

## Pathos and Total Pathos Semifull Line Graph

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### Abstract

In this paper, we introduce the concept of pathos semifull line graph and total pathos semifull line graph of a tree. We obtain some properties of these graphs. We study the characterization of graphs whose pathos and total pathos semifull line graph are planar, maximal outer planar, minimally nonouter planar, crossing number one, noneulerian and Hamiltonian.

*Key words:* Inner vertex number, pathos graph, semifull line graph.

### Introduction

All graphs considered here are finite, undirected without loops or multiple edges. Any undefined term or notation in this paper may be found in Harary<sup>2</sup>.

The concept of pathos of a graph  $G$  was introduced by<sup>1</sup> as a collection of minimum number of edge disjoint open paths whose union is  $G$ . The path number of a graph  $G$  is the number of path of pathos. Stanton<sup>9</sup> and Harary<sup>2</sup> have calculated the path number of

certain classes of graphs like trees and complete graphs.

For a graph  $G(p, q)$  if  $B = \{u_1, u_2, u_3, \dots, u_r; r \geq 2\}$  is a block of  $G$ , then we say that point  $u_1$  and block  $B$  are incident with each other, as are  $u_2$  and  $B$  and so on. If two distinct blocks  $B_1$  and  $B_2$  are incident with a common cut vertex then they are called adjacent blocks.

The crossing number  $c(G)$  of  $G$  is the least number of intersection of pairs of edges in any embedding of  $G$  in the plane. Obviously

$G$  is planar if and only if  $c(G)=0$ . The *edgedegree* of an edge  $e = \{a, b\}$  is the sum of degrees of the end vertices  $a$  and  $b$ . Degree of a block is the number of vertices lies on a block. *Blockdegree*  $B_v$  of a vertex  $v$  is the number of blocks in which  $v$  lies. *Blockpath* is a path in which each edge in a path becomes a block. If two paths  $p_1$  and  $p_2$  contain a common cutvertex then they are adjacent paths and the *pathdegree*  $P_v$  of a vertex  $v$  is the number of paths in which  $v$  lies. Pendant pathos is a path  $p_i$  of pathos having unit length.

The inner vertex number  $i(G)$  of a planar graph  $G$  is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of  $G$  in the plane.

#### *Literature Review :*

A new concept of a graph valued functions called the edge semientire block graph  $E_b(G)$  of a plane graph  $G$  was introduced by Venkanagouda in<sup>11</sup> and is defined as the graph whose vertex set is the union of set of edges, set of blocks and set of regions of  $G$  in which two vertices are adjacent if and only if the corresponding edges of  $G$  are adjacent or corresponding blocks of  $G$  are adjacent or one corresponding to edge of  $T$  and other the block of  $G$  and edge lies on the block, one corresponding to edge of  $G$  and other the region and edge lies on the region.

The pathos edge semientire graph  $P_e(T)$  of a tree was introduced by in<sup>12</sup>. The pathos edge semientire graph  $P_e(T)$  of a tree  $T$  is the graph whose vertex set is the union of set of edges, regions and the set of pathos of

pathos in which two vertices are adjacent if and only if the corresponding edges of  $T$  are adjacent, edges lies on the region and edges lies on the path of pathos. Since the system of path of pathos for a tree  $T$  is not unique, the corresponding pathos edge semientire graph is also not unique.

The pathos edge semientire block graph of a tree  $T$  denoted by  $P_{vb}(T)$  is the graph whose vertex set is the union of the vertices, regions, blocks and path of pathos of  $T$  in which two vertices are adjacent if and only if they are adjacent vertices of  $T$  or vertices lie on the blocks of  $T$  or vertices lie on the regions of  $T$  or the adjacent blocks of  $T$ . Clearly the number of regions in a tree is one. This concept was introduced by Venkanagouda in<sup>3</sup>.

The block graph  $B(G)$  of a graph  $G$  is the graph whose vertex set is the set of blocks of  $G$  in which two vertices are adjacent if the corresponding blocks are adjacent. This graph was studied in<sup>2</sup>. The path graph  $P(T)$  of a tree is the graph whose vertex set is the set of path of pathos of  $T$  in which two vertices of  $P(T)$  are adjacent if the corresponding path of pathos have a common vertex.

The semifull graph  $F_s(G)$  of a graph  $G$  is the graph whose vertex set is the union of vertices, edges and blocks of  $G$  in which two vertices are adjacent if the corresponding members of  $G$  are adjacent or one corresponds to a vertex and other to a line incident with it or one corresponds to a block  $B$  of  $G$  and other to a vertex  $v$  of  $G$  and  $v$  is in  $B$ . This concept was introduced in<sup>5</sup>. Further in<sup>6</sup> studied the concept of semifull line graph  $F_l(G)$  and is defined as the graph whose vertex set is the union of

vertices, edges and blocks of  $G$  in which two vertices are adjacent if the corresponding vertices and edges of  $G$  are adjacent or one corresponds to a vertex and other to a line incident with it or one corresponds to a block  $B$  of  $G$  and other to a vertex  $v$  of  $G$  and  $v$  is in  $B$ . The pathos line graph of a tree is introduced in<sup>7</sup> and is defined as the graph whose vertex set is the union of the set of edges and paths of  $T$  in which two vertices are adjacent if and only if the corresponding edges of  $T$  are adjacent and the edge lies on the path of pathos of  $T$ .

The following will be useful in the proof of our results.

*Theorem A<sup>6</sup>.* If  $G$  is a  $(p, q)$  connected graph, whose vertices have degree  $d_i$  and if  $b_i$  is the number of blocks to which vertex  $v_i$  belongs in  $G$ , then the semifull line graph  $F_1(G)$  of  $G$  has  $q + \sum b_i + 1$  vertices and  $2q + \sum b_i + \frac{1}{2} \sum d_i^2$  edges.

*Theorem B<sup>7</sup>.* The pathos line graph  $P_1(T)$  of a tree  $T$  is planar if and only if  $\Delta(T) \leq 4$ .

*Theorem C<sup>1</sup>.* A graph is planar if and only if it has no subgraph homeomorphic to  $K_5$  or  $K_{3,3}$ .

*Theorem D<sup>1</sup>.* Every maximal outerplanar graph  $G$  with  $p$  vertices has  $(2p-3)$  edges.

*Theorem E<sup>1</sup>.* A connected graph  $G$  is eulerian if and only if each vertex in  $G$  has even degree.

*Theorem F<sup>2</sup>.* A nontrivial graph is bipartite if and only if all its cycles are even.

*Theorem G<sup>2</sup>.* If  $G$  is a  $(p, q)$  graph whose vertices have degree  $d_i$  then  $L(G)$  has  $q$  vertices

and  $q_L$  edges where  $q_L = -q + \frac{1}{2} \sum d_i^2$ .

*Theorem H<sup>2</sup>.* The line graph  $L(G)$  of a graph  $G$  has crossing number one if and only if  $G$  is planar and 1 or 2 holds:

1. The maximum degree  $d(G)$  is 4 and there is unique non cutvertex of degree.
2. The maximum degree  $d(G)$  is 5, every vertex of degree 4 is a cutvertex and there is a unique vertex of degree 5 and has at most 3 edges in any block.

*Pathos Semifull Line graph :*

We now define the following graph valued function.

The Pathos semifull line graph  $P_{FL}(T)$  of a tree  $T$  is the graph whose vertex set is the union of vertices, edges, blocks and path of pathos of  $T$  in which two vertices are adjacent if the corresponding vertices and edges of  $T$  are adjacent or one corresponds to a vertex and other to an edge incident with it or one corresponds to a block  $B$  of  $T$  and other to a vertex  $v$  of  $T$  and  $v$  is in  $B$  or one corresponds to path of pathos  $P_i$  of  $T$  and other to an edge  $e$  of  $T$  and  $e$  is on the path  $P_i$ . Since the path of pathos of  $T$  is not unique then the corresponding  $P_{FL}(T)$  is also not unique.

## Results

*Remark 1.* If  $T$  is connected tree then the pathos semifull line graph  $P_{FI}(T)$  is also connected and conversely.

*Remark 2.* The semifull line graph  $F_1(T)$  of  $T$  is a spanning subgraph of the semifull line graph  $P_{FI}(T)$  of  $T$ .

*Remark 3.* For any tree  $T$ ,  $P_{FI}(T) = F_1(T) \cup P(T)$  where  $P(T)$  is the pathos graph of a tree  $T$ .

*Remark 4.* If every edge of a tree  $T$  is of edgedegree  $n$ , then the corresponding vertex  $P_{FI}(T)$  having degree  $n + 1$ .

*Remark 5.* If degree of a vertex in  $T$  is  $n$ , then the degree of the corresponding vertex in  $P_{FI}(T)$  is  $3n$ .

*Theorem 1.* If  $T$  be a  $(p, q)$  tree whose vertices have degree  $d_i$  and  $b_i$  is the number of blocks to which a vertex  $v_i$  belongs in  $T$ , then the  $P_{FI}(T)$  has  $q + \sum b_i + k + 1$  vertices and  $3q + b_i + \frac{1}{2} \sum d_i^2$  edges.

*Proof.* By Remark 2, semifull line graph  $F_1(T)$  is spanning subgraph of  $P_{FI}(T)$ . The number of vertices of  $P_{FI}(T)$  equal to the number of vertices of  $F_1(T)$  and the number of path of pathos of  $T$  which is  $k$ . Thus  $P_{FI}(T)$  has  $q + \sum b_i + k + 1$  vertices.

Further by Remark 2,  $F_1(T)$  is a spanning subgraph of  $P_{FI}(T)$ . The number of

edges in  $P_{FI}(T)$  is the sum of the number of edges in  $F_1(T)$  and the number of edges in each path of pathos of  $T$  which is  $q$ . By Theorem A,  $F_1(T)$  has  $2q + \sum b_i + \frac{1}{2} \sum d_i^2$  edges. Thus the number of edges in  $P_{FI}(T) = 2q + \sum b_i + \frac{1}{2} \sum d_i^2 + q = 3q + \sum b_i + \frac{1}{2} \sum d_i^2$ .

*Theorem 2.* For any tree  $T$ , pathos semifull line graph  $P_{FI}(T)$  is non-separable if and only if  $T$  has no pendent pathos.

*Proof.* Suppose  $P_{FI}(T)$  is non-separable. Assume that  $T$  contains at least one pendent pathos  $p_i$  of  $T$ . By the definition of  $F_1(T)$ , each edge is adjacent to exactly two of its end vertices  $e_i = (v_i, v_{i+1})$  and each edge  $e_i = b_i$  is a block. Hence the blockvertex is also adjacent to exactly two vertices. The adjacency between the vertices is same as that of  $T$ . Clearly  $F_1(T)$  is non-separable. If  $T$  contains a pendent pathos  $p_i = \{e_i\}$ , then it is adjacent to the corresponding vertex  $e'_i$  in  $P_{FI}(T)$ , and  $e'_i$  becomes a cut vertex, a contradiction.

Converse part is obvious.

*Theorem 3.* The pathos semifull line graph  $P_{FI}(T)$  of a tree  $T$  is planar if and only if  $\Delta(T) \leq 3$  for every vertex  $v$  of  $T$ .

*Proof.* Suppose the pathos semifull line graph  $P_{FI}(T)$  is planar. Assume  $\Delta(T) \leq 4$ . If there exists a vertex  $v$  of degree 4 clearly  $v$  lies on four blocks and four edges of  $T$ . Then  $T$  has the subgraph homeomorphic to  $K_{1,4}$ , it can be drawn in the plane such that  $P_{FI}(T)$  has a subgraph homeomorphic to  $K_5$ . By

Theorem C,  $P_{F_1(T)}$  is non-planar, a contradiction.

Conversely suppose  $\Delta(T) \leq 3$ . then by the Theorem B, the pathos line graph  $P_L(T)$  is planar. Each block vertex is lies on exactly two vertices. This gives a  $P_{F_1(T)}$  planar.

*Theorem 4.* For any tree  $T$ , pathos semifull line graph  $P_{F_1(T)}$  is outer planar if and only if  $T$  is a path  $P_3$ .

*Proof.* Suppose  $P_{F_1(T)}$  is outer planar. Assume that  $T$  is path  $P_n$ ,  $n \geq 4$ , without loss of generality, we take  $n=4$ . By definition of  $F_1(T)$ , each edges are adjacent and edges are incident to the vertices and the vertices lies on block B. Clearly  $F_1(T)$  is planar and all vertices lies on entire region since the pathos vertices  $p_i$  is adjacent to all edges  $e_1, e_2, e_3$  of  $T$  to form a graph with  $e_3$  as inner vertex. Hence  $i(G) = 1$ , a contradiction.

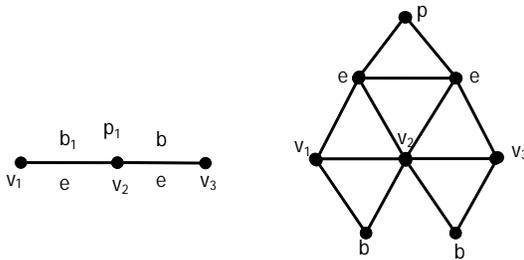


Figure 2.2

Conversely suppose  $T$  is path  $P_3$ . By the definition of semifull line graph  $F_1(P_3)$  is also planar. In  $P_3$  there is only one pathos vertex which is adjacent to two vertices  $e_1$

and  $e_2$  of  $F_1(P_3)$  gives  $P_{F_1(T)}$  outer planar as shown in figure 2.2.

*Theorem 5.* For any tree  $T$ , pathos semifull line graph  $P_{F_1(T)}$  is maximal outer planar if and only if  $P_3$ .

*Proof.* Suppose  $P_{F_1(T)}$  is maximal outer planar then  $F_1(T)$  is connected. By the Theorem 4,  $T$  is  $P_3$ .

Conversely suppose  $T$  is  $P_3$ . By Theorem 4,  $P_{F_1(T)}$  is outer planar. We now prove that  $P_{F_1(T)}$  maximal outer planar. By Theorem D, every maximal outer planar graph  $G$  with  $p$  vertices has  $(2p-3)$  edges. In  $P_{F_1(T)}$ ,  $p=8$ ,  $q=13$ . For  $p=8$ , then  $2 \times 8 - 3 = 13$  edges which is maximal outerplanar.

*Theorem 6.* For any tree  $T$ ,  $P_{F_1(T)}$  is minimally nonouter planar if and only if  $T$  is  $P_4$ .

*Proof.* Suppose  $P_{F_1(T)}$  is minimally nonouter planar. Assume that  $T$  is  $P_5$  and let  $P_5: v_1e_1v_2e_2v_3e_3v_4e_4v_5$  be a path. Since in a path, each edge is a block. We have  $e_i' = \{v_i, v_{i+1}\} = b_i'$ . By definition of  $F_1(T)$ , each  $e_i'$  and  $b_i'$  are adjacent to  $v_i$  and  $v_{i+1}$ . Also each  $e_i'$  and  $e_{i+1}'$  are adjacent to each other. Clearly  $F_1(T)$  is outer planar. In  $P_{F_1(T)}$  the pathos vertex  $p_i$  is adjacent to all  $e_i'$  and the internal edges  $e_2$  and  $e_3$  becomes the internal vertices  $e_2'$  and  $e_3'$  in  $P_{F_1(T)}$ , which gives  $i[P_{F_1(T)}] = 2$ , a contradiction.

Conversely, suppose  $T$  is  $P_4$ . Let  $P_4 :$

$v_1e_1v_2e_2v_3e_3v_4$  be a path. Let  $e_1', e_2', e_3'$  be the vertices of  $P_{FI}(T)$  corresponds to the edges  $e_1, e_2, e_3$  of  $T$ .

Let  $e_i = \{v_i, v_{i+1}\} = b_i$ . By the definition  $F_1(T)$ , each  $e_i$  is adjacent to  $v_i, v_{i+1}$  and  $e_{i+1}$ . Also  $b_i$  is adjacent to  $v_i$  and  $v_{i+1}$ . Clearly  $F_1(T)$  is outer planar. In  $P_{FI}(T)$ , the pathos vertex  $p_1$  is adjacent to  $e_1, e_2, e_3$  to form a graph with one internal vertex  $e_2$  this gives  $i[P_{FI}(T)] = 1$ .

*Theorem 7.* The pathos semifull line graph  $P_{FI}(T)$  is tree  $T$  has crossing number one if and only if  $\Delta(G) \leq 3$  for every vertex of  $T$  and has a unique vertex of degree 4.

*Proof.* Suppose  $P_{FI}(T)$  has crossing number one. Assume  $\Delta(G) \geq 4$ . We have the following cases.

**Case 1.** Suppose  $T$  has vertex  $v$  of degree 5. By the definition of line graph, the edges incident to the vertex together with the cut vertex form  $\langle K_5 \rangle$  in  $L(T)$ . In  $P_{FI}(T)$  the vertex  $v$  is adjacent to all vertices of  $e_1', e_2', e_3', e_4'$  corresponds to the edges  $e_1, e_2, e_3, e_4$  of  $T$  to form a graph with crossing number  $c[P_{FI}(T)] > 1$ , a contradiction.

**Case 2.** Suppose  $T$  has two vertices  $v_1, v_2$  of degree 4, then  $T$  contains two  $K_{1,4}$  as sub graphs. By the definition,  $P_{FI}(T)$  contains a subgraph homeomorphic to  $K_5$ . Hence in  $P_{FI}(T)$  has at least two  $K_5$  sub graphs. Clearly  $c[P_{FI}(T)] > 1$ , a contradiction. Thus  $T$  has exactly one cut vertex of degree 4.

Conversely, suppose  $T$  holds both the

conditions of the theorem. For  $\Delta(G) \leq 3$  by Theorem 3,  $P_{FI}(T)$  is planar. Let  $v \in T$  be unique vertex of  $T$  such that  $\text{degree}(v) = 4$ . By definition of  $P_{FI}(T)$ ,  $P_{FI}(K_{1,4})$  is a graph homeomorphic to  $K_5$ . Clearly  $c[P_{FI}(T)] = 1$ .

*Theorem 8.* For any tree  $T$ , pathos semifull line graph  $P_{FI}(T)$  is non-complete graph.

*Proof.* For  $T = P_2$ ,  $F_1(T)$  is  $K_4-x$ . In  $P_{FI}(T)$ , the pathos vertex  $p_1$  is adjacent to  $e_1$  to get a separable graph which is non-complete. For  $T = P_n, n \geq 3$ , there is no adjacency between the block vertices. Hence  $P_{FI}(T)$  is always non-complete graph.

*Theorem 9.* For any tree  $T$ , pathos semifull line graph  $P_{FI}(T)$  is not a bipartite graph.

*Proof.* By the definition of  $F_1(T)$ , each edge and its end vertices always form  $K_3$  as induced subgraph. Clearly  $F_1(T)$  is not bipartite and hence  $P_{FI}(T)$  is also not a bipartite graph.

*Theorem 10.* For any tree  $T$  with  $P$  pathos semifull line graph  $P_{FI}(T)$  is noneulerian.

*Proof.* Suppose  $T$  is a tree. In a tree there exist at least two pendent vertices. By Remark 5, degree of the corresponding vertices in  $P_{FI}(T)$  is odd. Hence  $P_{FI}(T)$  is noneulerian.

*Theorem 11.* The pathos semifull line graph  $P_{FI}(T)$  of a tree  $T$  is Hamiltonian if  $T$  is a path  $P_n, n$ .

*Proof.* Suppose  $T$  be a path  $P_n: v_1 e_1 v_2 e_2 \dots e_{n-1} v_n$  and  $b_1 = e_1, b_2 = e_2, \dots, b_{n-1} = e_n$  be the blocks of  $T$  then  $T$  has exactly one path of pathos  $P_1$ . Now the vertex set of  $P_{Fl}(T)$  is  $V[P_{Fl}(T)] = \{e'_1, e'_2, \dots, e'_{n-1}\} \cup \{b'_1, b'_2, \dots, b'_{n-1}\} \cup \{v_1, v_2, \dots, v_n\}$  corresponding edge set  $e_1, e_2, \dots, e_{n-1}$ , blocks set  $b_1, b_2, \dots, b_{n-1}$  of  $T$ , respectively. In  $P_{Fl}(T)$ , the vertices  $b_i$  and  $e_i$  are adjacent to the vertices  $v_i$  and  $v_{i+1}$  which are end vertices of  $e_i$ . Also in  $P_{Fl}(T)$ ,  $e_i$  is adjacent to  $e_{i+1} \forall i$ . Clearly in  $P_{Fl}(T)$ , the Hamiltonian cycle  $p_1 e'_1 v_1 b'_1 v_2 b'_2 v_3 b'_3 v_4 \dots b'_{n-1} v_n e'_{n-1} e'_{n-2} \dots e'_2 p_1$  exists as shown in figure 2.3. Hence  $P_{Fl}(T)$  is Hamiltonian.

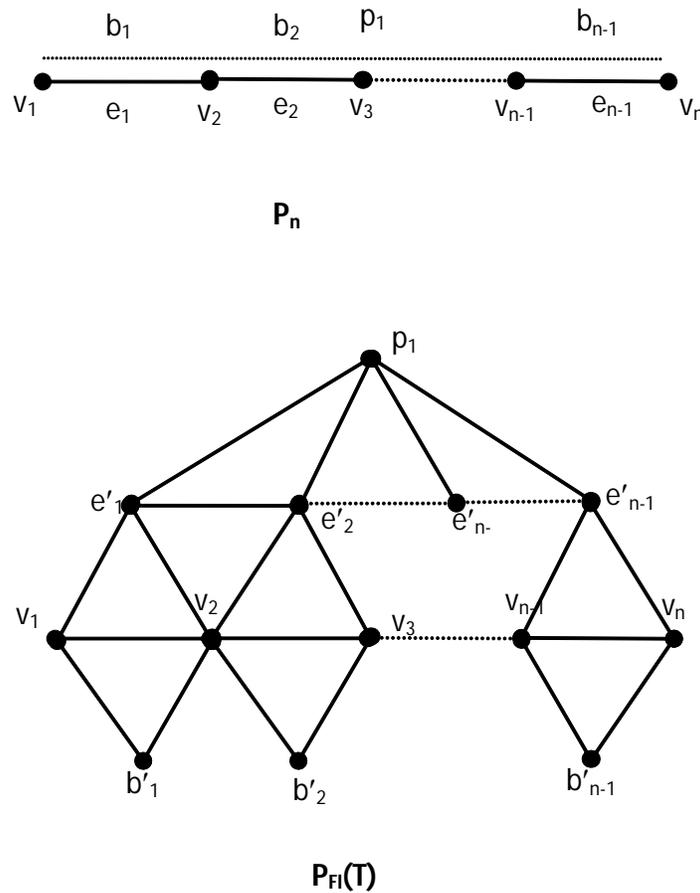


Figure 2.3

*Total pathos semifull line graph  $[T_{FI}(T)]$ :*

In this section we now define another graph valued function.

*Definition.* The Total pathos semifull line graph  $T_{FI}(T)$  of a tree  $T$  is the graph whose vertex set is the union of vertices, edges, blocks and path of pathos of  $T$  in which two vertices are adjacent if the corresponding vertices of  $T$  are adjacent or edges of  $T$  are adjacent or path of pathos are adjacent or one corresponds to a vertex and other to an edge incident with it or one corresponds to a block  $B$  of  $T$  and other to a vertex  $v$  of  $T$  and  $v$  is in  $B$  or one corresponds to path of pathos  $p_i$  of  $T$  and other to an edge  $e$  of  $T$  and  $e$  is on the path  $p_i$ . Since the path of pathos of  $T$  is not unique then the corresponding  $T_{FI}(T)$  is also not unique.

*Remark 6.* For any tree  $T$ ,  $P_{FI}(T)$  is spanning sub graph of  $T_{FI}(T)$ .

*Theorem 12.* If  $T$  is a  $(p, q)$  connected tree whose vertices have degree  $d_i$  and if  $b_i$  is the number of blocks to which  $v_i$  belongs in  $T$ , then the total pathos semifull line graph  $T_{FI}(T)$  has  $q + \sum b_i + k + 1$  vertices and  $3q + \sum b_i + \frac{1}{2}\sum d_i^2 + \frac{1}{2}\sum P_{d_i}(P_{d_i} - 1)$  edges where  $P_{d_i}$  is the path degree of vertex  $v_i$ .

*Proof.* By Remark 6,  $P_{FI}(T)$  is a spanning subgraph of  $T_{FI}(T)$ . The number of vertices of  $T_{FI}(T)$  equals number of vertices of  $P_{FI}(T)$ . By Theorem 1, the number of vertices of  $P_{FI}(T)$  is  $q + \sum b_i + k + 1$ . Hence

the number of vertices of  $T_{FI}(T)$  is  $q + \sum b_i + k + 1$ .

Further by the definition of  $T_{FI}(T)$ , the number of edges of  $T_{FI}(T)$  is the sum of the number of edges of  $P_{FI}(T)$  and the number obtained by the path degree of vertex. By the Theorem 1 the number of edges in  $P_{FI}(T)$  is  $3q + \sum b_i + \frac{1}{2}\sum d_i^2$  and the number of edges obtained by the path degree is  $\frac{1}{2}\sum P_{d_i}(P_{d_i} - 1)$ . Hence in  $T_{FI}(T)$ , the number of edges is  $3q + \sum b_i + \frac{1}{2}\sum d_i^2 + \frac{1}{2}\sum P_{d_i}(P_{d_i} - 1)$ .

*Theorem 13.* The total pathos semifull line graph  $T_{FI}(T)$  of a tree is planar if and only if  $T$  is  $K_{1,3}$ .

*Proof.* Suppose  $T_{FI}(T)$  is planar we have the following cases.

*Case 1.* Assume that  $T$  be a tree with  $\Delta(T) \leq 3$  a unique vertex of degree 4. By Theorem 7,  $P_{FI}(T)$  has crossing number 1 and hence  $T_{FI}(T)$  is non-planar which is a contradiction.

*Case 2.* Assume that  $T$  be a tree with  $\Delta(T) \leq 3 \forall v \in T$ . By Theorem 3,  $P_{FI}(T)$  is planar and it contains at least two path pathos. These two pathos vertices are lies on different regions in  $T_{FI}(T)$  while joining the edge two these two pathos vertices a becomes non-planar, a contradiction.

Conversely, suppose  $T$  is  $K_{1,3}$ , by Theorem 3,  $P_{FI}(T)$  is planar.  $K_{1,3}$  contains two path of pathos. The corresponds pathos

vertices of  $T_{FI}(T)$  lies on one region, clearly  $T_{FI}(T)$  is planar.

*Theorem 14.* The total pathos semifull line graph  $T_{FI}(T)$  is always noneulerian.

*Proof.* Suppose  $T$  be a tree which contains at least two pendent vertices. By Remark 5, degree of the vertices  $v_i^l v_j^l(T)$  in  $T_{FI}(T)$  odd, by Theorem E,  $T_{FI}(T)$  is noneulerian.

*Theorem 15.* For any tree  $T$ ,  $T_{FI}(T)$  is always nonhamiltonian.

*Proof.* Suppose  $T$  be a tree other than path. By Theorem 11, for any non- path tree  $T$ ,  $P_{FI}(T)$  is nonhamiltonian. Hence  $T_{FI}(T)$  is nonhamiltonian.

*Theorem 16.* For any tree  $T$ ,  $T_{FI}(T)$  has crossing number one if and only if  $T$  is  $K_{1,3}(P_n)$ .

*Proof.* Suppose  $T_{FI}(T)$  has crossing number one we have the following cases.

*Case 1.* Assume that  $T$  is  $K_{1,4}$ . By Theorem 7,  $P_{FI}(T)$  has crossing number one. Also  $K_{1,4}$  contains two path of pathos. In  $T_{FI}(T)$  both pathos vertices lies on different region, while joining the edges between them two pathos vertices gives crossing number at least two, a contradiction.

*Case 2.* Assume that  $T$  is  $K_{1,3}(P_n, P_n)$ . By Theorem 7,  $P_{FI}(T)$  has crossing number

one. Also  $T$  contains two path of pathos. In  $T_{FI}(T)$ , both pathos vertices lies on different regions. While joining the edge between these two pathos vertices gives crossing number at least two, a contradiction.

Conversely suppose  $T$  is  $K_{1,3}(P_n)$ . By the definition  $P_{FI}(T)$ ,  $P_{FI}(K_{1,3})$  is planar and  $P_{FI}[K_{1,3}(P_n)]$  is also planar. In  $K_{1,3}(P_n)$  there are two path of pathos. In  $P_{FI}(T)$  both pathos vertices lies on difference and is planar. In  $T_{FI}(T)$ , these two pathos vertices joint to form crossing number one.

## Conclusion

In this paper, we introduced the concept of the pathos semifull line graph and total pathos semifull line graph of a tree. We characterized the graphs whose pathos semifull line graph and total pathos semifull line graph are planar, noneulerian, Hamiltonian and crossing number one.

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