

Pathos and Total Pathos Semifull Line Graph

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Abstract

In this paper, we introduce the concept of pathos semifull line graph and total pathos semifull line graph of a tree. We obtain some properties of these graphs. We study the characterization of graphs whose pathos and total pathos semifull line graph are planar, maximal outer planar, minimally nonouter planar, crossing number one, noneulerian and Hamiltonian.

Key words: Inner vertex number, pathos graph, semifull line graph.

Introduction

All graphs considered here are finite, undirected without loops or multiple edges. Any undefined term or notation in this paper may be found in Harary².

The concept of pathos of a graph G was introduced by¹ as a collection of minimum number of edge disjoint open paths whose union is G . The path number of a graph G is the number of path of pathos. Stanton⁹ and Harary² have calculated the path number of

certain classes of graphs like trees and complete graphs.

For a graph $G(p, q)$ if $B = \{u_1, u_2, u_3, \dots, u_r; r \geq 2\}$ is a block of G , then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut vertex then they are called adjacent blocks.

The crossing number $c(G)$ of G is the least number of intersection of pairs of edges in any embedding of G in the plane. Obviously

G is planar if and only if $c(G)=0$. The *edgedegree* of an edge $e = \{a, b\}$ is the sum of degrees of the end vertices a and b . Degree of a block is the number of vertices lies on a block. *Blockdegree* B_v of a vertex v is the number of blocks in which v lies. *Blockpath* is a path in which each edge in a path becomes a block. If two paths p_1 and p_2 contain a common cutvertex then they are adjacent paths and the *pathdegree* P_v of a vertex v is the number of paths in which v lies. Pendant pathos is a path p_i of pathos having unit length.

The inner vertex number $i(G)$ of a planar graph G is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of G in the plane.

Literature Review :

A new concept of a graph valued functions called the edge semientire block graph $E_b(G)$ of a plane graph G was introduced by Venkanagouda in¹¹ and is defined as the graph whose vertex set is the union of set of edges, set of blocks and set of regions of G in which two vertices are adjacent if and only if the corresponding edges of G are adjacent or corresponding blocks of G are adjacent or one corresponding to edge of T and other the block of G and edge lies on the block, one corresponding to edge of G and other the region and edge lies on the region.

The pathos edge semientire graph $P_e(T)$ of a tree was introduced by in¹². The pathos edge semientire graph $P_e(T)$ of a tree T is the graph whose vertex set is the union of set of edges, regions and the set of pathos of

pathos in which two vertices are adjacent if and only if the corresponding edges of T are adjacent, edges lies on the region and edges lies on the path of pathos. Since the system of path of pathos for a tree T is not unique, the corresponding pathos edge semientire graph is also not unique.

The pathos edge semientire block graph of a tree T denoted by $P_{vb}(T)$ is the graph whose vertex set is the union of the vertices, regions, blocks and path of pathos of T in which two vertices are adjacent if and only if they are adjacent vertices of T or vertices lie on the blocks of T or vertices lie on the regions of T or the adjacent blocks of T . Clearly the number of regions in a tree is one. This concept was introduced by Venkanagouda in³.

The block graph $B(G)$ of a graph G is the graph whose vertex set is the set of blocks of G in which two vertices are adjacent if the corresponding blocks are adjacent. This graph was studied in². The path graph $P(T)$ of a tree is the graph whose vertex set is the set of path of pathos of T in which two vertices of $P(T)$ are adjacent if the corresponding path of pathos have a common vertex.

The semifull graph $F_s(G)$ of a graph G is the graph whose vertex set is the union of vertices, edges and blocks of G in which two vertices are adjacent if the corresponding members of G are adjacent or one corresponds to a vertex and other to a line incident with it or one corresponds to a block B of G and other to a vertex v of G and v is in B . This concept was introduced in⁵. Further in⁶ studied the concept of semifull line graph $F_l(G)$ and is defined as the graph whose vertex set is the union of

vertices, edges and blocks of G in which two vertices are adjacent if the corresponding vertices and edges of G are adjacent or one corresponds to a vertex and other to a line incident with it or one corresponds to a block B of G and other to a vertex v of G and v is in B . The pathos line graph of a tree is introduced in⁷ and is defined as the graph whose vertex set is the union of the set of edges and paths of T in which two vertices are adjacent if and only if the corresponding edges of T are adjacent and the edge lies on the path of pathos of T .

The following will be useful in the proof of our results.

Theorem A⁶. If G is a (p, q) connected graph, whose vertices have degree d_i and if b_i is the number of blocks to which vertex v_i belongs in G , then the semifull line graph $F_1(G)$ of G has $q + \sum b_i + 1$ vertices and $2q + \sum b_i + \frac{1}{2} \sum d_i^2$ edges.

Theorem B⁷. The pathos line graph $P_1(T)$ of a tree T is planar if and only if $\Delta(T) \leq 4$.

Theorem C¹. A graph is planar if and only if it has no subgraph homeomorphic to K_5 or $K_{3,3}$.

Theorem D¹. Every maximal outerplanar graph G with p vertices has $(2p-3)$ edges.

Theorem E¹. A connected graph G is eulerian if and only if each vertex in G has even degree.

Theorem F². A nontrivial graph is bipartite if and only if all its cycles are even.

Theorem G². If G is a (p, q) graph whose vertices have degree d_i then $L(G)$ has q vertices

and q_L edges where $q_L = -q + \frac{1}{2} \sum d_i^2$.

Theorem H². The line graph $L(G)$ of a graph G has crossing number one if and only if G is planar and 1 or 2 holds:

1. The maximum degree $d(G)$ is 4 and there is unique non cutvertex of degree.
2. The maximum degree $d(G)$ is 5, every vertex of degree 4 is a cutvertex and there is a unique vertex of degree 5 and has at most 3 edges in any block.

Pathos Semifull Line graph :

We now define the following graph valued function.

The Pathos semifull line graph $P_{FL}(T)$ of a tree T is the graph whose vertex set is the union of vertices, edges, blocks and path of pathos of T in which two vertices are adjacent if the corresponding vertices and edges of T are adjacent or one corresponds to a vertex and other to an edge incident with it or one corresponds to a block B of T and other to a vertex v of T and v is in B or one corresponds to path of pathos P_i of T and other to an edge e of T and e is on the path P_i . Since the path of pathos of T is not unique then the corresponding $P_{FL}(T)$ is also not unique.

Results

Remark 1. If T is connected tree then the pathos semifull line graph $P_{FI}(T)$ is also connected and conversely.

Remark 2. The semifull line graph $F_I(T)$ of T is a spanning subgraph of the semifull line graph $P_{FI}(T)$ of T .

Remark 3. For any tree T , $P_{FI}(T) = F_I(T) \cup P(T)$ where $P(T)$ is the pathos graph of a tree T .

Remark 4. If every edge of a tree T is of edgedegree n , then the corresponding vertex $P_{FI}(T)$ having degree $n + 1$.

Remark 5. If degree of a vertex in T is n , then the degree of the corresponding vertex in $P_{FI}(T)$ is $3n$.

Theorem 1. If T be a (p, q) tree whose vertices have degree d_i and b_i is the number of blocks to which a vertex v_i belongs in T , then the $P_{FI}(T)$ has $q + \sum b_i + k + 1$ vertices and $3q + b_i + \frac{1}{2} \sum d_i^2$ edges.

Proof. By Remark 2, semifull line graph $F_I(T)$ is spanning subgraph of $P_{FI}(T)$. The number of vertices of $P_{FI}(T)$ equal to the number of vertices of $F_I(T)$ and the number of path of pathos of T which is k . Thus $P_{FI}(T)$ has $q + \sum b_i + k + 1$ vertices.

Further by Remark 2, $F_I(T)$ is a spanning subgraph of $P_{FI}(T)$. The number of

edges in $P_{FI}(T)$ is the sum of the number of edges in $F_I(T)$ and the number of edges in each path of pathos of T which is q . By Theorem A, $F_I(T)$ has $2q + \sum b_i + \frac{1}{2} \sum d_i^2$ edges. Thus the number of edges in $P_{FI}(T) = 2q + \sum b_i + \frac{1}{2} \sum d_i^2 + q = 3q + \sum b_i + \frac{1}{2} \sum d_i^2$.

Theorem 2. For any tree T , pathos semifull line graph $P_{FI}(T)$ is non-separable if and only if T has no pendent pathos.

Proof. Suppose $P_{FI}(T)$ is non-separable. Assume that T contains at least one pendent pathos p_i of T . By the definition of $F_I(T)$, each edge is adjacent to exactly two of its end vertices $e_i = (v_i, v_{i+1})$ and each edge $e_i = b_i$ is a block. Hence the blockvertex is also adjacent to exactly two vertices. The adjacency between the vertices is same as that of T . Clearly $F_I(T)$ is non-separable. If T contains a pendent pathos $p_i = \{e_i\}$, then it is adjacent to the corresponding vertex e'_i in $P_{FI}(T)$, and e'_i becomes a cut vertex, a contradiction.

Converse part is obvious.

Theorem 3. The pathos semifull line graph $P_{FI}(T)$ of a tree T is planar if and only if $\Delta(T) \leq 3$ for every vertex v of T .

Proof. Suppose the pathos semifull line graph $P_{FI}(T)$ is planar. Assume $\Delta(T) \leq 4$. If there exists a vertex v of degree 4 clearly v lies on four blocks and four edges of T . Then T has the subgraph homeomorphic to $K_{1,4}$, it can be drawn in the plane such that $P_{FI}(T)$ has a subgraph homeomorphic to K_5 . By

Theorem C, $P_{FI}(T)$ is non-planar, a contradiction.

Conversely suppose $\Delta(T) \leq 3$. then by the Theorem B, the pathos line graph $P_L(T)$ is planar. Each block vertex is lies on exactly two vertices. This gives a $P_{FI}(T)$ planar.

Theorem 4. For any tree T , pathos semifull line graph $P_{FI}(T)$ is outer planar if and only if T is a path P_3 .

Proof. Suppose $P_{FI}(T)$ is outer planar. Assume that T is path P_n , $n \geq 4$, without loss of generality, we take $n=4$. By definition of $F_1(T)$, each edges are adjacent and edges are incident to the vertices and the vertices lies on block B. Clearly $F_1(T)$ is planar and all vertices lies on entire region since the pathos vertices p_i is adjacent to all edges e_1, e_2, e_3 of T to form a graph with e_3 as inner vertex. Hence $i(G) = 1$, a contradiction.

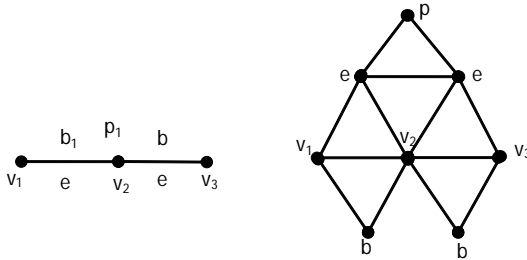


Figure 2.2

Conversely suppose T is path P_3 . By the definition of semifull line graph $F_1(P_3)$ is also planar. In P_3 there is only one pathos vertex which is adjacent to two vertices e_1

and e_2 of $F_1(P_3)$ gives $P_{FI}(T)$ outer planar as shown in figure 2.2.

Theorem 5. For any tree T , pathos semifull line graph $P_{FI}(T)$ is maximal outer planar if and only if P_3 .

Proof. Suppose $P_{FI}(T)$ is maximal outer planar then $F_1(T)$ is connected. By the Theorem 4, T is P_3 .

Conversely suppose T is P_3 . By Theorem 4, $P_{FI}(T)$ is outer planar. We now prove that $P_{FI}(T)$ maximal outer planar. By Theorem D, every maximal outer planar graph G with p vertices has $(2p-3)$ edges. In $P_{FI}(T)$, $p=8$, $q=13$. For $p=8$, then $2 \times 8 - 3 = 13$ edges which is maximal outerplanar.

Theorem 6. For any tree T , $P_{FI}(T)$ is minimally nonouter planar if and only if T is P_4 .

Proof. Suppose $P_{FI}(T)$ is minimally nonouter planar. Assume that T is P_5 and let

$P_5: v_1e_1v_2e_2v_3e_3v_4e_4v_5$ be a path. Since in a path, each edge is a block. We have $e_i' = \{v_i, v_{i+1}\} = b_i'$. By definition of $F_1(T)$, each e_i' and b_i' are adjacent to v_i and v_{i+1} . Also each e_i' and e_{i+1}' are adjacent to each other. Clearly $F_1(T)$ is outer planar. In $P_{FI}(T)$ the pathos vertex p_i is adjacent to all e_i' and the internal edges e_2 and e_3 becomes the internal vertices e_2' and e_3' in $P_{FI}(T)$, which gives $i[P_{FI}(T)] = 2$, a contradiction.

Conversely, suppose T is P_4 . Let $P_4 :$

$v_1e_1v_2e_2v_3e_3v_4$ be a path. Let e_1', e_2', e_3' be the vertices of $P_{FI}(T)$ corresponds to the edges e_1, e_2, e_3 of T .

Let $e_i = \{v_i, v_{i+1}\} = b_i$. By the definition $F_1(T)$, each e_i is adjacent to v_i, v_{i+1} and e_{i+1} . Also b_i is adjacent to v_i and v_{i+1} . Clearly $F_1(T)$ is outer planar. In $P_{FI}(T)$, the pathos vertex p_1 is adjacent to e_1, e_2, e_3 to form a graph with one internal vertex e_2 this gives $i[P_{FI}(T)] = 1$.

Theorem 7. The pathos semifull line graph $P_{FI}(T)$ is tree T has crossing number one if and only if $\Delta(G) \leq 3$ for every vertex of T and has a unique vertex of degree 4.

Proof. Suppose $P_{FI}(T)$ has crossing number one. Assume $\Delta(G) \geq 4$. We have the following cases.

Case 1. Suppose T has vertex v of degree 5. By the definition of line graph, the edges incident to the vertex together with the cut vertex form $\langle K_5 \rangle$ in $L(T)$. In $P_{FI}(T)$ the vertex v is adjacent to all vertices of e_1', e_2', e_3', e_4' corresponds to the edges e_1, e_2, e_3, e_4 of T to form a graph with crossing number $c[P_{FI}(T)] > 1$, a contradiction.

Case 2. Suppose T has two vertices v_1, v_2 of degree 4, then T contains two $K_{1,4}$ as sub graphs. By the definition, $P_{FI}(T)$ contains a subgraph homeomorphic to K_5 . Hence in $P_{FI}(T)$ has at least two K_5 sub graphs. Clearly $c[P_{FI}(T)] > 1$, a contradiction. Thus T has exactly one cut vertex of degree 4.

Conversely, suppose T holds both the

conditions of the theorem. For $\Delta(G) \leq 3$ by Theorem 3, $P_{FI}(T)$ is planar. Let $v \in T$ be unique vertex of T such that $\deg(v) = 4$. By definition of $P_{FI}(T)$, $P_{FI}(K_{1,4})$ is a graph homeomorphic to K_5 . Clearly $c[P_{FI}(T)] = 1$.

Theorem 8. For any tree T , pathos semifull line graph $P_{FI}(T)$ is non-complete graph.

Proof. For $T = P_2$, $F_1(T)$ is $K_4 - x$. In $P_{FI}(T)$, the pathos vertex p_1 is adjacent to e_1 to get a separable graph which is non-complete. For $T = P_n$, $n \geq 3$, there is no adjacency between the block vertices. Hence $P_{FI}(T)$ is always non-complete graph.

Theorem 9. For any tree T , pathos semifull line graph $P_{FI}(T)$ is not a bipartite graph.

Proof. By the definition of $F_1(T)$, each edge and its end vertices always form K_3 as induced subgraph. Clearly $F_1(T)$ is not bipartite and hence $P_{FI}(T)$ is also not a bipartite graph.

Theorem 10. For any tree T with P pathos semifull line graph $P_{FI}(T)$ is noneulerian.

Proof. Suppose T is a tree. In a tree there exist at least two pendent vertices. By Remark 5, degree of the corresponding vertices in $P_{FI}(T)$ is odd. Hence $P_{FI}(T)$ is noneulerian.

Theorem 11. The pathos semifull line graph $P_{FI}(T)$ of a tree T is Hamiltonian if T is a path P_n , n .

Proof. Suppose T be a path P_n : $v_1 e_1 v_2 e_2 \dots e_{n-1} v_n$ and $b_1 = e_1, b_2 = e_2, \dots, b_{n-1} = e_{n-1}$ be the blocks of T then T has exactly one path of pathos P_1 . Now the vertex set of $P_{Fl}(T)$ is $V[P_{Fl}(T)] = \{e'_1, e'_2, \dots, e'_{n-1}\} \cup \{b'_1, b'_2, \dots, b'_{n-1}\} \cup \{v_1, v_2, \dots, v_n\}$ corresponding edge set e_1, e_2, \dots, e_{n-1} , blocks set b_1, b_2, \dots, b_{n-1} of T , respectively. In $P_{Fl}(T)$, the vertices b_i and e_i are adjacent to the vertices v_i and v_{i+1} which are end vertices of e_i . Also in $P_{Fl}(T)$, e_i is adjacent to $e_{i+1} \forall i$. Clearly in $P_{Fl}(T)$, the Hamiltonian cycle $p_1 e'_1 v_1 b'_1 v_2 b'_2 v_3 b'_3 v_4 \dots b'_{n-1} v_n e'_{n-1} e'_{n-2} \dots e'_2 p_1$ exists as shown in figure 2.3. Hence $P_{Fl}(T)$ is Hamiltonian.

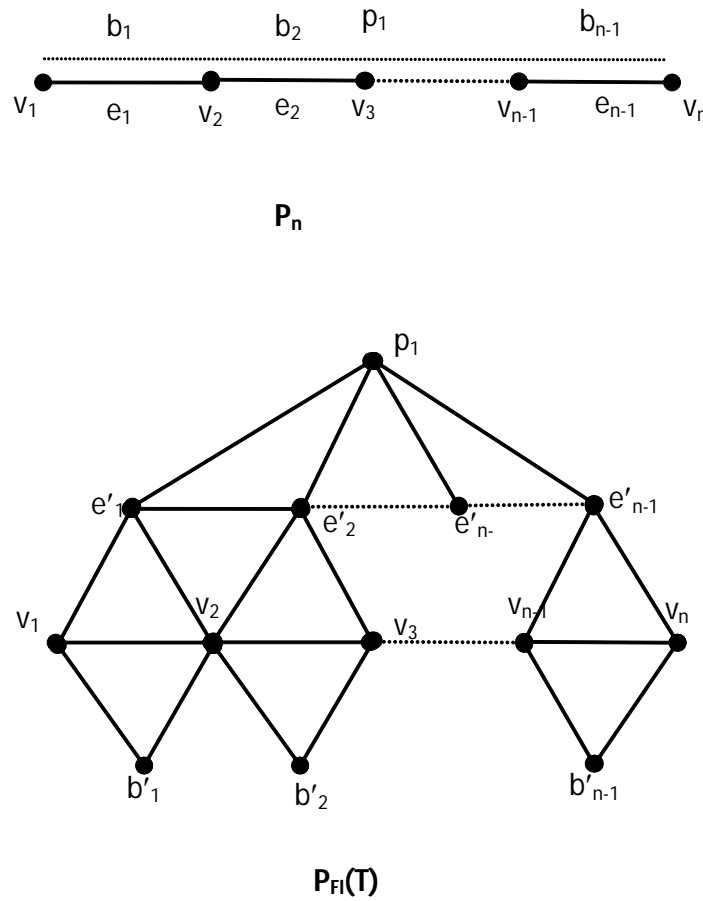


Figure 2.3

Total pathos semifull line graph $[T_{FI}(T)]$:

In this section we now define another graph valued function.

Definition. The Total pathos semifull line graph $T_{FI}(T)$ of a tree T is the graph whose vertex set is the union of vertices, edges, blocks and path of pathos of T in which two vertices are adjacent if the corresponding vertices of T are adjacent or edges of T are adjacent or path of pathos are adjacent or one corresponds to a vertex and other to an edge incident with it or one corresponds to a block B of T and other to a vertex v of T and v is in B or one corresponds to path of pathos p_i of T and other to an edge e of T and e is on the path p_i . Since the path of pathos of T is not unique then the corresponding $T_{FI}(T)$ is also not unique.

Remark 6. For any tree T , $P_{FI}(T)$ is spanning sub graph of $T_{FI}(T)$.

Theorem 12. If T is a (p, q) connected tree whose vertices have degree d_i and if b_i is the number of blocks to which v_i belongs in T , then the total pathos semifull line graph $T_{FI}(T)$ has $q + \sum b_i + k + 1$ vertices and $3q + \sum b_i + \frac{1}{2}\sum d_i^2 + \frac{1}{2}\sum P_{d_i}(P_{d_i} - 1)$ edges where P_{d_i} is the path degree of vertex v_i .

Proof. By Remark 6, $P_{FI}(T)$ is a spanning subgraph of $T_{FI}(T)$. The number of vertices of $T_{FI}(T)$ equals number of vertices of $P_{FI}(T)$. By Theorem 1, the number of vertices of $P_{FI}(T)$ is $q + \sum b_i + k + 1$. Hence

the number of vertices of $T_{FI}(T)$ is $q + \sum b_i + k + 1$.

Further by the definition of $T_{FI}(T)$, the number of edges of $T_{FI}(T)$ is the sum of the number of edges of $P_{FI}(T)$ and the number obtained by the path degree of vertex. By the Theorem 1 the number of edges in $P_{FI}(T)$ is $3q + \sum b_i + \frac{1}{2}\sum d_i^2$ and the number of edges obtained by the path degree is $\frac{1}{2}\sum P_{d_i}(P_{d_i} - 1)$. Hence in $T_{FI}(T)$, the number of edges is $3q + \sum b_i + \frac{1}{2}\sum d_i^2 + \frac{1}{2}\sum P_{d_i}(P_{d_i} - 1)$.

Theorem 13. The total pathos semifull line graph $T_{FI}(T)$ of a tree is planar if and only if T is $K_{1,3}$.

Proof. Suppose $T_{FI}(T)$ is planar we have the following cases.

Case 1. Assume that T be a tree with $\Delta(T) \leq 3$ a unique vertex of degree 4. By Theorem 7, $P_{FI}(T)$ has crossing number 1 and hence $T_{FI}(T)$ is non-planar which is a contradiction.

Case 2. Assume that T be a tree with $\Delta(T) \leq 3 \forall v \in T$. By Theorem 3, $P_{FI}(T)$ is planar and it contains at least two path pathos. These two pathos vertices are lies on different regions in $T_{FI}(T)$ while joining the edge two these two pathos vertices a becomes non-planar, a contradiction.

Conversely, suppose T is $K_{1,3}$, by Theorem 3, $P_{FI}(T)$ is planar. $K_{1,3}$ contains two path of pathos. The corresponds pathos

vertices of $T_{FI}(T)$ lies on one region, clearly $T_{FI}(T)$ is planar.

Theorem 14. The total pathos semifull line graph $T_{FI}(T)$ is always noneulerian.

Proof. Suppose T be a tree which contains at least two pendent vertices. By Remark 5, degree of the vertices $v_i^l v_j^l(T)$ in $T_{FI}(T)$ odd, by Theorem E, $T_{FI}(T)$ is noneulerian.

Theorem 15. For any tree T , $T_{FI}(T)$ is always nonhamiltonian.

Proof. Suppose T be a tree other than path. By Theorem 11, for any non- path tree T , $P_{FI}(T)$ is nonhamiltonian. Hence $T_{FI}(T)$ is nonhamiltonian.

Theorem 16. For any tree T , $T_{FI}(T)$ has crossing number one if and only if T is $K_{1,3}(P_n)$.

Proof. Suppose $T_{FI}(T)$ has crossing number one we have the following cases.

Case 1. Assume that T is $K_{1,4}$. By Theorem 7, $P_{FI}(T)$ has crossing number one. Also $K_{1,4}$ contains two path of pathos. In $T_{FI}(T)$ both pathos vertices lies on different region, while joining the edges between them two pathos vertices gives crossing number at least two, a contradiction.

Case 2. Assume that T is $K_{1,3}(P_n, P_n)$. By Theorem 7, $P_{FI}(T)$ has crossing number

one. Also T contains two path of pathos. In $T_{FI}(T)$, both pathos vertices lies on different regions. While joining the edge between these two pathos vertices gives crossing number at least two, a contradiction.

Conversely suppose T is $K_{1,3}(P_n)$. By the definition $P_{FI}(T)$, $P_{FI}(K_{1,3})$ is planar and $P_{FI}[K_{1,3}(P_n)]$ is also planar. In $K_{1,3}(P_n)$ there are two path of pathos. In $P_{FI}(T)$ both pathos vertices lies on difference and is planar. In $T_{FI}(T)$, these two pathos vertices joint to form crossing number one.

Conclusion

In this paper, we introduced the concept of the pathos semifull line graph and total pathos semifull line graph of a tree. We characterized the graphs whose pathos semifull line graph and total pathos semifull line graph are planar, noneulerian, Hamiltonian and crossing number one.

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