plate slightly vary from their mean values v_0 and T_w respectively. Consider the unsteady flow of a visco-elastic[1] incompressible and conducting fluid through a porous medium occupying a semi-infinite region bounded by a vertical infinite porous plate when the temperature of the plate varies with time about a non-zero constant mean and the temperature at the free stream is constant. The porous medium is considered tobe homogeneous. Also we assume that the fluid properties are not affected by the temperature and concentration variations except that of the density ρ in the body force term and the influence of the density variations in the momentum and energy equations is negligible. We also consider a transverse magnetic field of constant field strength B_0 applied perpendicular to main flow direction *i.e.*, x –axis. Consequently,a resisting body force Lorentz force ($\vec{J} \times \vec{B} = -\sigma B_0^2 u^*$) of electromagnetic origin comes in to play affecting the flow. Further let the free stream velocity U(t) be defined as $U^*(t) = U_0 (1 + \varepsilon e^{i\omega t})$ where U_0- is the mean of the free stream oscillation.

We now introduce the following non-dimensional quantities:

$$y=\frac{y^* v_0}{\upsilon}, t=\frac{t^* v_0^2}{4\upsilon}, u=\frac{u^*}{U_0}, \omega=\frac{4v\omega^*}{v_0^2},$$

$$U(t)=\frac{u^*(t^*)}{U_0}, T=\frac{T^*-T_\infty}{T_w-T_\infty}, P_r=\frac{\mu C_p}{k}$$

$$k_p=\frac{K_p^* v_0^2}{\upsilon^2}, K_0=\frac{K_0^* v_0^2}{\rho\upsilon^2}, G_r=\frac{\upsilon g\beta(T_w-T_\infty)}{U_0 v_0^2},$$

$$M^2=\frac{\sigma B_0^2\upsilon}{\rho v_0^2}, K_c=\frac{K_c^*\upsilon}{v_0^2}, S_c=\frac{v}{D}$$

$$C=\frac{C^*-C_\infty}{C_w-C_\infty}, S=\frac{S^*\upsilon}{v_0^2}, G_c=\frac{\upsilon g\beta^*(C_w-C_\infty)}{U_0 v_0^2} \tag{2.7}$$

In view of equations (2.6) and (2.7), equations (2.2)- (2.4) become :

$$\frac{1}{4}\frac{\partial u}{\partial t}-\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\left(M^2+\frac{1}{k_p}\right)$$

$$\left(1+\varepsilon e^{i\omega t}-u\right)-K_0\left\{\frac{1}{4}\frac{\partial^3 u}{\partial t\partial y^2}-\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial^3 u}{\partial y^3}\right\}$$

$$+\frac{1}{4}\varepsilon i\omega e^{i\omega t}+G_r T+G_c C \tag{2.8}$$

$$\frac{P_r}{4}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial y^2}+P_r\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial T}{\partial y}+P_r ST \tag{2.9}$$

$$\frac{S_c}{4}\frac{\partial C}{\partial t}=\frac{\partial^2 C}{\partial y^2}+S_c\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial C}{\partial y}-S_c K_c C \tag{2.10}$$

where G_r the Grashoff number for heat transfer, *P*, the Prandtl number, K_0, the non-Newtonian parameter, G_c the Grashoff number for mass transfer, S_c the Schmidt number, M the magnetic parameter, k_p the porosity parameter and K_c the chemical reaction parameter.

The corresponding boundary conditions in non-dimensional form are

$y = 0: u = 0, T = 1+\varepsilon e^{i\omega t}, C = 1$

$y \to \infty: u \to U(t)=1+e^{i\omega t}, T\to 0, C\to 0$ (2.11)

3. Method of solution :

To sove the equations (2.8) to (2.10) we use a technique of domain perturbation (steady and unsteady) following Josheph[15], Mahato[16] and many others.

In view of the above assumption it is justified to assume :

$$u(y, t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t} \quad (3.1)$$

$$T(y, t) = T_0(y) + \varepsilon T_1(y)\, e^{i\omega t} \quad (3.2)$$

$$C(y, t) = C_0(y) + \varepsilon C_1(y)\, e^{i\omega t} \quad (3.3)$$

Substituting equations (3.1) – (3.3) into equations (2.8) – (2.10) and equating the co-efficients of like power of ε, we get

Zeroth order equations:

$$K_0 u_0''' + u_0'' + u_0' - \left(M^2 + \frac{1}{k_p}\right)u_0 = -G_r$$

$$T_0 - G_c C_0 - \quad (3.4)$$

$$T_0'' + P_r T_0' + P_r S T_0 = 0 \quad (3.5)$$

$$C_0'' + S_c C_0' - K_c S_c C_0 = 0 \quad (3.6)$$

First order equations:

$$K_0 u_1''' + \left(1 - \frac{K_0 i\omega}{4}\right)u_1'' + u_1' - \left(M^2 + \frac{1}{k_p} + \frac{i\omega}{4}\right)u_1$$

$$= -G_r T_1 - G_c C_1 - K_0 u_0''' - u_0' - \left(M^2 + \frac{1}{k_p} + \frac{i\omega}{4}\right) \quad (3.7)$$

$$T_1'' + P_r T_1' + P_r\left(S - \frac{i\omega}{4}\right)T_1 = -P_r T_0' \quad (3.8)$$

$$C_1'' + S_c C_1' - \left(\frac{i\omega}{4} + K_c\right)S_c C_1 = -S_c C_0' \quad (3.9)$$

where prime denotes differentiation with respect to y.

The corresponding boundary conditions are reduced to :

$$y = 0: u_0 = 0,\ u_1 = 0, T_0 = 1, T_1 = 1, C_0 = 1,\ C_1 = 0,$$
$$y \to \infty: u_0 \to 1,\ u_1 \to 1,\ T_0 \to 0, T_1 \to 0,\ C_0 \to 0,\ C_1 \to 0 \quad (3.10)$$

Solving equations (3.5), (3.6), (3.8) and (3.9) under the boundary conditions (3.10), we get

$$T_0 = e^{m_1 y} \quad (3.11)$$

$$C_0 = e^{m_3 y} \quad (3.12)$$

$$T_1 = \left(1 + \frac{4i}{\omega}m_1\right)e^{m_2 y} - \frac{4i}{\omega}m_1 e^{m_1 y} \quad (3.13)$$

$$C_1 = \frac{4i m_3}{\omega}\left(e^{m_4 y} - e^{m_3 y}\right) \quad (3.14)$$

The equations (3.4) and (3.7) are still of third order where $K_0 \neq 0$ and reduce to second order differential equation when $K_0=0$ *i.e.*, for a Newtonian fluid case. Hence, the presence ofelastic parameter increases the order of differential equation. While from physical consideration only two boundary conditions are available.Since the elastic parameter (K_0) is very small for incompressible fluid (Walters[1]), therefore, u_0 and u_1 can be expanded in powers of K_0 as

$$u_0(y) = u_{00}(y) + K_0 u_{01}(y) + O(K_0^2)$$
$$u_1(y) = u_{10}(y) + K_0 u_{11}(y) + O(K_0^2) \quad (3.15)$$

Introducing equation (3.15) into equations (3.4) and (3.7), we obtain the following systems of equations.

$$u''_{00} + u'_{00} - \left(M^2 + \frac{1}{k_p}\right) u_{00} = -\left(M^2 + \frac{1}{k_p}\right) - G_r T_0 - G_c C_0 \quad (3.16)$$

$$u''_{10} + u'_{10} - \left(M^2 + \frac{1}{k_p} + \frac{i\omega}{4}\right) u_{10} = -G_r T_1 - G_c C_1 - u'_{00} - \left(M^2 + \frac{1}{k_p} + \frac{i\omega}{4}\right) \quad (3.17)$$

$$u''_{01} + u'_{01} - \left(M^2 + \frac{1}{k_p}\right) u_{01} = -u'''_{00} \quad (3.18)$$

$$u''_{11} + u'_{11} - \left(M^2 + \frac{1}{k_p} + \frac{i\omega}{4}\right) u_{11} = -u'''_{00} - u'_{01} - u'''_{10} + \frac{i\omega}{4} u''_{10} \quad (3.19)$$

With the corresponding boundary conditions:

$$y = 0,\ u_{00} = 0,\ u_{01} = 0,\ u_{10} = 0,\ u_{11} = 0$$
$$y \to \infty:\ u_{00} \to 1,\ u_{01} \to 0,\ u_{10} \to 1, u_{11} \to 0 \quad (3.20)$$

Solving equations (3.16) – (3.19) under the boundary condition (3.20), we get

$$u_{00} = (A_3 + A_4 - 1)\, e^{m_5 y} - A_3 e^{m_1 y} - A_4 e^{m_3 y} + 1 \quad (3.21)$$

$$u_{01} = A_9\, e^{m_5 y} + A_7 e^{m_1 y} + A_8 e^{m_3 y} \quad (3.22)$$

$$u_{10} = A_{19}\, e^{m_6 y} + A_{17} e^{m_1 y} + A_{18} e^{m_3 y} + A_{10} e^{m_2 y} + A_{14} e^{m_5 y} + A_{12} e^{m_4 y} + 1 \quad (3.23)$$

$$u_{11} = A_{38}\, e^{m_6 y} + A_{35} e^{m_5 y} + A_{28} e^{m_4 y} + A_{36} e^{m_3 y} + A_{26} e^{m_2 y} + A_{37} e^{m_1 y} \quad (3.24)$$

Finally, the velocity u(y, t) is given by

$$u(y,\, t) = (A_3 + A_4 - 1)\, e^{m_5 y} - A_3 e^{m_1 y} - A_4 e^{m_3 y} + 1 + K_0 \left(A_9 e^{m_5 y} + A_7 e^{m_1 y} + A_8 e^{m_3 y}\right)$$
$$+ \varepsilon \Big[\Big\{ A_{19} e^{m_6 y} + A_{17} e^{m_1 y} + A_{18} e^{m_3 y} + A_{10} e^{m_2 y} + A_{14} e^{m_5 y} + A_{12} e^{m_4 y} + 1 \Big\}$$
$$+ K_0 \Big\{ A_{38} e^{m_6 y} + A_{35} e^{m_5 y} + A_{28} e^{m_4 y} + A_{36} e^{m_3 y} + A_{26} e^{m_2 y} + A_{37} e^{m_1 y} \Big\} \Big] e^{i\omega t} \quad (3.25)$$

where A_1 *to* A_{38}, m_1 *to* m_6 are given in the Appendix.

Skin friction :

$$\tau = \frac{\tau_w^*}{\rho U v} = \left[\frac{\partial u}{\partial y} - K_0 \left(\frac{1}{4} \frac{\partial^2 u}{\partial t \partial y} - \left(1 + \varepsilon e^{i\omega t}\right) \frac{\partial^2 u}{\partial y^2} \right) \right]_{y=0} \quad (3.26)$$

Nusselt Number:

$$N_u = \frac{q\upsilon}{v_0 k (T_w - T_\infty)} = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$
$$= -\left[m_1 + \varepsilon e^{i\omega t} \left\{ \left(1 + \frac{4i}{\omega} m_1\right) m_2 - \frac{4i}{\omega} m_1^2 \right\} \right] \quad (3.27)$$

Sherwood Number :

$$S_h = \frac{mv}{v_0 D(C_w - C_\infty)} = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$= -\left[m_3 + \varepsilon e^{i\omega t} I_1 \left\{m_4 - m_3\right\}\right] \quad (3.28)$$

Here τ_w^* is the shear stress componentof the visco-elastic fluid andthe symbols q and m represent heat and mass flux and they are given by:

$$q = -k\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0} \quad \text{and m} = -D\left(\frac{\partial C^*}{\partial y^*}\right)_{y^*=0}$$

4. Results and Discussion

The effect of the various flow, heat and mass transfer parameters in a magnetohydrodynamic natural convection flow of a visco-elastic fluid (Walters B′ fluid model) has been discussed.The solutions of the non-linear equations proceeds through a multi state perturbation technique, which has been justified earlier with $\varepsilon$and K_0 as parameters

Fig. 2 depicts the velocity distribution for various values of G_r, G_c, K_0, k_p, K_c, P_r, S_c, ω, ωt and M. It is seen that an increase in G_r and G_c increases the velocity near the plate (curves I, II and IX), due to free convection current which accelerates the velocity. The increase isinsignificant in case of thermal buoyancy (G_r), whereas it is quite significant in case of mass buoyancy (G_c). Further, it is seen that the presence of transverse magnetic field reduces the velocity due to magnetic interaction, producing Lorentz force which opposes the motion (curves I and XI). From the curves I, XIII and XIV it is observed that the velocity increases in case of endothermic chemical reaction ($K_c < 0$) but the effect is reversed in case of exothermic reaction ($K_c > 0$). It is also observed that the fluid velocity increases due to increase in the elastic parameter (K_0) and opposite effect is observed in case of phase angle (ωt) and Schmidt number (S_c). It is seen that permeability parameter (curve I, $k_p = 0.4$) and (curve-*VI*, k_p =100) contributes very significantly in increasing the velocity. This observation is compatible to the physics of the problem as because the presence of porous matrix restricts the passage of flow and hence contributes to enhancing the velocity.It is further observed that in case of higher Prandtl number fluid, the velocity increases.

The curves I and VII (Fig. 2) show the effect of variation of time t. The curves I and VII correspond, respectively, to t = 0.157 and t = 0.052 due to variations in frequency parameter ω phase angle ωt. The impact oflapse of time,is quite significant in increasing the velocity of the fluid particle in the entire flow domain *i.e.*, as t increase velocity increases at all points. Consequently, lapse of time contributes to thickening of the boundary layer and designing MHD devices requiring fluid flow control.

From fig. 3 it is observed that an increase in time t decreases the temperature at any point. Thus, an increase in time t gives rise to thinning of thermal boundary layer thickness. This is quite consistent as because with the elapse of time, the heat energy gets disspated contributing to higher velocity and lowering the temperature in the flow domain.

Further, from Fig.3 it is seen that frequency parameter (ω) and heat source (S) increase the thickness of thermal boundary layer (Curves I, III, V), but the reverse effect is observed in case of phase angle (ωt) and Prandtl number P_r.

Fig. 4 exhibits the concentration distribution in the flow field. For heavier species, *i.e.*, for higher values of (S_c) concentration decreases at all the layers in the flow domain. Similar effects are also observed in case of chemical reaction parameter (K_c), frequency parameter (ω) and phase angle *(ωt)*

Curve	G_r	P_r	k_p	w	wt	K_0	G_c	M	K_c	S_c	S
I	5	7	0.4	5	π/4	0.2	2	1	1	0.22	1
II	7	7	0.4	5	π/4	0.2	2	1	1	0.22	1
III	-5	7	0.4	5	π/4	0.2	2	1	1	0.22	1
IV	-7	7	0.4	5	π/4	0.2	2	1	1	0.22	1
V	5	5	0.4	5	π/4	0.2	2	1	1	0.22	1
VI	5	7	100	5	π/4	0.2	2	1	1	0.22	1
VII	5	7	0.4	15	π/4	0.2	2	1	1	0.22	1
VIII	5	7	0.4	5	π/3	0.2	2	1	1	0.22	1
IX	5	7	0.4	5	π/4	0.0	5	1	1	0.22	1
X	5	7	0.4	5	π/4	0.2	-2	1	1	0.22	1
XI	5	7	0.4	5	π/4	0.2	2	5	1	0.22	1
XII	5	7	0.4	5	π/4	0.2	2	1	1	0.22	1.5
XIII	5	7	0.4	5	π/4	0.2	2	1	-.04	0.22	1
XIV	5	7	0.4	5	π/4	0.2	2	1	0	0.22	1
XV	5	7	0.4	5	π/4	0.2	2	1	1	0.78	1

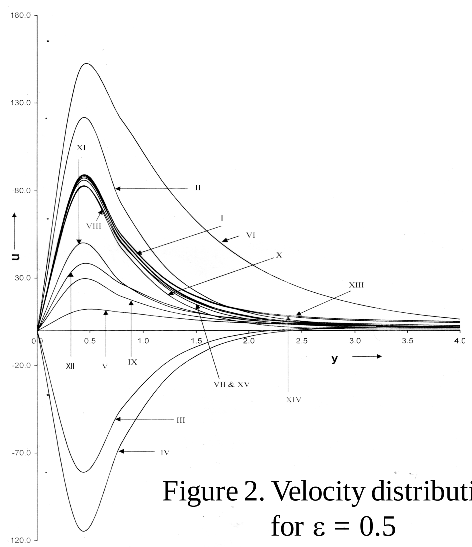

Figure 2. Velocity distribution for ε = 0.5

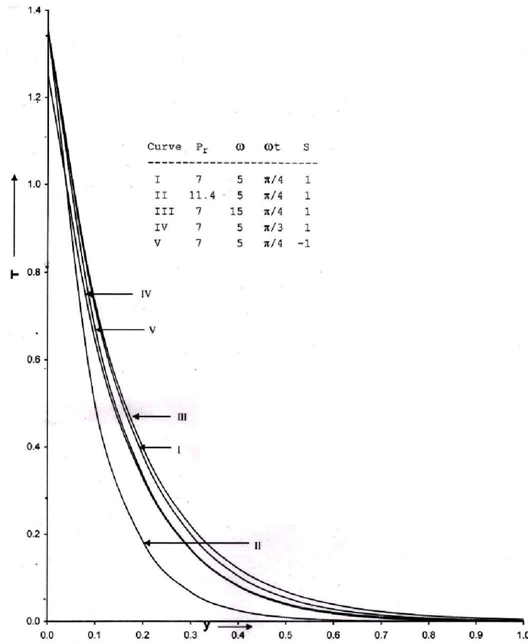

Figure 3. Temperature profile for ε = 0.5

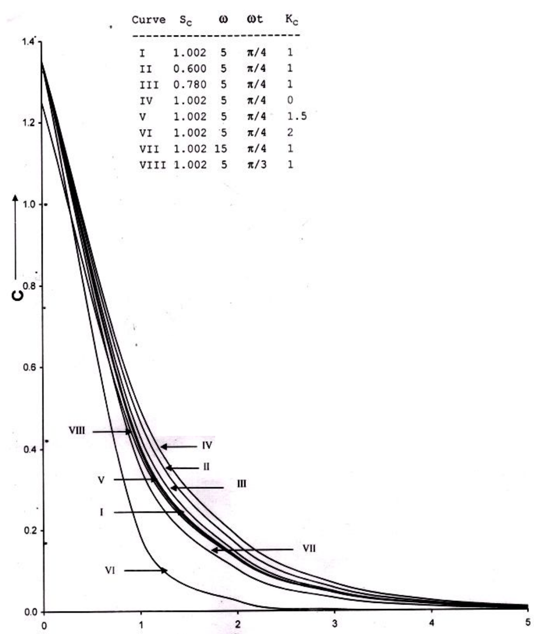

Figure 4. Concentration profile for ε = 0.5

From Table-1 it is observed that the coefficient of skin friction (τ) decreases due to higher value of Prandtl number and other parameters such as elastic parameter, chemical reaction parameterand Grashoff numbers for heat transfer but reverse effect is observed in case of Grashoff number for mass transfer, permeability parameter, magnetic parameter, Schmidt number, heat source parameter and phase angle.

Table 1 Skin friction (τ)

G_r	P_r	k_p	ω	$\omega\,\tau$	K_0	G_c	M	K_c	S_c	S	S_f
5	5	0.4	5	$\pi/4$	0.2	2	1	1	0.22	1	-45745750
7	5	0.4	5	$\pi/4$	0.2	2	1	1	0.22	1	-642-38980
-5	5	0.4	5	$\pi/4$	0.2	2	1	1	0.22	1	467.28510
-7	5	0.4	5	$\pi/4$	0.2	2	1	1	0.22	1	652.23110
5	7	0.4	5	$\pi/4$	0.2	2	1	1	0.22	1	3.68054
5	5	100	5	$\pi/4$	0.2	2	1	1	0.22	1	-402.36430
5	5	0.4	15	$\pi/4$	0.2	2	1	1	0.22	1	-373.53470
5	5	0.4	5	$\pi/3$	0.2	2	1	1	0.22	1	-329.03480
5	5	0.4	5	$\pi/4$	0	2	1	1	0.22	1	260.68100
5	5	0.4	5	$\pi/4$	0.2	5	5	1	0.22	1	-463.40480
5	5	0.4	5	$\pi/4$	0.2	2	5	1	0.22	1	-379.47400
5	5	0.4	5	$\pi/4$	0.2	2	0	1	0.22	1	-472.94450
5	5	0.4	5	$\pi/4$	0.2	2	1	-0.04	0.22	1	-454.76710
5	5	0.4	5	$\pi/4$	0.2	2	1	0	0.78	1	-455.12590
5	5	0.4	5	$\pi/4$	0.2	2	1	1	0.22	0.5	-460.36140
5	5	0.4	5	$\pi/4$	0.2	2	1	1	0.22	0.5	-85.58006

From Table-2 it is observed that the rate of heat transfer (N_u) at the surface increases in case of higher Prandtl number (P_r) and frequency of oscillation (ω) but the reverse effect is observed in case of heat source parameter (S) and phase angle (ωt).

Table 2. Nusselt number *(N_u)*

P_r	ω	$\omega\,\tau$	S	N_u
7	5	$\pi/4$	1	8.81121
11.4	5	$\pi/4$	1	14.68664
5	15	$\pi/4$	1	9.37648
5	5	$\pi/3$	1	8.34028
5	5	$\pi/4$	0	9.92890
5	5	$\pi/4$	-1	10.95107

From Table 3. it is observed that the rate of mass transfer (S_h) at the surface increases due to increase in Schmidt number, chemical reaction parameter and frequency parameter but decreases with an increase in phase angle.

Table 3. Sherwood number (S_h)

S_c	ω	$\omega\tau$	K_c	S_h
1.002	5	$\pi/4$	1	1.69103
0.600	5	$\pi/4$	1	1.17297
0.780	5	$\pi/4$	1	1.41045
1.002	5	$\pi/4$	0	1.35615
1.002	5	$\pi/4$	1.5	1.75429
1.002	5	$\pi/4$	2	1.78982
1.002	15	$\pi/4$	1	1.88002
1.002	5	$\pi/3$	1	1.52351

5. Conclusion

i) Magnetic field, chemical reaction parameter and heat source parameter in case of high Prandtl number fluid have a retarding effect on the velocity.

ii) Flow characteristic in the present study is more dependent on mass buoyancy effect (G_c) rather than thermal buoyancy effect (G_r).

iii) Thinning of thermal boundary layer occurs for higher Prandtl number fluid.

iv) The higher rate of mass transfer is experienced in case of heavier species in the presence of exothermic reaction.

v) The elapse of time contributes to extend the domain of velocity boundary layer and reduce the thermal bounding layer.

Appendix

$$m_1 = -\left(\frac{P_r + \sqrt{P_r^2 - 4P_r S}}{2}\right), \quad m_2 = -\left(\frac{P_r + \sqrt{P_r^2 - 4P_r\left(S - \frac{i\omega}{4}\right)}}{2}\right),$$

$$m_3 = -\left(\frac{S_c + \sqrt{S_c^2 + 4K_c S_c}}{2}\right), \quad m_4 = -\left(\frac{S_c + \sqrt{S_c^2 + 4S_c\left(K_c + \frac{i\omega}{4}\right)}}{2}\right),$$

$$m_5 = -\left(\frac{1+\sqrt{1+4\left(M^2 + \frac{1}{k_p}\right)}}{2}\right), \quad m_6 = -\left(\frac{1+\sqrt{1+4\left(M^2 + \frac{1}{k_p}\right)+\frac{i\omega}{4}}}{2}\right),$$

$$A_1 = m_1^2 + m_1 - \left(M^2 + \frac{1}{k_p}\right), \quad A_2 = m_3^2 + m_3 - \left(M^2 + \frac{1}{k_p}\right),$$

$$A_3 = \frac{G_r}{A_1}, \quad A_4 = \frac{G_c}{A_2}, \quad A_5 = m_5^2 + m_5 - \left(M^2 + \frac{1}{k_p}\right)$$

$$A_6 = \frac{(1 - A_3 - A_4)m_5^3}{A_1}, \quad A_7 = \frac{A_3 m_1^3}{A_1}, \quad A_8 = \frac{A_4 m_3^3}{A_2} \quad A_9 = -(A_7 + A_8),$$

$$A_{10} = \frac{-G_r\left(1 + \frac{4i}{\omega} m_1\right)}{m_2^2 + m_2 - \left(M^2 + \frac{1}{k_p} + \frac{i\omega}{4}\right)}, \quad A_{11} = \frac{G_r m_1 \frac{4i}{\omega}}{A_1 - \frac{i\omega}{4}}, \quad A_{12} = \frac{-G_c\left(1 + \frac{4i}{\omega} m_3\right)}{m_4^2 + m_4 - \left(M^2 + \frac{1}{k_p} + \frac{i\omega}{4}\right)}$$

$$A_{13} = \frac{G_c m_3 \frac{4i}{\omega}}{A_2 - \frac{i\omega}{4}}, A_{14} = -\frac{(A_3 + A_4 - 1)m_5}{A_5 - \frac{i\omega}{4}} \quad A_{15} = \frac{A_3 m_1}{A_1 - \frac{i\omega}{4}}, A_{16} = \frac{A_4 m_3}{A_2 - \frac{i\omega}{4}}, A_{17} = A_{11} + A_{15},$$

$A_{18} = A_{13} + A_{16}$, $A_{19} = -(1 + A_{17} + A_{18} + A_{10} + A_{14} + A_{12})$

$$A_{20} = m_6^2 + m_6 - \left(M^2 + \frac{1}{k_p}\right), \quad A_{21} = m_2^2 + m_2 - \left(M^2 + \frac{1}{k_p}\right), A_{22} = m_4^2 + m_4 - \left(M^2 + \frac{1}{k_p}\right)$$

$$A_{23}=\frac{\left(\frac{i\omega}{4}m_6^2-m_6^3\right)A_{19}}{A_{20}-\frac{i\omega}{4}},\ A_{24}=\frac{\left(\frac{i\omega}{4}m_1^2-m_1^3\right)A_{17}}{A_1-\frac{i\omega}{4}}\ \ A_{25}=\frac{\left(\frac{i\omega}{4}m_3^2-m_3^3\right)A_{18}}{A_2-\frac{i\omega}{4}},\ A_{26}=\frac{\left(\frac{i\omega}{4}m_2^2-m_2^3\right)A_{10}}{A_{21}-\frac{i\omega}{4}}$$

$$A_{27}=\frac{\left(\frac{i\omega}{4}m_5^2-m_5^3\right)A_{14}}{A_5-\frac{i\omega}{4}},\ A_{28}=\frac{\left(\frac{i\omega}{4}m_4^2-m_4^3\right)A_{12}}{A_{22}-\frac{i\omega}{4}}\ \ A_{29}=\frac{-\left(A_3+A_4-1\right)m_5^3}{A_5-\frac{i\omega}{4}},\ A_{30}=\frac{A_3m_1^3}{A_1-\frac{i\omega}{4}}$$

$$A_{31}=\frac{A_4m_3^3}{A_2-\frac{i\omega}{4}},\ A_{32}=\frac{-A_9m_5}{A_5-\frac{i\omega}{4}},\ A_{33}=-\frac{A_7m_1}{A_1-\frac{i\omega}{4}},\ A_{34}=-\frac{A_8m_3}{A_2-\frac{i\omega}{4}}$$

$A_{35} = A_{27} + A_{29} + A_{32}$, $A_{36} = A_{25} + A_{31} + A_{34}$

$A_{37} = A_{22} + A_{30} + A_{33}$, $A_{38} = -(A_{35} + A_{28} + A_{36} + A_{26} + A_{37})$

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