Ultra Scientist Vol. 28(3)A, 183-186 (2016).
www.ultrascientist.org
ISSN 2231-3478 (Print)
2319-8052 (Online)

On quasi $\bar{g}$-open and quasi $\bar{g}$-closed functions

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(Acceptance Date 24th July, 2016)

Abstract

In this paper, we introduce a new type of open function namely quasi $\bar{g}$-open function. Further we obtain its characterizations and basic properties.

Key words : $\bar{g}$-open set, $\bar{g}$-closed set, $\bar{g}$-interior, $\bar{g}$-closure, $\bar{g}$-quasiopen function.

2000 Mathematics Subject Classification: 54c10, 54c08, 54c05.

1. Introduction and Preliminaries

Functions stand among the important notions in the whole of mathematical science. Many different open functions have been introduced over the years. The importance is significant in various areas of mathematics and sciences. The notion of $\bar{g}$-closed sets were introduced and studied by Manoj *et al.*[5]. In this paper, we will continue the study of related functions by considering $\bar{g}$-open sets and $\bar{g}$-open functions. We further introduce and characterize the concept of quasi $\bar{g}$-open functions.

Throughout this paper, spaces mean topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f : (X, \tau) \rightarrow (Y, \sigma)$ denotes a function f of a space (X,) into a space (Y, σ). Let A be a subset of space X. Thenthe closure and the interior of A are denoted by cl(A) and int(A) respectively.

Definition 1.1: A subset A of a topological space (X, τ) is called semi-open[2] (resp. semi- closed) if $A \subseteq cl(int(A))$ (resp. $int(cl(A)) \subseteq A$).

The semi-closure[1] of a subset A of X (denoted by scl(A)) is defined to be the intersection of all semi-closed sets containing A.

Definition 1.2: A subset A of a topological space (X, τ) is called

(i)sg-closed[3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X. The Complement of sg-closed set is called sg-open.

(ii) $\hat{\hat{g}}$-closed[4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U issg-open in X. The complement of $\hat{\hat{g}}$-closed set is called $\hat{\hat{g}}$-open.

(iii) $\overline{g}$-closed[5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{\hat{g}}$-open in X. The complement of $\overline{g}$-closed set is called $\overline{g}$-open.

The union (resp. intersection) of all $\overline{g}$-open (resp. $\overline{g}$-closed) sets, each contained in (resp. containing) a set A in a space X is called the $\overline{g}$-interior (resp. $\overline{g}$-closure) of A and is denoted by $\overline{g}$-Int(A) (resp. $\overline{g}$-cl(A))[5].

*Definition 1.3:*A function $f :(X, \tau) \to (Y, \sigma)$ is called

(i) $\overline{g}$-irresolute[5] (resp. $\overline{g}$-continuous[5]) if the inverse image of every $\overline{g}$-closed (resp. closed) set in Y is $\overline{g}$-closed in X.

(ii) $\overline{g}$-open[5] (resp. $\overline{g}$-closed[5]) if f(V) is $\overline{g}$-open (resp. $\overline{g}$-closed) in Y for every open (resp. closed) subset of X.

(iii) $\overline{g}$*-closed[5] if the image of every $\overline{g}$-closed subset of X is $\overline{g}$-closed in Y.

Definition 1.4: Let x be a point of (X, τ) and N be a subset of X. Then N is called a $\overline{g}$-neighborhood (briefly $\overline{g}$-nbd)[5] of x if there exists a $\overline{g}$-open set G such that $x \in G$ and $G \subset N$.

2. Quasi -open Functions :

In this section we introduce the following definitions.

*Definition 2.1:*A functionf : $(X, \tau) \to (Y, \sigma)$ is called quasi $\overline{g}$-open if the image of every $\overline{g}$-open set in Xis open in Y.

If the function f is bijective then the concept of quasi $\overline{g}$-openness and $\overline{g}$-continuity coincide.

*Theorem 2.2:*A function $f : X \to Y$ is quasi $\overline{g}$-open iff for every subset A of X, $f(\overline{g}\text{-Int}(A)) \subseteq \text{Int}(f(A))$.

Lemma 2.3: If a function $f : X \to Y$ is quasi $\overline{g}$-open, then $\overline{g}\text{-Int}(f^{-1}(A)) \subseteq f^{-1}(\text{Int}(A))$ for every subset A of Y.

Proof: Let A be any arbitrary subset of Y. Then $\overline{g}\text{-Int}(f^{-1}(A))$ is a $\overline{g}$-open set in X and f is quasi $\overline{g}$-open, then $f(\overline{g}\text{-Int}(f^{-1}(A)) \subseteq \text{Int}(f(f^{-1}(A)) \subseteq \text{Int}(A)$. Thus $\overline{g}\text{-Int}(f^{-1}(A)) \subseteq f^{-1}(\text{Int}(A))$.

Theorem 2.4: For a function $f : X \to Y$, the following are equivalent

(i) f is quasi $\overline{g}$-open,

(ii) For each subset A of X, $f(\overline{g}\text{-Int}(A)) \subseteq \text{Int}(f(A))$.

(iii) For each $x \in X$ and each $\overline{g}$-nbd A of x in X, there exists a neighborhood A of x in X, there exists a neighborhood B of f(x) in Y such that $B \subseteq f(A)$.

Theorem 2.5 : A function $f : X \rightarrow Y$ is quasi $\overline{g}$-open iff for any subset B of Y and for any $\overline{g}$-closed set A of X containing $f^{-1}(B)$ there exists a closed set C of Y containing B such that $f^{-1}(C) \subseteq A$.

*Theorem 2.6:*A function $f : X \rightarrow Y$ is quasi $\overline{g}$-open iff for any subset $f^{-1}(cl(B)) \subseteq \overline{g}\text{-cl}(f^{-1}(B))$ for every subset B of Y.

Theorem 2.7: Let$f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions and $gof : X \rightarrow Z$ is quasi $\overline{g}$-open. If g is continuous injective, then f is quasi $\overline{g}$-open.

3. *Quasi $\overline{g}$-closed Functions :*

In this section we introduce the following definitions.

*Definition 3.1:*A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called quasi $\overline{g}$-closed if the image of each $\overline{g}$-closed set in X is open in Y.

Clearly every quasi $\overline{g}$-closed function is closed and $\overline{g}$-closed.

Remark 3.2: Every $\overline{g}$-closed (resp. closed) function need not be quasi $\overline{g}$-closed as shown by the following example.

Example 3.3: LetX =Y={a,b,c}, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, \{c\}, \{a, c\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by identity mapping then f is -closed and closed but not quasi $\overline{g}$-closed.

Lemma 3.4: If a function $f : X \rightarrow Y$ is quasi $\overline{g}$-closed, then $f^{-1}(\text{Int}(A)) \subseteq \overline{g}\text{-Int}(f^{-1}(A))$ for every subset A of Y.

*Theorem 3.5 :*A function $f : X \rightarrow Y$ is quasi $\overline{g}$-closed iff for any subset A of Y and for any $\overline{g}$-open set G of X containing $f^{-1}(A)$, there exists an open set U of Y containing A such that $f^{-1}(U) \subseteq G$.

Theorem 3.6: If$f : X \rightarrow Y$and $g : Y \rightarrow Z$ are two quasi $\overline{g}$-closed function, then their composition $gof : X \rightarrow Z$ is a quasi -closed function.

*Proof:*Proof is definition based.

Theorem 3.7: Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be any two functions then

(i) If f is $\overline{g}$-closed and g is quasi $\overline{g}$-closed, then gof is closed.

(ii) If f is quasi $\overline{g}$-closed and g is $\overline{g}$-closed, then gof is $\overline{g}$*-closed.

(iii) If f is $\overline{g}$*-closed and g is quasi $\overline{g}$-closed, then gof is quasi $\overline{g}$-closed.

Theorem 3.8: Let$f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions such that their composition $gof : X \rightarrow Z$ is quasi $\overline{g}$-closed

(i) If f is $\bar{g}$-irresolute surjective, then g is closed.

(ii) If g is $\bar{g}$-continuous injective, then f is $\bar{g}$*-closed.

Theorem 3.9: Let X and Y be two topological spaces. Then the function $g : X \to Y$ is a quasi $\bar{g}$-closed if and only if g(X) is closed in Y and $g(V) \setminus g(X \setminus V)$ is open in g(X) whenever V is $\bar{g}$-open in X.

Proof: Let $g : X \to Y$ is a quasi $\bar{g}$-closed function. Since X is $\bar{g}$-closed g(X) is closed in Y and $g(V) \setminus g(X \setminus V) = g(V) \cap g(X) \setminus g(X \setminus V)$ is open in g(X) when V is $\bar{g}$-open in X.

Conversely, let g(X) is closed in Y, $g(V) \setminus g(X \setminus V)$ is open in g(X) when V is $\bar{g}$-open in X and let F be closed in X. Then $g(F) = g(X) \setminus (g(X \setminus F) \setminus g(F))$ is closed in g(X) and hence, closed in Y.

Corollary 3.10: Let X and Y be two topological spaces. Then a surjection function $g : X \to Y$ is quasi $\bar{g}$-closed if and only if $g(V) \setminus g(X \setminus V)$ is open in Y whenever V is $\bar{g}$-open in X.

Corollary 3.11: Let X and Y be two topological spaces and let $g : X \to Y$ be a $\bar{g}$-continuous, quasi $\bar{g}$-closed surjective function. Then the topology on Y is $\{g(V) \setminus g(X \setminus V) : V$ is $\bar{g}$-open in $X\}$.

Definition 3.12: A topological space (X, τ) is said to be $\bar{g}$-normal if for any pair of disjoint $\bar{g}$-closed subsets F_1 and F_2 of X, there exists disjoint open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Theorem 3.13: Let X and Y be topological spaces with X is $\bar{g}$-normal. If $g : X \to Y$ is a $\bar{g}$-continuous quasi $\bar{g}$-closed surjective function. Then Y is normal.

References

1. Crossley S.G. and Hildebrand S.K., Semi closure, *Texas J. Sci., 22*, 99-112 (1971).
2. Levine N., Semi open sets and semi continuity in topological spaces, *Amer. Math. Monthly, 70*, 36-41 (1963).
3. Bhattacharya P. and Lahiri B.K., Semi generalized closed sets in a topology, *Indian J. Math, 29* (3), 375 (1987).
4. Garg M., Agarwal S. and Goal C. K., On $\hat{\hat{g}}$-closed sets in topological spaces, *Acta Ciencia Indica*, XXXIII (4)M, 1643 - 1652 (2007).
5. Garg M., Rathor S.S., A generalized closed sets in topological spaces, Communicated.