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C-Field Cosmological models with bulk viscosity and variable G in flat FRW space-time

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Abstract

Cosmological models with variable gravitational constant (G) and bulk viscosity in C-field cosmology for flat FRW space-time are investigated. To get the deterministic model of the universe, we have assumed $G = R^n$ where R is the scale factor and n is a constant. Under this assumption, we have investigated two C-field cosmological models for two cases: (i) $\zeta\theta$ = constant, (ii) ζ = constant, $\theta = \frac{3\dot{R}}{R}$ where ζ is the coefficient of bulk viscosity, θ the expansion in the model. The physical and geometrical aspects of the models and the effect of bulk viscosity related with the observation are discussed.

Key words: C-field Cosmology, bulk viscosity, variable G.

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1. Introduction

In the early universe, all the investigation dealing with the physical processes use a model of the universe, usually called the 'big-bang' model. However, the big-bang model is known to have the following problems: (i) The model has singularity in the past and possible one in future. The singularity signals mathematically inconsistency and physical incompleteness. (ii) The conservation of energy is violated in the big-bang model. (iii) The big-bang models based on reasonable equations of state lead to a very small particle horizon in the early epochs of the universe. This fact gives rise to the horizon problem in the universe. (iv) No consistent scenario exists within the frame work of big-bang model that explains the origin, evolution and characteristic of structures in the universe at small scales. (v) Flatness problem.

If a model explains successfully the creation of positive energy matter without violating the conservation of energy then it is necessary to have some degree of freedom which acts as a negative energy mode. Thus a negative energy field provides a natural way for creation of matter. It is worthwhile to mention that classical singularity theorems cease to be operational when positivity of energy density is not guaranteed. Thus, the introduction of a negative energy field may solve two of the five difficulties faced by big-bang model. Hoyle and Narlikar[1] adopted a field theoretic approach introducing a massless and chargeless scalar field to account for creation of matter. In C-field theory, there is no big-bang type singularity. Narlikar[2] has pointed out that matter creation is accomplished at the expense of negative-energy C-field. Narlikar and Padmanabhan[3] have obtained a solution of Einstein's field equations which admit relation and a negative energy massless scalar creation field as a source. They have shown that cosmological model based on this solution satisfies all the observational tests and is a viable alternative to the standard big-bang model and free from singularity and particle horizon and also provides a natural explanation to the flatness problem. Vishwakarma and Narlikar[4] have discussed modeling repulsive gravity with creation of matter. Bali and Tikekar[5] have investigated C-field cosmological models for dust distribution in flat FRW space-time with variable gravitational constant. Recently Bali and Kumawat[6] have investigated C-field cosmological models based on Hoyle-Narlikar theory with variable gravitational constant using FRW space-time for positive and negative curvature for dust distribution. It has been argued for a long time in the early stage of cosmic expansion that the dissipative process may well account for the high degree of isotropy we observe today. Padmanabhan and Chitre[7] have investigated that presence of bulk viscosity leads to inflationary like solution. The effect of bulk viscosity on the cosmological evolution has been studied by number of authors viz. Misner[8,9], Belinski and Khalatnikov[10], Saha[11], Sahni and Starobinsky[12], Bali and Pradhan[13]. Dirac[14] originally suggested the variation of gravitational constant on the basis of his large number hypothesis. In an evolving universe, it is natural to consider G=G(t) as G couples geometry to matter. Pochoda and Schwarzschild[15], Gamow[16] studied the solar evolution in the presence of a time varying gravitational constant. Demarque *et al.*[17] considered an ansatz in which $G \alpha t^{-n}$ and showed that for $|n| < 0.1$,

$$\left|\frac{\dot{G}}{G}\right| < 2 \, x \, 10^{-11} \, yr^{-1}$$

Barrow[18] assumed the ansatz $G \alpha t^{-n}$ and obtained from helium abundances

$$\left|\frac{\dot{G}}{G}\right| < (2 \pm 9.3)h \;\; x \, 10^{-12} \, yr^{-1}$$

by assuming flat universe and $-5.9x10^{-3} < h < 7x10^{-3}$.

Copi *et al.*[19] obtained a constraint on the variation of G by using WMAP and the big-bang nucleosynthesis observations for

$$-3\,x\,10^{-13}\,yr^{-1} < \left|\frac{\dot{G}}{G}\right| < 4\,x\,10^{-13}\,yr^{-1}.$$

Subsequently, mathematically well known posed alternative theories of gravity were developed to generalize Einstein's general theory of relativity by including variable G and satisfying conservation equation. Many attempts have been proposed for the possible extension of general relativity with time dependent G (Brans and Dicke[20], Hoyle and Narlikar[21,22], Canuto *et al.*[23]).

In this paper, we have investigated two C-field cosmological models with bulk viscosity and variable gravitational constant in flat FRW space-time for dust distribution. In the first model, we have assumed $G = R^n$, R is scale factor and n is a constant, $\zeta\theta$ = constant, ζ being the coefficient of viscosity and θ the expansion in the model. In the second model, we have assumed $G = \dot{R}/R, \theta = 3\dot{R}/R$, ζ being a constant and R is the scale factor. The physical aspects of the models related with the observations are also discussed.

1. The metric and field equations :

We consider the flat FRW model as

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2\,d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (1)$$

Einstein's field equation by introduction of C-field is modified as

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi G\,(\underset{(m)}{T}{}_i^j + \underset{(c)}{T}{}_i^j) \quad (2)$$

The energy momentum tensor for viscous fluid is taken as

$$\underset{(m)}{T}{}_i^j = \rho\, v_i v^j - \zeta\theta(g_i^j + v_i\, v^j) \quad (3)$$

and

$$\underset{(c)}{T}{}_i^j = -f\left(C_i\, C^j - \frac{1}{2} g_i^j\, C^\alpha\, C_\alpha\right) \quad (4)$$

where $f > 0$ and $C_i = \dfrac{dC}{dx^i}$.

Here two cases arise

Case I. $\zeta\theta$ = constant = β(say) (5)

After using the condition (5), the field equation (2) for metric (1) leads to

$$\frac{3\dot{R}^2}{R^2} = 8\pi G\,(t)\left[\rho - 2\beta - \frac{1}{2} f\, \dot{C}^2\right] \quad (6)$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = 8\pi G(t)\left[\frac{1}{2} f\dot{C}^2 - \beta\right] \quad (7)$$

The conservation equation

$$[8\pi G\, T_i^j]_{;j} = 0 \quad (8)$$

leads to

$$8\pi\dot{G}\left[\rho - 2\beta - \frac{1}{2} f\dot{C}^2\right] + 8\pi G\left[\dot{\rho} - f\,\dot{C}\ddot{C} + 3\rho\frac{\dot{R}}{R} - 3\beta\frac{\dot{R}}{R} - 3f\,\dot{C}^2\,\frac{\dot{R}}{R}\right] = 0 \quad (9)$$

which yields $\dot{C}=1$ when used in the source equation.

Equations (6) and (7) lead to

$$\frac{\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}=4\pi G\,[\rho-3\beta] \qquad (10)$$

Using $\dot{C}=1$ in equation (7), we have

$$\frac{2\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}=4\pi Gf-8\pi G\beta \qquad (11)$$

To obtain the deterministic solution of equation (11), in terms of cosmic time t, we assume

$$G=R^n \qquad (12)$$

where n is a constant and R is the scale factor.

From equations (11) and (12), we have

$$2\ddot{R}+\frac{\dot{R}^2}{R}=K\,R^{n+1} \qquad (13)$$

where $4\pi(f-2\beta)=K.$

To get the solution of equation (13), we assume that $\dot{R}=F(R).$

This leads to $\ddot{R}=F\,F'$ with $F'=\frac{dF}{dR}$. Thus equation (13) leads to

$$\frac{dF^2}{dR}+\frac{F^2}{R}=K\,R^{n+1} \qquad (14)$$

which leads to

$$F^2=\frac{KR^{n+2}}{n+3}+\frac{L}{R} \qquad (15)$$

which again leads to

$$\frac{\sqrt{R}\,dR}{\sqrt{KR^{n+3}+L(n+3)}}=\frac{dt}{\sqrt{n+3}} \qquad (16)$$

where L is the constant of integration.

To obtain the determinate value of R, we assume that n = –3/2. Thus equation (16) leads to

$$\frac{\sqrt{R}\,dR}{\sqrt{R^{3/2}+\frac{3}{2}\frac{L}{K}}}=\sqrt{\frac{2K}{3}}\,dt \qquad (17)$$

From equation (17), we have

$$R^{3/2}=(at+b)^2-\frac{3L}{2K} \qquad (18)$$

where

$$a=\frac{1}{2}\sqrt{\frac{3K}{2}} \qquad (19)$$

$$b=\frac{3N}{4} \qquad (20)$$

and N is the constant of integration. Thus we have

$$G = R^{-3/2} = \left[(at+b)^2 - \frac{3L}{2K}\right]^{-1} \quad (21)$$

From equations (10), (18) and (21), we have

$$8\pi\rho = \frac{8a^2(at+b)^2 - \frac{4La^2}{K}}{\left[(at+b)^2 - \frac{3L}{2K}\right]} + 24\pi\beta \quad (22)$$

Thus the metric (1) after using equation (18) leads to

$$ds^2 = dt^2 - \left[(at+b)^2 - \frac{3L}{2K}\right]^{4/3} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \quad 23(a)$$

which leads to

$$ds^2 = dt^2 - t^{8/3}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

by chosing

$$L = 0, a = 1, b = 0. \quad 23(b)$$

Now equation (9) leads to

$$8\pi[G\dot{\rho} + \dot{G}\rho] - 16\pi\dot{G}\beta - 4\pi\dot{G}f\dot{C}^2 - 8\pi Gf\dot{C}\ddot{C} + 24\pi G\frac{\rho\dot{R}}{R} - 24\pi G\frac{\beta\dot{R}}{R} - 24\pi Gf\dot{C}^2\frac{\dot{R}}{R} = 0 \quad (24)$$

Equations (18), (21), (22) and (24) lead to

$$\dot{C}^2\left[(at+b)^2 - \frac{3L}{2K}\right]^3 = \frac{1}{4\pi f}\int 48\beta\pi a(at+b)\left[(at+b)^2 - \frac{3}{2}\frac{L}{K}\right]^2 dt + \frac{1}{4\pi f}\int\left[16a^3(at+b)^3 - \frac{24L}{K}a^3 (at+b)\right]\left[(at+b)^2 - \frac{3}{2}\frac{L}{K}\right]dt \quad (25)$$

Thus equation (25) leads to

$$\dot{C}^2 = \left(\frac{2\beta}{f} + \frac{2a^2}{3\pi f}\right) \quad (26)$$

$a = \frac{1}{2}\sqrt{\frac{3K}{2}}$ and $K = 4\pi(f - 2\beta)$, then equation (26) leads to

$$\dot{C}^2 = 1 \quad (27)$$

which leads

$$\dot{C} = 1$$

and $C = t$. (28)

Here we find $\dot{C} = 1$, which agrees with the value used in source equation. Thus creation field C is proportional to time t and the metric (1) for the constraints mentioned above, leading to

$$ds^2 = dt^2 - \left[(at+b)^2 - \frac{3L}{2K}\right]^{4/3} [dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad 29(a)$$

which leads to

$$ds^2 = dt^2 - t^{8/3}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \quad 29(b)$$

by choosing a = 1, b = 0, L = 0. The model 29(b) is the same model as obtained by Bali and Tikekar[5].

The homogeneous mass density ρ, the gravitational constant G, the scale factor $\bar{a}(t)$ and the deceleration parameter q for the model (23) are given by

$$8\pi\rho = \frac{8a^2(at+b)^2 - \frac{4L}{K}a^2}{\left[(at+b)^2 - \frac{3L}{2K}\right]} + 24\pi\beta \quad (30)$$

$$G = \left[(at+b)^2 - \frac{3L}{2K}\right]^{-1} \quad (31)$$

$$\bar{a}(t) = \left[(at+b)^2 - \frac{3L}{2K}\right]^{2/3} \quad (32)$$

$$q = -\left[\frac{1}{2} - \frac{9L}{8K(at+b)^2}\right] \quad (33)$$

Case – II. Let ζ = constant and

$$\theta = \frac{3\dot{R}}{R} \quad (34)$$

For this case the field equation (2) for metric (1) leads to

$$\frac{3\dot{R}^2}{R^2} = 8\pi G(t)\left[\rho - \frac{6\zeta\dot{R}}{R} - \frac{1}{2}f\dot{C}^2\right] \quad (35)$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = 8\pi G(t)\left[\frac{1}{2}f\dot{C}^2 - \frac{3\zeta\dot{R}}{R}\right] \quad (36)$$

The conservation equation

$$[8\pi G\, T_i^j]_{;j} = 0$$

leads to

$$8\pi\dot{G}\left[\rho - \frac{6\zeta\dot{R}}{R} - \frac{1}{2}f\dot{C}^2\right] + 8\pi G\left[\dot{\rho} - \frac{6\zeta\ddot{R}}{R} - \frac{3\zeta\dot{R}^2}{R^2} - f\,\dot{C}\ddot{C} + \frac{3\dot{R}}{R}\rho - 3f\dot{C}^2\frac{\dot{R}}{R}\right] = 0 \quad (37)$$

which yields $\dot{C} = 1$ when used in the source equation.

Equations (35) and (36) lead to

$$\frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} = 4\pi G\left[\rho - \frac{9\zeta\dot{R}}{R}\right] \quad (38)$$

Using $\dot{C} = 1$ in equation (36), we have

$$\frac{2\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+24\pi G\frac{\zeta\dot{R}}{R}=4\pi Gf \quad (39)$$

To obtain deterministic solution of equation (39), we assume

$$G=\frac{\dot{R}}{R} \quad (40)$$

where R is a function of t alone.

Equations (39) and (40) lead to

$$\frac{2\ddot{R}}{R}+(K+1)\frac{\dot{R}^2}{R^2}=2\ell\frac{\dot{R}}{R} \quad (41)$$

where $24\pi\zeta=K$ and $4\pi f=2\ell$.

Equation (41) leads to

$$\frac{2\ddot{R}}{\dot{R}}+(K+1)\frac{\dot{R}}{R}=2\ell \quad (42)$$

From equation (42), we have

$$\dot{R}R^{\frac{K+1}{2}}=e^{\ell t} \quad (43)$$

Equation (43) leads to

$$R^{\frac{K+3}{2}}=\frac{e^{\ell t}}{\ell}\left(\frac{K+3}{2}\right)+N\left(\frac{K+3}{2}\right) \quad (44)$$

where N is constant of integration. Thus we have

$$G=\frac{\dot{R}}{R}=\frac{e^{\ell t}}{\left[\left(\frac{K+3}{2}\right)\frac{e^{\ell t}}{\ell}+N\left(\frac{K+3}{2}\right)\right]} \quad (45)$$

From equations (38), (44) and (45) using $24\pi\zeta=K$, we have

$$8\pi\rho=\frac{2N\ell\left(\frac{K+3}{2}\right)+3e^{\ell t}(K+2)}{\left[\left(\frac{K+3}{2}\right)\frac{e^{\ell t}}{\ell}+N\left(\frac{K+3}{2}\right)\right]} \quad (46)$$

Thus the metric (1) after using equation (44) leads to

$$ds^2=dt^2-\left[\frac{e^{\ell t}}{\ell}\left(\frac{K+3}{2}\right)+N\left(\frac{K+3}{2}\right)\right]^{\frac{4}{K+3}}$$

$$(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2) \quad (47a)$$

which leads to

$$ds^2=dt^2-e^{\frac{4t}{K+3}}[dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)] \quad (47b)$$

In absence of bulk viscosity *i.e.* $K\to 0$ then the model leads to

$$ds^2=dt^2-e^{\frac{4t}{3}}[dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)] \quad (47c)$$

by choosing N = 0 and $\ell=1$.

Now equation (37), using $24\pi\zeta=K$, and $4\pi f=2\ell$ leads to

$$8\pi[G\dot{\rho}+\dot{G}\rho]-2K\dot{G}\frac{\dot{R}}{R}-2\ell\dot{G}\dot{C}^2-2KG\frac{\ddot{R}}{R}-KG\frac{\dot{R}^2}{R^2}$$

$$-4G\dot{C}\ddot{C}+24\pi G\rho\frac{\dot{R}}{R}-12\ell G\dot{C}^2\frac{\dot{R}}{R}=0 \quad (48)$$

Substituting equation (44), (45) and (46) into equation (48), we have

$$\frac{d\dot{C}^2}{dt} + \frac{\left[N\ell\left(\frac{K+3}{2}\right) + 6e^{\ell t}\right]\dot{C}^2}{\left[\left(\frac{K+3}{2}\right)\frac{e^{\ell t}}{\ell} + N\left(\frac{K+3}{2}\right)\right]}$$

$$= \frac{N^2\ell\left(\frac{K+3}{2}\right)^2 + \frac{N}{2}(K+15)\left(\frac{K+3}{2}\right)e^{\ell t} + \frac{e^{2\ell t}}{\ell}(3K+9)}{\left[\left(\frac{K+3}{2}\right)\frac{e^{\ell t}}{\ell} + N\left(\frac{K+3}{2}\right)\right]^2} \quad (49)$$

To get the deterministic value of $\dot{C}$, we assume N = 0. Thus equation (49) leads to

$$\frac{d\dot{C}^2}{dt} + \frac{12\ell}{(K+3)}\dot{C}^2 = \frac{12\ell}{(K+3)} \quad (50)$$

From equation (50), we have

$$\dot{C}^2 = 1 \quad (51)$$

which leads to

$$C = t \quad (52)$$

Here we find $\dot{C} = 1$ which agrees with the value used in the source equation. Thus creation field C is proportional to time t and metric (1) for the constraints mentioned above, leading to

$$ds^2 = dt^2 - \left[\frac{e^{\ell t}}{\ell}\left(\frac{K+3}{2}\right)\right]^{\frac{4}{K+3}} (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \quad (53)$$

The homogeneous mass density ρ, the gravitational constant G, the scale factor $\bar{a}(t)$ and the deceleration parameter (q) for the model (47) are given by

$$8\pi\rho = \frac{2N\ell\left(\frac{K+3}{2}\right) + 3(K+2)e^{\ell t}}{\left[\left(\frac{K+3}{2}\right)\frac{e^{\ell t}}{\ell} + N\left(\frac{K+3}{2}\right)\right]} \quad (54)$$

$$G = \frac{\ell e^{\ell t}}{\left[\left(\frac{K+3}{2}\right)e^{\ell t} + \ell N\left(\frac{K+3}{2}\right)\right]} \quad (55)$$

$$\bar{a}(t) = \left[\frac{e^{\ell t}}{\ell}\left(\frac{K+3}{2}\right) + N\left(\frac{K+3}{2}\right)\right]^{2/K+3} \quad (56)$$

$$q = -\left[\frac{N\ell\left(\frac{K+3}{2}\right)e^{\ell t} + e^{2\ell t}}{e^{2\ell t}}\right] \quad (57)$$

In the absence of bulk viscosity, the above mentioned quantities lead to

$$8\pi\rho = 3N\ell + 6e^{\ell t} \quad (58)$$

$$G = \frac{\ell e^{\ell t}}{\frac{3}{2}e^{\ell t} + \frac{3\ell N}{2}} \quad (59)$$

$$\bar{a}(t) = \left(\frac{3e^{\ell t}}{2\ell} + \frac{3N}{2}\right)^{2/3} \quad (60)$$

$$q = -\left[\frac{\frac{3N\ell}{2}e^{\ell t} + e^{2\ell t}}{e^{2\ell t}}\right] \qquad (61)$$

Discussion and Conclusion

For the model (23), the spatial volume increases as time increases. Thus inflationary scenario exists. The matter density decreases as time increases. Since the deceleration parameter q<0, hence the model (23) represents an accelerating universe. $\left|\frac{\dot{G}}{G}\right| \simeq \frac{1}{t} \simeq H.$ $G \to \infty$ when $t \to 0$ and $G \to 0$ when $t \to \infty$. The creation field C increases with time and $\dot{C} = 1$. These results match with the astronomical observations and theoretical results.

For the model (53), the matter density decreases with time. The spatial volume increases with time. Thus inflationary scenario also exists for the model (53). $\left|\frac{\dot{G}}{G}\right| \simeq \frac{1}{t} \simeq H.$ $G \to \infty$ when $t \to 0$ and $G \to$ finite quantity when $t \to \infty$. The creation field C increases with time and $\dot{C} = 1$. Since the deceleration parameter q < 0, hence the model (53) represents an accelerating universe.

The coordinate distance to the horizon $r_H(t)$ is the maximum distance a null ray could have traveled at time t from the infinite past *i.e.*

$$r_H(t) = \int_{\infty}^{t} \frac{dt}{R(t)}$$

We could extend the proper time t to (–∞) in the past because of the non-singular nature of the space-time. Now

$$r_H(t) = \int_{0}^{t} \frac{dt}{\alpha t}$$

This integral diverges at lower limit showing that the models (23) and (53) are free from horizon.

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