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Investigation on Weak form of Generalized Closed sets in Ideal Topological Spaces

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Abstract

This paper proposes a new class of closed sets in ideal spacescalled $I_{\tilde{\tilde{g}}s}$-closed set and shows that this class of closed sets lies betweenthe class of I_g-closed sets and the class of sgI-closed sets. Furthermore, it shows that the necessary and sufficient conditions for a set to be a $I_{\tilde{\tilde{g}}s}$-closed set and investigatesthe properties of $I_{\tilde{\tilde{g}}s}$-open sets in idealtopological spaces.

Key words: A_s^*, $I_{\tilde{\tilde{g}}s}$-closed set, I_g-closed set, I_{gs}-closed sets, semi-⋆-closed set, sgI-closed sets.

2010 Mathematics subject classification: 54A05, 54D15.

1. Introduction

The concept of ideals in topological spaces was initiated byKuratowski[7] and Vaidyanathaswamy[14]. In 1990, Jankovic and Hamlett[4] also studied the properties of ideal topological spaces. After that I_g-closedset was introduced by Noiri *et al.*[6,11]. Also Khan and Noiri[5] introducedand studied the properties of sgI-closed sets in ideal topological spaces. Weakform of open sets called semi-open sets and also the first step of generalizingclosed sets was done by Levine[9,10]. Recently, Jayaparthasarathy *et al.*[8] introduced a new class of closed sets called $\tilde{\tilde{g}}s$-closed sets and its propertiesalso discussed. In this paper we introduce and characterize the properties of $I_{\tilde{\tilde{g}}s}$-closed set, and this class of closed sets lies between the classes of I_g-closedsets and the class of sgI-closed sets. We also introduce $I_{\tilde{g}s}$-open sets andprove the necessary and sufficient conditions of $I_{\tilde{g}s}$-open sets. Further, weinvestigate the relationship between $I_{\tilde{g}s}$-closed sets and other existing sets inideal topological spaces.

2. Preliminaries :

This section discuss some basic properties about ideal topological spaces and weak form of open sets in topological spaces which are useful in sequel.

Definition 2.1[9] Let (X, τ) be a topological space. A subset A of X is called semi-open if $A \subseteq Cl(Int(A))$. The complement of semi-open set iscalled semi-closed.

Definition 2.2 A subset A of a topological space (X, τ) is called a

i) generalized closed (briefly g-closed) set[9] if $Cl(A) \subseteq U$ wheneverand $A \subseteq U$ and U is open in (X, τ).
ii) semi-generalized closed (briefly sg-closed) set[3] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ).
iii) $\hat{g}$-closed set (= ω-closed)[12] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U issemi-open in (X, τ).
iv) $*g$-closed set[13] if $Cl(A) \subseteq U$ whenever and U is $\hat{g}$-open in(X, τ).
v) #g-semi-closed set (briefly #gs-closed)[13] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is $*g$-open in (X, τ).
vi) $\tilde{g}s$-closed set[13] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is #gs-openin (X, τ).
vii) $\tilde{\tilde{g}}s$-closed set[8] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tilde{g}s$-openin (X, τ).

The complement of the above mentioned sets are called their respective generalized open sets.

Definition 2.3[7,14] An idealIon a topological space (X, τ) is a non-emptycollection of subsets ofXsatisfying the following two conditions:

(i) If A∈I and B⊂A, then B∈I

(ii) If A∈I and B∈I, then A∪B∈I.

Let (X, τ) be a topological space andIan ideal of subsets of X. An idealtopological space is a topological space (X, τ)with an idealIonXand is denoted by (X, τ, I).

Definition 2.4[1,4] For a subset A of X,

(i) A*(I, τ) = {x∈X: U∩A∉I for every U∈ τ(X, x)} is called the local function of A with respect to ideal I and topology τ, where τ (X, x) ={U∈τ: x∈U}.

(ii) A_s^*(I, τ) ={x ∈ X: U ∩ A ∉ I for every U∈SO(X, x)} is called the semi-local function of A with respect to ideal I and topology τ, where SO(X,x)={U∈SO(X): x∈U}.

In [1,4] very ideal topological space (X, τ, I), there exists a topology τ* finer than τ, defined by τ*={U⊆X:Cl*(X–U)=X–U} which is generated by the base β(I, τ) ={U–J: U∈τ andJ∈I} and $Cl^*(A) = A \cup A^*$ is a Kuratowski closure operator for the topology τ^*. A subset A of an ideal topological space (X, τ, I) is said to be ⋆-closed[4] if $A^* \subseteq A$ and $Int^*(A)$ denote the interior of the set A in (X, τ^*, I). Also we define Cl_s^* [5] by $Cl_s^*(A)$ $A \cup A_s^*$ and there exists a topology $\tau_s^*(I)$ finer than τ and τ^*, defined by $\tau_s^*(I) = \{U \subseteq X: Cl_s^*(X-U) = X-U\}$ which is generated by the base $\beta_s(I, \tau) = \{U-J: U \in \tau$ and $J \in I\}$. A subset A of an ideal topological space (X, τ, I) is said to be semi-⋆-closed[5] if $A_s^* \subseteq A$ and is said to be ⋆-semi dense[5] if $A \subseteq A_s^*$. Also note that[1] $A_s^* = sCl(A_s^*) \subseteq A^* \subseteq Cl_s^*(A)$ and $Int_s^*(A)$ denote the interior of the set A in (X, τ_s^*,I).

Definition 2.5 A subsetAof an ideal topological space (X, τ, I) is said to be

(i) Igeneralized closed (briefly Ig-closed)[6] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(ii) semi generalized I (briefly sgI)-closed[5] if $A_s^* \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

(iii) $I_{\hat{g}}$-closed[2] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

Theorem 2.6[5] Let $\{A_a : a \in \Omega\}$ be a locally finite family of sets of an ideal space (X, τ, I). Then $\bigcup_{a\in\Omega}(A_a)_s^* = (\bigcup_{a\in\Omega} A_a)_s^*$.

Theorem 2.7[5] Let (X, τ, I) be an ideal space and $A \subseteq X$. If $A \subseteq B \subseteq A_s^*$, then $A_s^* = B_s^*$ and Bis ⋆-semi dense in itself.

Theorem 2.8[5] Let AandBbe two subsets of an ideal space (X, τ, I). Then $(A \cap B)_s^* \subseteq A_s^* \cup B_s^*$.

3. Weak open sets in ideal topological spaces :

This section introduces a weak form of closed sets in ideal spacesand investigates some properties of it.

Definition 3.1 A subsetAof an ideal topological space (X, τ, I) is saidto be $I_{\tilde{\tilde{g}}s}$-closed if $A_s^* \subseteq U$ whenever $A \subseteq U$ and U is $\tilde{\tilde{g}}s$-open in X. The complement of $I_{\tilde{\tilde{g}}s}$-closed set is called $I_{\tilde{\tilde{g}}s}$-open set.

Example 3.2 Let X = {a, b, c, d}, τ O(X) ={ϕ, X,{a},{b, c},{a, b, c}} and I={ϕ, {a}}. Then $I_{\tilde{\tilde{g}}s}$-closed sets are ϕ, X, {a}, {d}, {a, d}, {b, c},{b, d},{c, d},{a, b, c}, {a, b, d}, {a, c, d}and{b, c, d}.

Remark 3.3 In an ideal topological space (X, τ, I), then the following are trivially true.

(i) If $A_s^* = \emptyset$ for every $A \subseteq X$, then A is $I_{\tilde{\tilde{g}}s}$-closed.

(ii) A_s^* is an $I_{\tilde{\tilde{g}}s}$-closed for every $A \subseteq X$.

Theorem 3.4 Let (X, τ, I) be an ideal topological space. Then the following are true.

(i) Every I_g-closed set is $I_{\tilde{\tilde{g}}s}$-closed, but not converse.

(ii) Every $I_{\hat{g}}$-closed set is $I_{\tilde{\tilde{g}}s}$-closed, but not converse.

(iii) Every $*$-closed set is $I_{\tilde{\tilde{g}}s}$-closed, but not converse.
(iv) Every semi-$*$-closed set is $I_{\tilde{\tilde{g}}s}$-closed, but not converse.
(v) Every $I_{\tilde{\tilde{g}}s}$-closed is sgI, but not converse.
(vi) If $A \subseteq X$ is an element of I, then A is $I_{\tilde{\tilde{g}}s}$-closed.

Proof :

(i) Let A be an I_g-closed set and let $A \subseteq U$ where U is open set. Since $A_s^* \subseteq A^* \subseteq U$ and every open set is $\tilde{g}s$-open, then A is $I_{\tilde{\tilde{g}}s}$-closed.
(ii) Proof is similar to part (i).
(iii) Let A be a$*$-closed set, then $A^* \subseteq A$. Let $A \subseteq U$ where U is $\tilde{g}s$-open. Hence $A_s^* \subseteq U$, whenever $A \subseteq U$ and U is $\tilde{g}s$-open.
(iv) Proof is trivially by part (i) and part (iii).
(v) Let A be an $I_{\tilde{\tilde{g}}s}$-closed and $A \subseteq U$ where U is semi-open. Since every semi-open set is $\tilde{g}s$-open, then we have, $A_s^* \subseteq U$, whenever $A \subseteq U$ and U is semi-open.
(vi) Let $A \subseteq U$ where U is $\tilde{g}s$-open. If $A \in I$, then $A_s^* = \emptyset \subseteq A \subseteq U$. Hence A is $I_{\tilde{\tilde{g}}s}$-closed.

Example 3.5 Consider X={a, b, c}, τ O(X) ={∅, X,{a},{b},{a, b}}and I={∅,{a}}. Then {b} is $I_{\tilde{\tilde{g}}s}$-closed set, but it is not I_g-closed and $I_{\hat{g}}$-closed. Again consider X={a, b, c},τ O(X) ={∅, X,{a}} and I={∅,{a},{b},{a, b}}. Then {a, c} is $I_{\tilde{\tilde{g}}s}$-closed set, but it is not $*$-closed. Also consider X = {a, b, c, d}, τ O(X) = {∅, X,{a},{b, c},{a, b, c}} and I ={∅,{a}}. Then {a, b, d} is $I_{\tilde{\tilde{g}}s}$-closed set, but it is not semi-$*$-closed. Also {a, b} is sgI-closed set, but it is not $I_{\tilde{\tilde{g}}s}$-closed set.

Theorem 3.6 Let (X, τ, I) be an ideal topological space and A⊆X. Then the following statements are equivalent:
(i) A is $I_{\tilde{\tilde{g}}s}$-closed.
(ii) $Cl_s^*(A) \subseteq U$ for every $\tilde{g}s$-open set U containing A.
(iii) For all $x \in Cl_s^*(A)$, $sCl(\{x\}) \cap A \neq \emptyset$.
(iv) $Cl_s^*(A) - A$ contains no non empty $\tilde{g}s$-closed set.
(v) $A_s^* - A$ contains no non empty $\tilde{g}s$-closed set.

Proof **(i) ⇒ (ii)**: Let A be an $I_{\tilde{\tilde{g}}s}$-closed set. Then $A_s^* \subseteq U$ whenever $A \subseteq U$ and U is $\tilde{g}s$-open in X and implies $Cl_s^*(A) = A \cup A_s^* \subseteq U$ whenever $A \subseteq U$ and U is $\tilde{g}s$-open in X.

(ii) ⇒ (iii): Let $x \in Cl_s^*(A)$ and suppose $sCl(\{x\}) \cap A = \emptyset$. Then $A \subseteq X - sCl(\{x\})$, where $X - sCl(\{x\})$ is semi-open set. Since every semi-openset is $\tilde{g}s$-open and by hypothesis, $Cl_s^*(A) \subseteq X - sCl(\{x\})$. This contradicts the fact that $x \in Cl_s^*(A)$. Hence, $sCl(\{x\}) \cap A \neq \emptyset$.

(iii)⇒(iv): Suppose $F \subseteq Cl_s^*(A) - A$, where F is a $\tilde{g}s$-closed set containinga pointx. Since $F \subseteq X - A$ and $\{x\} \subseteq F$, we have $sCl(\{x\}) \subseteq F$ and $sCl(\{x\}) \cap A = \emptyset$. Since $x \in Cl_s^*(A)$ and by hypothesis, $sCl(\{x\}) \cap A \neq \emptyset$. This is a contradiction.

(iv)⇒(v): Since $Cl_s^*(A) - A = (A \cup A_s^*) - A = (A \cap A^c) \cup (A_s^* \cap A^c) = A_s^* \cap A^c = A_s^* - A$. Therefore $A_s^* - A$ contains no non empty $\tilde{g}s$-closed set.

(v)⇒(i): Let $A \subseteq U$ and U be a $\tilde{g}s$-open. Therefore $X - U \subseteq X - A$ and $A_s^* \cap (X - U) \subseteq A_s^* \cap (X - A) = A_s^* - A$. Therefore, $A_s^* \cap (X - U) \subseteq A_s^* - A$. Since A_s^* is semi-closed set, so $A_s^* \cap (X - U)$ is a $\tilde{g}s$-closed set contained in $A_s^* - A$. Therefore $A_s^* \cap (X - U) = \emptyset$ and hence $A_s^* \subseteq U$.

Theorem 3.7 Let (X, τ, I) be an ideal topological space and $A \subseteq X$. Then the following statements are equivalent:

(i) A is $I_{\tilde{g}s}$-closed.
(ii) $sCl(A_s^*) \subseteq U$ for every $\tilde{g}s$-open set U containing A.
(iii) For all $x \in sCl(A_s^*)$, $sCl(\{x\}) \cap A \neq \emptyset$.
(iv) $sCl(A_s^*) - A$ contains no non empty $\tilde{g}s$-closed set.
(v) $A_s^* - A$ contains no non empty $\tilde{g}s$-closed set.

Proof (i)⇒(ii): Let A be an $I_{\tilde{g}s}$-closed set. Then $A_s^* \subseteq U$ whenever $A \subseteq U$ and U is $\tilde{g}s$-open in X. By Definition 2.4, we have the desired result.

(ii)⇒(iii): Let $x \in sCl(A_s^*)$ and suppose $sCl(\{x\}) \cap A = \emptyset$. Then $A \subseteq X - sCl(\{x\})$, where $X - sCl(\{x\})$ is semi-open set. Since every semi-openset is $\tilde{g}s$-open and by hypothesis, $sCl(A_s^*) \subseteq X - sCl(\{x\})$,this contradictsthe fact that $x \in sCl(A_s^*)$. Hence $sCl(\{x\}) \cap A \neq \emptyset$.

(iii)⇒(iv): Suppose $F \subseteq sCl(A_s^*) - A$, where F is a $\tilde{g}s$-closed set containinga point x. Since $F \subseteq X - A$ and $\{x\} \subseteq F$, we have $sCl(\{x\}) \subseteq F$ and $sCl(\{x\}) \cap A = \emptyset$. Since $x \in sCl(A_s^*)$ and by hypothesis, $sCl(\{x\}) \cap A \neq \emptyset$. This is a contradiction.

(iv)⇒(v): Assume that $F \subseteq A_s^* - A$, where F is $\tilde{g}s$-closed set and $F \neq \emptyset$. This gives $F \subseteq sCl(A_s^*) - A$. This contradicts the hypothesis.

(v)⇒(i): Let $A \subseteq U$ and U be a $\tilde{g}s$-open. Therefore $X - U \subseteq X - A$ and $A_s^* \cap (X - U) \subseteq A_s^* \cap (X - A) = A_s^* - A$. Therefore $A_s^* \cap (X - U) \subseteq A_s^* -$A. Since A_s^* is semi-closed set, so $A_s^* \cap (X - U)$ is a $\tilde{g}s$-closed set contained in $A_s^* -$A. Therefore $A_s^* \cap (X - U) = \emptyset$ and hence $A_s^* \subseteq U$.

Theorem 3.8 Let (X, τ, I) be an ideal topological space. Then every $I_{\tilde{g}s}$-closed and $\tilde{g}s$-open set is semi-⋆-closed.

Proof By hypothesis, $A_s^* \subseteq A$ whenever $A \subseteq A$ and A is $\tilde{g}s$-open set. Hence A is semi-⋆-closed.

Theorem 3.9 Let $\{A_i : i \in \Omega\}$ be a locally finite family of $I_{\tilde{g}s}$-closed sets of an ideal topological space (X, τ, I). Then $\bigcup_{i\in\Omega} A_i$ is $I_{\tilde{\tilde{g}}s}$-closedset.

Proof Let $\bigcup_{i\in\Omega} A_i \subseteq U$ where U is $\tilde{g}s$-open. Since each A_i is $I_{\tilde{\tilde{g}}s}$-closed set, then for eachi i ∈ Ω, $(A_i)_s^* \subseteq U$ and $\bigcup_{i\in\Omega} (A_i)_s^* \subseteq U$. By theorem 2.6, $(\bigcup_{i\in\Omega} A_i)_s^* \subseteq U$ and hence $\bigcup_{i\in\Omega} A_i$ is $I_{\tilde{\tilde{g}}s}$-closed set.

Example 3.10 Intersection of two $I_{\tilde{\tilde{g}}s}$-closed sets need not be $I_{\tilde{\tilde{g}}s}$-closed. Consider X={a, b, c, d}, τ O(X) = {∅, X,{a},{b, c}, {a, b, c}} and I ={∅,{a}}. Then {b, c} and {c, d}are $I_{\tilde{\tilde{g}}s}$-closed sets, but {b, c} ∩ {c, d} = {c} is not a $I_{\tilde{\tilde{g}}s}$-closed set.

Theorem 3.11 LetAbe an $I_{\tilde{\tilde{g}}s}$-closed set andB be $\tilde{g}s$-closed set of an ideal topological space (X, τ, I). Then $A\cap B$ is $I_{\tilde{\tilde{g}}s}$-closedset.

Proof Let U be a $\tilde{g}s$-open subset of X containing $A\cap B$. Then $A\subseteq U\cup(X-B)$. Since A is $I_{\tilde{\tilde{g}}s}$-closed, then $A_s^* \subseteq U \cap (X-B)$ and $B \cap A_s^* \subseteq U$. By theorem 2.8, $(A\cap B)_s^* \subseteq A_s^* \cap B_s^* \subseteq A_s^* \cap B \subseteq$, because B is $\tilde{g}s$-closed. Thus $A\cap B$ is $I_{\tilde{\tilde{g}}s}$-closed.

Theorem 3.12 Let (X, τ, I) be an ideal topological space and A⊆X. Then A is $I_{\tilde{\tilde{g}}s}$-closed if and only if $A=F–N$ where F is semi-⋆-closed and N contains no nonempty $\tilde{g}s$-closed set.

Proof If A is $I_{\tilde{\tilde{g}}s}$-closed, then by theorem 3.7, $N=A^*_s$-A contains no non-empty $\tilde{g}s$-closed set. If $F = Cl_s^*(A)$, then F is semi-⋆-closed such that $F - N = (A \cup A_s^*) - (A_s^* - A) = (A \cup A_s^*) \cap (A_s^* \cap A^c)^c = (A \cup A_s^*) \cap ((A_s^*)^c \cup A) = (A \cup A_s^*) \cap (A \cup (A_s^*)^c) = A \cup (A_s^* \cap (A_s^*)^c) = A$.
Conversely, suppose $A=F–N$ where F is semi-⋆-closed and Ncontains no nonempty $\tilde{g}s$-closed set. Let U be a $\tilde{g}s$-open set such that $A\subseteq U$. Then $F–N \subseteq U$ implies $F \cap (X-U) \subseteq N$. Now $A \subseteq F$ and $F_s^* \subseteq F$ then $A_s^* \subseteq F_s^*$ and so $A_s^* \cap (X-U) \subseteq F_s^* \cap (X-U) \subseteq F \cap (X-U) \subseteq N$. By hypothesis, since $A_s^* \cap (X-U)$ is $\tilde{g}s$-closed, $A_s^* \cap (X-U) = \emptyset$ and so $A_s^* \subseteq U$. Hence A is $I_{\tilde{\tilde{g}}s}$-closed.

Theorem 3.13 Let (X, τ, I) be an ideal topological space and A and B be subsets of X such that $A \subseteq B \subseteq sCl(A_s^*)$ and A is $I_{\tilde{\tilde{g}}s}$-closed, then B is $I_{\tilde{\tilde{g}}s}$-closed.

Proof Since A is $I_{\tilde{\tilde{g}}s}$-closed, then by theorem 3.7, sCl(A⋆s)–A contains no nonempty $\tilde{g}s$-closed set. Since sCl(B_s^*)–B⊆sCl(A_s^*)–A, sCl(B_s^*)–B contains no nonempty $\tilde{g}s$-closed set, by theorem 3.7. Hence B is $I_{\tilde{\tilde{g}}s}$-closed.

Theorem 3.14 Let (X, τ, I) be an ideal topological space. Then every $\tilde{g}s$-closed is $I_{\tilde{\tilde{g}}s}$-closed, but not conversely.

Proof Assume A is $\tilde{g}s$-closed and since $sCl(A_s^*) \subseteq sCl(A)$, then $sCl(A_s^*) \subseteq sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tilde{g}s$-open. By theorem 3.7, we get thedesired result.

Example 3.15 Consider X={a, b, c}, τ O(X) = {∅, X, {a},{b},{a, b}} and I={∅,{a}}. Then {a, b} is $I_{\tilde{g}s}$-closed set, but it is not $\tilde{g}s$-closed.

Theorem 3.16 Let (X, τ, I) be an ideal topological space and $A \subseteq X$ is $I_{\tilde{g}s}$-closed and ⋆-semi dense set in itself, then A is $\tilde{g}s$-closed.

Proof SupposeA is $I_{\tilde{g}s}$-closed and ⋆-semi dense set. Let $A \subseteq U$ where U is $\tilde{g}s$-open, then theorem 3.7 (ii), $sCl(A_s^*) \subseteq U$. Since Ais ⋆-semi dense initself, $A \subseteq A_s^*$ and implies $sCl(A) \subseteq sCl(A_s^*) \subseteq U$ whenever $A \subseteq U$ and U is $\tilde{g}s$-open. Hence Ais $\tilde{g}s$-closed.

Theorem 3.17 Let (X, τ, I) be an ideal space where I={∅}. Then A is $I_{\tilde{g}s}$-closed if and only if A is $\tilde{g}s$-closed.

Proof Since I={∅}, $A_s^* = Cl(A) \supseteq A$. This implies Ais ⋆-semi dense and so it is $\tilde{g}s$-closed. Converse is theorem 3.14.

Theorem 3.18 Let (X, τ, I) be an ideal topological space and A and B be subsets of X such that $A \subseteq B \subseteq A_s^*$ and A is $I_{\tilde{g}s}$-closed. Then A and B are $\tilde{g}s$-closed.

Proof Let A and B be subsets of X such that $A \subseteq B \subseteq A{*}s$ and A is $I_{\tilde{g}s}$-closed. Since $A \subseteq B \subseteq A_s^* \subseteq sCl(A_s^*)$, by Theorem 3.13, we get B is $I_{\tilde{g}s}$-closed. Since $A \subseteq B \subseteq A_s^*$, then $B_s^* = A_s^*$. Then Aand B are ⋆-semi dense and both are $\tilde{g}s$-closed, by theorem 3.16.

Theorem 3.19 Let (X, τ, I) be an ideal topological space and $A \subseteq X$. Then A is $I_{\tilde{g}s}$-closed if and only if $A \cup (X - A_s^*)$ is $I_{\tilde{g}s}$-closed.

Proof Assume A is $I_{\tilde{g}s}$-closed. Let U be a $\tilde{g}s$-open set such that $(A \cup (X - A_s^*)) \subseteq U$. Then $X - U \subseteq X - (A \cup (X - A_s^*)) = A_s^* - A$. Since A is $I_{\tilde{g}s}$-closed, by Theorem 3.7 (v) $A_s^* - A$ contains no non-empty $\tilde{g}s$-closed set. This implies $X - U = \emptyset$ or $X = U$. Thus X is the only $\tilde{g}s$-open set containing $A \cup (X - A_s^*)$. Thus $[A \cup (X - A_s^*)]_s^* \subseteq X$. Thus $A \cup (X - A_s^*)$ is $I_{\tilde{g}s}$-closed. Conversely, assume $A \cup (X - A_s^*)$ is $I_{\tilde{g}s}$-closed and let U be a $\tilde{g}s$-open set such that $A \subseteq U$. Let F be any $\tilde{g}s$-closed set such that $F \subseteq A_s^* - A$. Since $A_s^* - A = X - (A \cup (X - A_s^*))$, we have $A \cup (X - A_s^*) \subseteq X - F$ and since $X–F$ is $\tilde{g}s$-open. Therefore, $(A \cup (X - A_s^*))_s^* = A_s^* \cup (X - A_s^*)_s^* \subseteq X - F$ and hence $F \subseteq X - A_s^*$. But $F \subseteq A_s^* - A$ implies $F = \emptyset$. Thus $A_s^* - A$ contains no non empty $\tilde{g}s$-closed set. By theorem 3.7(v), A is $I_{\tilde{g}s}$-closed.

Theorem 3.20 Let (X, τ, I) be an ideal space and $A \subseteq X$. Then $A \cup (X - A_s^*)$ is $I_{\tilde{\tilde{g}}s}$-closed if and only if $A_s^* - A$ is $I_{\tilde{\tilde{g}}s}$-closed.

Proof Proof is trivially from the fact that $X - (A_s^* - A) = A \cup (X - A_s^*)$.

Theorem 3.21 Let (X, τ, I) be an ideal topological space and if every $\tilde{g}s$-open set in X is semi-⋆-closed. Then every subset of X is $I_{\tilde{\tilde{g}}s}$-closed.

Proof Assume that every $\tilde{g}s$-open set is semi-⋆-closed and let $A \subseteq X$ and U be $\tilde{g}s$-open set such that $A \subseteq U$. Then $A_s^* \subseteq U_s^* \subseteq U$. Hence Ais $I_{\tilde{\tilde{g}}s}$-closed.

Theorem 3.22 Let A be a $I_{\tilde{\tilde{g}}s}$-closed set and B be $\tilde{g}s$-closed set of an ideal topological space (X, τ, I). Then $A \cap B$ is $I_{\tilde{\tilde{g}}s}$-closedset.

Proof Let U be a $\tilde{g}s$-open subset of X containing $A \cap B$. Then $A \subseteq U \cup (X - B)$. Since A is $I_{\tilde{\tilde{g}}s}$-closed, then $A_s^* \subseteq U \cap (X - B)$ and $B \cap A_s^* \subseteq U$. By theorem 2.7, $(A \cap B)_s^* \subseteq A_s^* \cap B_s^* \subseteq A_s^* \cap B \subseteq U$, because B is $\tilde{g}s$-closed. Thus $A \cap B$ is $I_{\tilde{\tilde{g}}s}$-closed.

Theorem 3.23 Let (X, τ, I) be an ideal topological space and A⊆X be an $I_{\tilde{\tilde{g}}s}$-closed set, then the following are equivalent.

(i) Ais a semi-⋆-closed set.

(ii) $Cl_s^*(A) - A$ is a $\tilde{g}s$-closed set.

(iii) $A*–A$ is a $\tilde{g}s$-closed set.

Proof

(i)⇒(ii): If A is semi-⋆-closed, then $A_s^* \subseteq A$ and so $Cl_s^*(A) - A = (A \cup A_s^*) - A = \emptyset$. Hence $Cl_s^*(A) - A$ is a $\tilde{g}s$-closed set.

(ii)⇒(iii): Since $Cl_s^*(A) - A = A_s^* - A$ and so $A_s^* - A$ is $\tilde{g}s$-closed set.

(iii)⇒(i): If $A_s^* - A$ is a $\tilde{g}s$-closed set, since A is $I_{\tilde{\tilde{g}}s}$-closed set, then by theorem 3.6, $A_s^* - A = \emptyset$ and so A is semi-⋆-closed.

4. Properties of $I_{\tilde{\tilde{g}}s}$-open sets :

This section is to investigate the properties of $I_{\tilde{\tilde{g}}s}$-open sets in idealtopological spaces.

Theorem 4.1 Let (X, τ, I) be an ideal topological space and $A \subseteq B$. Then is $I_{\tilde{\tilde{g}}s}$-open if and only if $F \subseteq Int_s^*(A)$ whenever F is $\tilde{g}s$-closed and $F \subseteq A$.

Proof Suppose A is $I_{\tilde{\tilde{g}}s}$-open. If F is $\tilde{g}s$-closed and $F \subseteq A$, then $X–F \subseteq X–F$ and so $(X - A)_s^* \subseteq X - F$. Then $X - Int_s^*(A) = Cl_s^*(X - A) = (X - A) \cup (X - A)_s^* \subseteq X - F$ and so $F \subseteq Int_s^*(A)$. Conversely, let U be a $\tilde{g}s$-open set such that $X - A \subseteq U$. Then $X - U \subseteq A$ and so $X - U \subseteq Int_s^*(A)$. Therefore $Cl_s^*(X - A) \subseteq U$ and implies $(X - A)_s^* \subseteq U$. Therefore, $X–A$ is $I_{\tilde{\tilde{g}}s}$-closed. Hence A is $I_{\tilde{\tilde{g}}s}$-open.

Theorem 4.2 Let (X, τ, I) be an ideal

topological space and $A \subseteq X$. If A is $I_{\tilde{g}s}$-open and $Int_s^*(A) \subseteq B \subseteq A$, then B is $I_{\tilde{g}s}$-open.

Proof Since A is $I_{\tilde{g}s}$-open, then $X–A$ is $I_{\tilde{g}s}$-closed. By theorem 3.6, $Cl_s^*(X - A) - (X - A)$ contains no nonempty $\tilde{g}s$-closed set. Since $Int_s^*(A) \subseteq Int_s^*(B)$ which implies that $Cl_s^*(X - B) \subseteq Cl_s^*(X - A)$ and so $Cl_s^*(X - B) - (X - B) \subseteq Cl_s^*(X - A) - (X - A)$. Hence B is $I_{\tilde{g}s}$-open.

Theorem 4.3 Let (X, τ, I) be an ideal topological space and $A \subseteq X$, then the followingare equivalent.

(i) A is $I_{\tilde{g}s}$-closed.

(ii) $A \cup (X - A_s^*)$ is $I_{\tilde{g}s}$-closed.

(iii) $A_s^* - A$ is $I_{\tilde{g}s}$-open.

Proof **(i)⇒(ii):** Suppose A is $I_{\tilde{g}s}$-closed. If U is any $\tilde{g}s$-open set such that $A \cup (X - A_s^*) \subseteq U$, then $X - U \subseteq X - (A \cup (X - A_s^*)) = X \cap (A \cup (A_s^*)^c)^c = A_s^* \cap A^c = A_s^* - A$. Since A is $I_{\tilde{g}s}$-closed, by theorem 3.7(v),we get $X - U = \emptyset$ or $X = U$. Therefore $A \cup (X - A_s^*) \subseteq U \Rightarrow A \cup (X - A_s^*) \subseteq X$ and implies $(A \cup (X - A_s^*))_s^* \subseteq X_s^* \subseteq X = U$. Hence $A \cup (X - A_s^*)$ is $I_{\tilde{g}s}$-closed.

(ii)⇒(iii): Suppose $A \cup (X - A_s^*)$ is $I_{\tilde{g}s}$-closed. If F is any $\tilde{g}s$-closed set such that $F \subseteq A_s^* - A$, then $F \subseteq A_s^*$ and $F \nsubseteq A \Rightarrow X - A_s^* \subseteq X - F$ and $A \subseteq X - F$. Therefore $A \cup (X - A_s^*) \subseteq A \cup (X - F) = X - F$ and $X–F$ is $\tilde{g}s$-open. Since $(A \cup (X - A_s^*))_s^* \subseteq X - F \Rightarrow A_s^* \cup (X - A_s^*)_s^* \subseteq X - F$ and so $A_s^* \subseteq X - F \Rightarrow F \subseteq X - A_s^*$. Since $F \subseteq A_s^*$, it follows that $F = \emptyset$. Hence A is $I_{\tilde{g}s}$-closed.

(iii)⇒(i): Since $X - (A_s^* - A) = X \cap (A_s^* \cap A^c)^c = X \cap ((A_s^*)^c \cup A) = (X \cap (A_s^*)^c) \cup (X \cap A) = A \cup (X - A_s^*)$.

Conclusion

This paper introduces new weak closed sets in ideal topological spaces and investigate its basic properties. This set can be extended in different research fields such as extended topology, Fuzzy topology, Intuitionistic topology, digital topology etc.

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