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Generalization of the Fibonacci Sequence in Case of Third Order Recurrence Relation

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Abstract

In this paper we generate pair of integer sequences using third order recurrence relation

$$p_{n+3} = q_{n+2} + q_{n+1} + q_n \qquad n \geq 0$$
$$q_{n+3} = p_{n+2} + p_{n+1} + p_n \qquad n \geq 0$$

This process of constructing two sequences $\{p_i\}_{i=0}^{\infty}$ and $\{q_i\}_{i=0}^{\infty}$ is called 2-Fibonacci sequences[5].

Key words: Fibonacci sequence, recurrence relation.

The process of construction of the Fibonacci numbers is a sequential process[1,2]. Atanassov K.[3,4] consider two infinite sequence $\{a_n\}$ and $\{b_n\}$ which have given initial values a_1, a_2 and b_1, b_2. Sequences $\{a_n\}$ and $\{b_n\}$ are generated for every natural number $n \geq 2$ by the coupled equations,

$$a_{n+2} = b_{n+1} + b_n; b_{n+2} = a_{n+1} + a_n$$

In this paper we consider two infinite sequence $\{p_i\}_{i=0}^{\infty}$ and $\{q_i\}_{i=0}^{\infty}$ which have given three initial values a, c, e and b, d, f (which are real numbers). Sequences $\{\alpha_i\}_{i=0}^{\infty}$ and $\{\beta_i\}_{i=0}^{\infty}$ are generated for every natural numbers $n \geq 3$ by the coupled equations

$$p_{n+3} = q_{n+2} + q_{n+1} + q_n \qquad n \geq 0$$
$$q_{n+3} = p_{n+2} + p_{n+1} + p_n \qquad n \geq 0$$

If we set a = b, c = d, e = f then the sequence $\{p_i\}_{i=0}^{\infty}$ and $\{q_i\}_{i=0}^{\infty}$ will coincide with each other and with the sequence $\{F_i\}_{i=0}^{\infty}$, which is a generalized Fibonacci sequence.

where, $F_o(a,c,e)=a$, $F_1(a,c,e)=c$, $F_2(a,c,e)=e$,
$F_{n+3}(a,c,e)=F_{n+2}(a,c,e)+F_{n+1}(a,c,e)+F_n(a,c,e)$

2. The 2f-Sequences :

We are constructing two sequences $\{p_i\}_{i=0}^{\infty}$ and $\{q_i\}_{i=0}^{\infty}$ by the following way –

$p_0 = a,\ p_1 = c,\ p_2 = e;\quad q_o = b,\ q_1 = d,\ q_2 = f$

$p_{n+3} = q_{n+2} + q_{n+1} + q_n \quad n \geq 0$

$$q_{n+3} = p_{n+2} + p_{n+1} + p_n \quad n \geq 0 \qquad (2.1)$$

where, a,b,c,d,e,f are real numbers.

First we shall study the properties of the sequence $\{p_i\}_{i=0}^{\infty}$ and $\{q_i\}_{i=0}^{\infty}$ defined by equation (2.1). The first ten terms of the sequences defined in equation (2.1) are shown in table below[5] :

n	p_n	q_n
0	a	b
1	c	d
2	e	f
3	b + d + f	a + c + e
4	a + c + e + f + d	b + c + d + e + f
5	a + b + 2c + d + 2f + 2e	a + b + c + 2d + 2e + 2f
6	2a + 2b + 3c + 3d + 4e + 3f	2a + 2b + 3c + 3d + 3e + 4f
7	3a + 4b + 5c + 6d + 6e + 7f	4a + 3b + 6c + 5d + 7e + 6f
8	7a + 6b + 10c + 10d + 12e + 12f	6a + 7b + 10c + 10d + 12e + 12f
9	12a + 12b + 18c + 18d + 22e + 22f	12a + 12b + 18c + 19d + 21e + 22f

Theorem 1 : For every integer $n \geq 0$

(a) $p_{5.n} + q_1 = q_{5.n} + p_1$

(b) $p_{5.n+1} + q_2 = q_{5.n+1} + p_2$

(c) $p_{5.n+2} + q_3 = q_{5.n+2} + p_3$

(d) $p_{5.n+3} + q_4 = q_{5.n+3} + p_4$

We prove the above results by induction hypothesis.

Proof (a) Assume that the result is true for some integer $n \geq 1$.

Now by equation (2.1) we can write –

$p_{5.n+5} + q_1 = q_{5.n+4} + q_{5.n+3} + q_{5.n+2} + q_1$
$= p_{5.n+3} + p_{5.n+2} + p_{5.n+1} + q_{5.n+3} + q_{5.n+2} + q_1$

$$= p_{5.n+3} + p_{5.n+2} + q_{5.n+3} + q_{5.n+2} + p_{5.n+1} + q_1$$
$$= p_{5.n+3} + p_{5.n+2} + q_{5.n+3} + q_{5.n+2} + q_{5.n+1} + p_1 \quad \text{(by ind. hyp.)}$$
$$= p_{5.n+3} + p_{5.n+2} + p_{5.n+4} + p_1 \text{(By eq. 2.1)}$$
$$= p_{5.n+4} + p_{5.n+3} + p_{5.n+2} + p_1$$
$$= q_{5.n+5} + p_1 \quad \text{(By eq. 2.1)}$$

Hence the result is true for all integers $n \geq 0$.

Proof (b) : Assume that the result is true for some integer $n \geq 1$.

Now by equation (2.1) we can write –

$$p_{5.n+6} + q_2 = q_{5.n+5} + q_{5.n+4} + q_{5.n+3} + q_2$$
$$= p_{5.n+4} + p_{5.n+3} + p_{5.n+2} + q_{5.n+4} + q_{5.n+3} + q_2$$
$$= p_{5.n+4} + p_{5.n+3} + q_{5.n+4} + q_{5.n+3} + p_{5.n+2} + q_2$$
$$= p_{5.n+4} + p_{5.n+3} + q_{5.n+4} + q_{5.n+3} + q_{5.n+2} + p_2 \quad \text{(by ind. hyp.)}$$
$$= p_{5.n+4} + p_{5.n+3} + p_{5.n+5} + p_2 \text{ (By eq. 2.1)}$$
$$= p_{5.n+5} + p_{5.n+4} + p_{5.n+3} + p_2$$
$$= q_{5.n+6} + p_2 \quad \text{(By eq. 2.1)}$$

Hence the result is true for all integers $n \geq 0$.

Some results for particular value of sequences $\{\alpha_i\}$ and $\{\beta_i\}$ defined in equation (2.1).

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