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Non compatible mappings in fuzzy metric spaces

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Abstract

In this paper we have proved some common fixed point theorems for the class of four non compatible mappings in fuzzy metric spaces. These results are proved without exploiting the notion of continuity and without imposing any condition on t-norm.

1. Introduction and Preliminaries

The concept of fuzzy metric space has been introduced and generalized by many ways Deng[3], Kaleva and Seikkala[5]. George and Veeramani[4] modified the concept of fuzzy metric space introduced by Kramosil and Michalek[6]. They also obtained a Hausdorff topology for this kind of fuzzy metric space which has very important applications in quantum particle physics, particularly in connection with both string and ε^{∞} theory[9] and references mentioned therein). Many authors have proved fixed point and common fixed point theorems in fuzzy metric spaces[8], Pant[11], Singh and Chauhan[12], O'Regan and Abbas[10] obtained some necessary and sufficient conditions for the existence of common fixed point in fuzzy metric spaces. Recently, Cho *et al.*,[2] established some fixed point theorems for mappings satisfying generalized contractive condition in fuzzy metric space. Our results generalize several comparable results in existing literature Cho *et al.*[2], Beg and Abbas[1] and references mentioned therein.

2. Main Results

Theorem 2.1: Let (X, M, *) be a fuzzy metric space. Let A, B, S and T be maps from X into itself with $A^a(X) \subseteq T^t(X)$ and $B^b(X) \subseteq S^s(X)$ and there exists a constant $k \in (0, 1)$ such that

$$M(A^a x, B^b y, kt) \geq \Psi\{M(S^s x, T^t y, t), M(A^a x, S^s x, t), M(B^b y, T^t y, t), M(B^b y, S^s x, 2t), (A^a x, T^t y, t)\} \quad (1)$$

For all $x, y \in X$, $t > 0$ and $\Psi \in \varphi$. and a, b, s, $t \in N$. Then A, B, S and T have a unique common fixed point in X provided the pair {A, S} or {B, T} satisfies (EA) property. One of $A^a(X)$, $T^t(X)$, $B^b(X)$, $S^s(X)$ in a closed subset of X and the pairs {B, T} and {A, S} are weakly compatible.

Proof: Suppose that a pair {B, T} satisfies property (EA). Therefore there exists a sequence $\{x_n\}$in X such that $\lim_{n\to\infty} B^b x_n = z = \lim_{n\to\infty} T^t x_n$

Now $B^b(X) \subseteq S^s(X)$ implies that there exists a sequence $\{y_n\}$ in X such that

$B^b x_n = S^s y_n$. For $x=y_n$ and $y=x_n$,

(1) becomes.

$$M(A^a y_n, B^b x_n, kt) \geq \Psi\{M(S^s y_n, T^t x_n, t), M(A^a y_n, S^s y_n, t), M(B^b x_n, T^t x_n, t), M(B^b x_n, S^s y_n, 2t), M(A^a y_n, T^t x_n, t)\}$$

Taking limit $n \to \infty$ we obtain

$$\lim_{n\to\infty} M(A^a y_n, B^b x_n, kt) \geq \Psi\{\lim_{n\to\infty} M(S^s y_n, T^t x_n, t), \lim_{n\to\infty} M(A^a y_n, S^s y_n, t), \lim_{n\to\infty} M(B^b x_n, T^t x_n, t), \lim_{n\to\infty} M(B^b x_n, S^s y_n, 2t), \lim_{n\to\infty} M(A^a y_n, T^t x_n, t)\}$$

$$M(\lim_{n\to\infty} A^a y_n, z, kt) \geq \Psi\{M(z, z, t), M(\lim_{n\to\infty} A^a y_n, z, t), M(z, z, t), M(z, z, 2t), M(\lim_{n\to\infty} A^a y_n, z, t)\}$$

Since Ψ is increasing in each of its coordinate and Ψ (t, t, t, t, t) > t for all $t \in [0, 1]$

$$M(\lim_{n\to\infty} A^a y_n, z, kt) > M(\lim_{n\to\infty} A^a y_n, z, t)$$

Using by Mishra[7], we have $\lim_{n\to\infty} A^a y_n = z$,

Suppose that $S^s(X)$ in a closed subspace of X. Then $z = S^s u$ for some $u \in X$.

Now replacing x by u and y by x_{2n+1} in (1) we have

$$M(A^a{}_u, B^b x_{2n+1}, kt) \geq \Psi\{M(S^s{}_u, T^t x_{2n+1}, t), M(A^a{}_u, S^s{}_u, t), M(B^b x_{2n+1}, T^t x_{2n+1}, t), M(B^b x_{2n+1}, S^s{}_u, 2t), M(A^a{}_u, T^t x_{2n+1}, t)\}$$

Taking limits as $n \to \infty$ we obtain

$$M(A^a{}_u, z, kt) \geq \Psi\{M(z, z, t), M(A^a{}_u, z, t), M(z, z, t), M(z, z, 2t), M(A^a{}_u, z, t)\} > M(A^a{}_u, z, t)\}$$

Which implies that $A^a u = z$ Hence $A^a u = z = S^s u$. Since $A^a(X) \subseteq T^t(X)$ thee exist $v \in X$ such that $z = T^t v$. Following the argument similar to those given above we obtain $z = B^b v = T^t v$. Since u in coincidence point of the pair {A, S}, therefore $S^s A^a u = A^a S^s u$ and $A^A z = S^S z$ Now we claim that $A^a z = z$, if not, then using (1) we arrive at

$$\begin{aligned} M(A^a z, z, kt) &= M(A^a z, B^b v, kt) \\ &\geq \Psi\{M(S^s z, T^t v, t), M(A^a z, S^s z, t), M(B^b v, T^t v, t), M(B^b v, S^s z, 2t), M(A^a z, T^t v, t)\} \\ &\geq \Psi\{M(A^a z, z, t), M(A^a z, A^a z, t), M(z, z, t), M(z, A^a z, 2t), M(A^a z, z, t)\}, \\ &> M(A^a z, z, t) \end{aligned}$$

a Contradiction. Hence $z = A^a z = S^s z$. Similarly we can prove that $z = B^b z = T^t z$.

The uniqueness of z follows from (1).

Following theorem was proved in Cho *et al.*[2] .

Let (X, M, *) be a fuzzy metric space with t * t = t. Let A, B, S and T be map from X into itself with $A^a(X) \subseteq T^t(X)$ and $B^b(X) \subseteq S^s(X)$ and there exists a constant $k \in (0, 1)$ such that

$$M(A^a x, B^b y, kt) \geq \Psi\{M(S^s x, T^t y, t), M(A^a x, S^s x, t),$$

$M(B^b y, T^t y, t), M(B^b y, S^s x, 2t), M(A^a x, T^t y, t)\}$ (2)

For all $x, y, \in X$, $t > 0$ and $\Psi \in \varphi$. Then A, B, S, and T have a unique common fixed point in X provided the pair {A, S} and {B, T} are compatible of Type (II) and A or B are continuous or the pair {A, S} and {B, T} are compatible of type (I) and S or T are continuous.

Theorem 2.2: Let (X, M, *) be a fuzzy metric space. Let A, B, S and T be maps from X into it self such that.

$$M(A^a x, B^b y, kt) \geq \Psi\{M(S^s x, T^t y, t), M(A^a x, S^s x, t), M(B^b y, T^t y, t), M(B^b y, S^s x, 2t), M(A^a x, T^t y, t)\} \quad (3)$$

For all $x, y \in X$, $k \in (0, 1)$, $t > 0$ and a, b, s, t $\in N$ and $\Psi \in \Phi$.

Then A, B, S and T have a unique common fixed point in X provided the pair {A, S} and {B, T} satisfy common (EA) property, $T^t(X)$ and $S^s(X)$ are closed subset of X and the pairs {B, T} and {A, S} are weakly compatible.

Proof: Suppose that (A, S) and (B, T) satisfy a common (EA) property, there exist two sequences $\{x_n\}$ and $\{y_n\}$ such that

$$\lim_{n\to\infty} A^a x_n = \lim_{n\to\infty} S^s x_n = \lim_{n\to\infty} B^b y_n = \lim_{n\to\infty} T^t y_n = z$$

For some z in X. Since $S^s(X)$ and $T^t(X)$ are closed subspace of X, therefore $z = S^s u = T^t v$ for Some $u, v, \in X$. Now we claim that $A^a u = z$.

For this replace x by u and y by y_n in (3), we obtain.

$$M(A^a u, B^b y_n, kt) \geq \Psi\{M(S^s u, T^t y_n, t), M(A^a u, S^s u, t), M(B^b y_n, T^t y_n, t), M(B^b y_n, S^s u, 2t), M(A^a u, T^t y_n, t)\}$$

Taking limit as $n \to \infty$ we have

$$M(A^a u, z, kt) \geq \Psi\{M(A^a u, z, t), M(A^a u, A^a u, t), M(z, z, t), M(z, A^a u, 2t), M(A^a u, z, t)\}$$

$M(A^a u, z, kt) > M(A^a u, z, kt)$

Hence $A^a u = Z = S^s u$.

Again using (3) we have

$$M(T^t v, B^b v, kt) = M(A^a u, B^b v, kt),$$
$$\geq \Psi\{M(S^s u, T^t v\, t), M(A^a u, S^s u, t), M(B^b v, T^t v, t), M(B^b v, S^s u, 2t), M(A^a u, T^t v, t)\}$$
$$= \Psi\{M(z, T^t v, t), M(z, z, t), M(B^b v, T^t v, t), M(B^b v, z, 2t), M(z, T^t v, t)\}$$
$$> M(T^t v, B^b v, t)$$

Which implies that $T^t v = B^b v$ and hence $A^a u = z = S^s u = B^b v = T^t v$ the rest of the proof follows as in theorem 2.1, observe that the corollaries 3.4, 3.5, 3.6, 3.7 as 3.8 in Cho *et al.*[2], can be easily improved in the light of theorem 2.1 and 2.2.

Corollary 2.3: Let (X, M, *) be a fuzzy metric space, where * is any continuous t-norm. Let A, B, R, S, H and T be mappings from X into it self with $A^a(X) \subseteq T^t H^h(X)$, $B^b(X) \subseteq S^s R^r(X)$ and there exists a constant $K \in (0, 1)$ such that

$$M(A^a x, B^b y, kt) \geq \Psi\{M(S^s R^r x, T^t H^h y, t), M(A^a x, S^s R^r x, t), M(B^b y, T^t H^h y, t), M(B^b y, S^s R^r x, 2t), M(A^a x, T^t H^h y, t)\}$$

For all $x, y, \in X$, $t > 0$ and $\Psi \in \Phi$. Then A, B, R, S, H and T have a unique common fixed point in X provided the pair {A, SR} or {B, TH} satisfies (EA) property, one of $A^a(X)$, $T^tH^h(X)$, $B^b(X)$, $S^sR^r(X)$ is closed subset of X and the pair {B, TH} and {A, SR} are weakly compatible.

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