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Characterization of induced paired domination number of a graph

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Abstract

The concept of induced paired domination number of a graph was introduced by D.S.Studer, T.W. Haynes and L.M. Lawson[11], with the following application in mind. In the guard application an induced paired dominating set represents a configuration of security guards in which each guard is assigned one other as a designated backup with in (as in a paired dominating set), but to avoid conflicts (such as radio interference) between a guard and his/her backup, we require that the backup each guard be unique. Since among the guards only designated partners are adjacent to each other, we reduce the possibility of conflicts in communication. A set $S \subseteq V$ is a induced -paired dominating set if S is a dominating set of G and the induced subgraph <S> is a set of independent edges. The induced - paired domination number $\gamma_{ip}(G)$ is the minimum cardinality taken over all induced paired dominating sets in G. The minimum number of colours required to colour all the vertices so that adjacent vertices do not receive the same colour and is denoted by $\chi(G)$. Mahadevan[3-5] G, characterized the classes of all graphs whose sum of induced paired domination number and chromatic number of order up to $2n - 5$. In this paper we characterize the classes of all graphs whose sum of induced paired domination number and chromatic number equals to $2n - 6$, for any $n \geq 4$.

Key words: Induced Paired domination number, Chromatic number

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1. Introduction

Throughout this paper, by a graph we mean a finite, simple, connected and undirected graph G(V, E). For notations and terminology, we follow Haynes *et al.*[11]. The number of vertices in G is denoted by n. Degree of a vertex v is denoted by deg(v). We denote a cycle on n vertices by C_n, a path of n vertices by P_n, complete graph on n vertices by K_n. If S is a subset of V, then <S> denotes the vertex induced subgraph of G induced by S. A subset S of V is called a dominating set of G if every vertex in V - S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality of all such dominating sets in G. A dominating set S is called a total dominating set, if the induced subgraph <S> has no isolated vertices. The minimum cardinality taken over all total dominating sets in G is called the total domination number and is denoted by $\gamma_t(G)$. One can get a comprehensive survey of results on various types of domination number of a graph[10]. The chromatic number $\chi(G)$ is defined as the minimum number of colors required to color all the vertices such that adjacent vertices receive the same color[1-5].

Recently many authors have introduced different types of domination parameters by imposing conditions on the dominating set and/ or its complement. D.S.Studer, T.W. Haynes and L.M. Lawson[11], introduced the concept of induced paired domination number of a graph with the following application in mind. In the guard application an induced paired dominating set represents a configuration of security guards in which each guard is assigned one, other as a designated backup with in (as in a paired dominating set), but to avoid conflicts (such as radio interference) between a guard and his/her backup. We require that the backup each guard be unique. Since among the guards only designated partners are adjancent to each other, we reduce the possibility of conflicts in communication[2-7].

A set $S \subseteq V$ is a induced -paired dominating set if S is a dominating set of G and the induced subgraph <S> is a set of independent edges. The induced - paired domination number $\gamma_{ip}(G)$ is the minimum cardinality taken over all induced paired dominating sets in G.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In[8], Paulraj Joseph J and Arumugam S proved that $\gamma + \kappa \leq p$, where denotes the vertex connectivity of the graph. In[7], Paulraj Joseph J and Arumugam S proved that $\gamma_c + \chi \leq p + 1$ and characterized the corresponding extremal graphs. They also proved similar results for γ and γ_t. In[6], Mahadevan G Selvam A, Iravithul Basira A characterized the extremal of graphs for which the sum of the complementary connected domination number and chromatic number. In[3], Mahadevan G, characterized the classes of all graphs whose sum of induced paired domination number and chromatic number of order up to 2n-5. Motivated by the above results, in this paper we characterize all graphs for which $\gamma_{ip}(G) + \chi(G) = 2n - 6$ for any $n \geq 4$.

We use the following preliminary results and notations for our consequent characterization:

Theorem[11] *1.1:* If G is a connected graph of order $n \geq 3$, then $\gamma_{ip}(G) \leq n - 1$ and equality holds if and only if G is isomorphic to P_3, C_3, P_5 or G' where G' is the graph as in the following Figure 1.1.

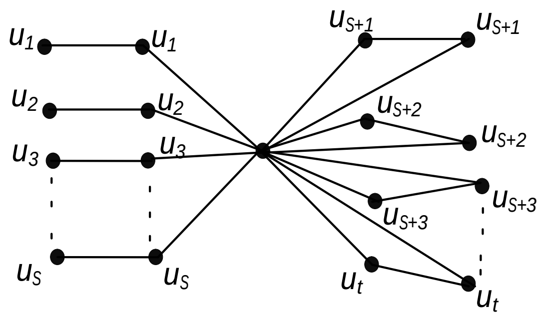

Figure 1.1 (where $s, t \geq 2$)

Notation 1.2 : $C_3(n_1, n_2, n_3)$ is a graph obtained from C_3 by attaching n_1 times the pendent vertex of P_{m1} (Path on m_1 vertices) to a vertex u_i of C_3 and attaching n_2 times the pendent vertex of P_{m2} (Path on m_2 vertices) to a vertex u_j for $i \neq j$ of C_3 and attaching n_3 times the pendent vertex of P_{m3} (Path on m_3 vertices) to a vertex u_k for $i \neq j$ k of C_3.

Notation 1.3: $K_4(u(P_{m1}, P_{m2}))$ is a graph obtained from K_4 by attaching the pendent vertex of P_{m1} (Path on m_1 vertices) and the pendent vertex of P_{m2} (Paths on m_2 vertices) to any vertex u of K_4.

Notation 1.4: $K_5(n_1P_{m1}, n_2P_{m2}, n_3P_{m3}, n_4P_{m4}, n_5P_{m5})$ is a graph obtained from K_5 by attaching n_1 times the pendent vertex of P_{m1} (Paths on m_1 vertices) to a vertex u_i of K_5 and attaching n_2 times the pendent vertex of P_{m2} (Paths on m_2 vertices) to a vertex u_j for $i \neq j$ of K_5 and attaching n_3 times the pendent vertex of P_{m3} (Paths on m_3 vertices) to a vertex u_k for $i \neq j \neq k$ of K_5 and attaching n_4 times the pendent vertex of P_{m4} (Paths on m_4 vertices) to a vertex u_l for $i \neq j \neq k \neq l$ of K_5 and attaching n_5 times the pendent vertex of P_{m5} (Paths on m_5 vertices) to a vertex u_m for $i \neq j \neq k \neq l \neq m$ of K_5.

Notation 1.5: $C_3(P_n)$ is the graph obtained from C_3 by attaching the pendant edge of P_n to any one vertices of C_3 and $K_n(P_m)$ is the graph obtained from K_n by attaching the pendant edge of P_m to any one vertices of K_n. For $n \leq p$, $K_p(n)$ is the graph obtained from K_p by adding a new vertex and join it with n vertices of K_p. $C_3(K_{1,n})$ is the graph obtained from C_3, by attaching the root vertex of $K_{1,n}$ to any one vertex of C_3.

2. Main Result

Theorem 2.1: For any connected graph G of order n, $n \geq 3$, $\gamma_{ip} + \chi = 2n-6$ if only if G $\cong K_8$, $K_4(P_4)$, P_6, C_6, $K_{1,4}$, $S^*(K_{1,3})$, $C_4(P_2)$, $K_4(P_3)$, $K_6(1)$, $K_6(2)$, $K_6(3)$, $K_6(4)$, $K_6(5)$, $K_4(u(P_3, P_3))$, $K_4(P_3, P_{-3}, 0, 0)$, $C_4(P_2, 0, P_2, 0)$, $C_4(P_3)$, $K_4(P_3, P_2, 0, 0)$, $K_4(P_2, P_2, 0, 0)$, $K_4(2P_2, 0, 0, 0)$ or any one of the graphs shown in Figure 2.1.

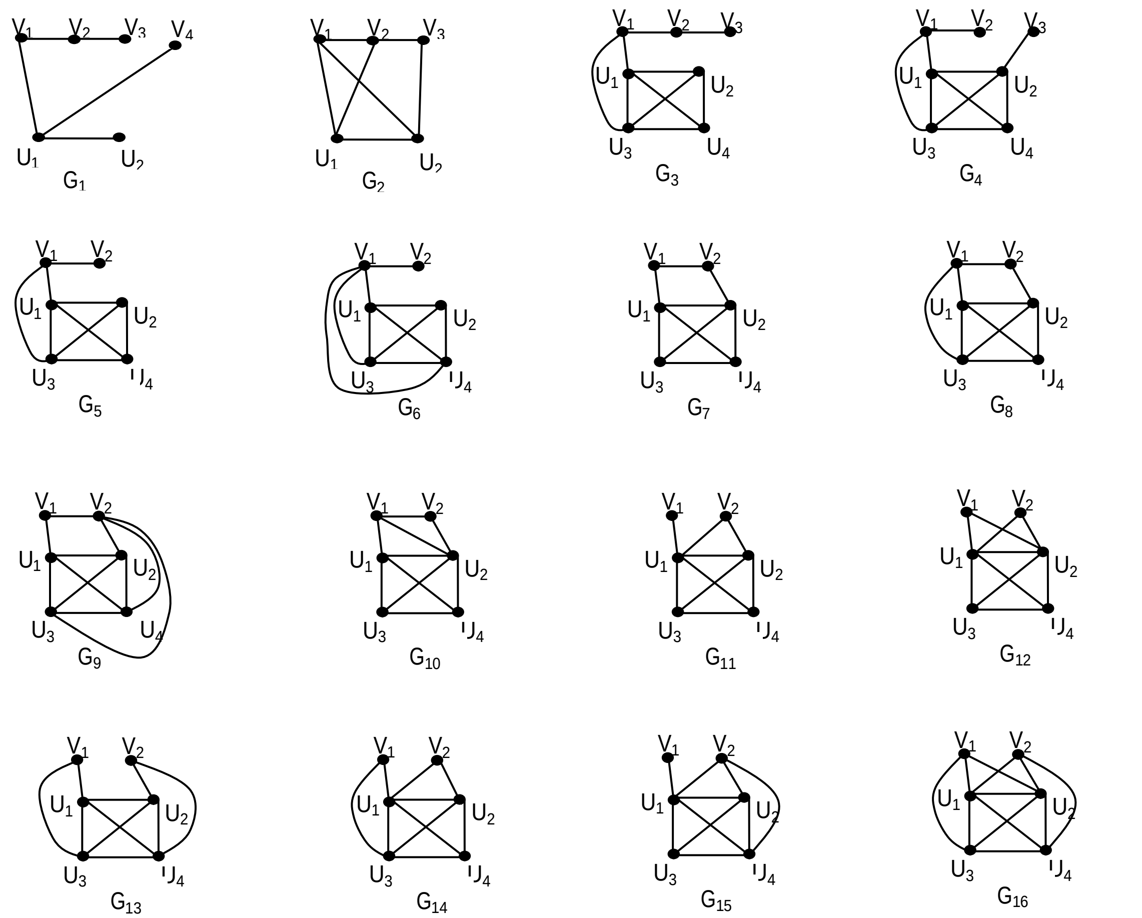

Figure 2.1

Proof: If G is any one the graphs given in the Figure 2.1, then it can be verified that $\gamma_{ip}(G)+\chi(G) = 10 = 2n - 6$. Conversely, let $\gamma_{ip}(G) + \chi(G) = 2n - 6$. Then the various possible cases are (i) $\gamma_{ip}(G) = n - 1$ and $\chi(G) = n - 5$ (ii) $\gamma_{ip}(G) = n - 2$ and $\chi(G) = n - 4$ (iii) $\gamma_{ip}(G) = n - 3$ and $\chi(G) = n - 3$ (iv) $\gamma_{ip}(G) = n - 4$ and $\chi(G) = n - 2$ (v) $\gamma_{ip}(G) = n - 5$ and $\chi(G) = n - 1$ (vi) $\gamma_{ip}(G) = n - 6$ and $\chi(G) = n$.

Case i. $\gamma_{ip}(G) = n - 1$ and $\chi(G) = n - 5$.

Since $\gamma_{ip}(G) = n - 1$, By theorem 1.1, G is isomorphic to P_3, C_3, P_5, or G'. Since $\chi(G) = n - 5$, G is not isomorphic to P_3, C_3 P_3, P_5, or G'. Hence no graph exists.

Case ii. $\gamma_{ip}(G) = n - 2$ and $\chi(G) = n - 4$.

Since $\chi(G) = n - 4$, G contains a clique K on n - 4 vertices or does not contain a clique K on n-4 vertices. Let G contains a clique K on n - 4 vertices. Let $S = \{v_1, v_2, v_3, v_4\}$. Then the induced subgraph <S> has the following possible cases. <S> = K_4, $\overline{K}_4$, P_4, C_4, $K_{1,3}$, $K_2 \cup K_2$, $K_3 \cup K_1$, $\{K_4 - e\}$,) $C_3(1, 0, 0)$, $P_3 \cup K_1$, $K_2\ \overline{K}_2$.

Subcase i: Let <S> = K_4.

Since G is connected, there exists a vertex u_i of K_{n-4} which is adjacent to any one of $\{v_1, v_2, v_3, v_4\}$. Let u_i be adjacent to v_1 for some i in K_{n-4}. Then $\{v_1, u_i\}$ is an γ_{ip} set of G, so that $\gamma_{ip} = 2$ and n= 4, which is a contradiction. Hence no graph exists.

Subcase ii. Let <S> = $\overline{K}_4$.

Let $\{v_1,v_2,v_3,v_4\}$ be the vertices of $\overline{K}_4$. Since G is connected, two vertices of the $\overline{K}_4$ are adjacent to one vertex say u_i and the remaining two vertices of $\overline{K}_4$ are adjacent to one vertex say u_j for $i \neq j$. In this case $\{u_i, u_j\}$ for $i \neq j$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and n = 4, which is a contradiction. Hence no graph exists. Since G is connected, one vertex of $\overline{K}_4$ is adjacent to u_i and the remaining three vertices of $\overline{K}_4$ are adjacent to vertex say u_j for $i \neq j$. In this case $\{u_i, u_j\}$ for $i \neq j$ forms a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and n = 4 , which is a contradiction. Hence no graph exists. Let $\{v_1, v_2, v_3, v_4\}$ be the vertices of $\overline{K}_4$. Since G is connected, all the vertices of are adjacent to one vertex say u_i in the vertices of K_{n-4}. In this case $\{u_i, u_j\}$ for $i \neq j$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and n = 4, which is a contradiction. Hence no graph exists. Since G is connected, two vertices of $\overline{K}_4$ is adjacent to u_i and one vertex is adjacent to u_j for $i \neq j$ and the remaining one vertex is adjancent to a vertex say u_k for $i \neq j \neq k$. In this case γ_{ip} set does not exist. If u_i is adjacent to v_1 and u_j for $i \neq j$ is adjacent to v_2 and u_k for $i \neq j \neq k$ is adjacent to v_3 and u_s for $i \neq j \neq k \neq s$ is adjacent to v_4. In this case γ_{ip} set does not exists.

Subcase iii. Let <S> = $P_4 = v_1v_2v_3v_4$.

Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1(or v_4) or v_2 (or v_3). If u_i is adjacent to v_1, then $\{u_i, u_j, v_2, v_3\}$ forms a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and n = 6. Hence $K = K_2 = u_1u_2$. If u_1 is adjacent to v_1. If $\deg(v_1)=2$, $\deg(v_2) =\deg(v_3)=2$, $\deg(v_4) = 1$, then G P_6. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_4. If $\deg(v_1) = 2 = \deg(v_2)$, $\deg(v_3) = 2 = \deg(v_4)$, then $G \cong C_6$. If u_i is adjacent to v_2, then $\{u_j, u_k, v_2, v_3\}$ for some u_j and u_k for $i \neq j \neq k$ in K_{n-4} forms a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and n = 6 and hence $K = K_2 = u_1u_2$. Hence no graph exists.

Subcase iv: Let <S> = $K_2 \cup K_2$.

Let v_1, v_2 be the vertices of K_2 and v_3, v_4 be the vertices of K_2. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to any one of $\{v_1, v_2\}$ and any one of $\{v_3, v_4\}$.

Let u_i be adjacent to v_1 and v_3. In this case $\{u_j, u_k, v_1, v_2, v_3, v_4\}$ for some u_j and u_k for $i \neq j \neq k$ in K_{n-4} forms an γ_{ip} set of G so that $\gamma_{ip} = 6$ and $n = 8$ and hence $K = K_4 = <u_1, u_2, u_3, u_4>$. Let u_3 be adjacent to v_1 and v_3. If $deg(v_1) = 2 = deg(v_3)$, $deg(v_2) = 1 = deg(v_4)$ then $G \cong K_4(u(P_3, P_3))$. Since G is connected ,there exists a vertex u_i in k_{n-4} which is adjacent to v_3 and u_j for $i \neq j$ is adjacent to v_1 and u_k for $i \neq j \neq k$ and u_s for $i \neq j \neq k \neq s$. In this case $\{v_1, v_2, v_3, v_4, u_k, u_s\}$ is a γ_{ip} set of G so that $\gamma_{ip} = 6$ and $n = 8$ and hence $K = K_4 = <u_1, u_2, u_3, u_4>$. Let u_3 be adjacent to v_3 and u_1 be adjacent to v_1. If $deg(v_1) = 2 = deg(v_3)$, $deg(v_2) = 1 = deg(v_4)$ then $G \cong K_4(P_3, P_3, 0, 0)$.

Subcase v: $<S>=K_2 \cup \overline{K}_2$.

Let v_1,v_2 be the vertices of $\overline{K}_2$ and v_3, v_4 be the vertices of K_2. Since G is connected, there exists a vertex u_i in K_{n-4}, which is adjacent to v_1 and v_2 and any one of $\{v_3,v_4\}$. Let u_i be adjacent to v_1, v_2 , v_3. In this case $\{u_i,v_3\}$ is a γ_{ip} set of G so that $\gamma_{ip} = 2$ and $n = 4$, which is a contradiction . Hence no graph exists. Since G is connected there exists a vertex u_i in K_{n-4} which is adjacent to v_1 and there exists a vertex u_j for $i \neq j$ in K_{n-4} is adjacent to v_2 and v_3. In this case γ_{ip} does not exist. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent v_1 and u_j for $i \neq j$ and u_k for $i \neq j \neq k$ is adjacent to v_3. In this case $\{u_i, u_j, v_3, v_4\}$ forms a γ_{ip} set of G. So that $\gamma_{ip}=4$ and $n =6$, which is a contradiction. Hence no graph exists.

Subcase vi: $<S> = P_3 \cup K_1$.

Let v_1, v_2, v_3 be the vertices of P_3 and v_4 be the vertex of K_1. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to any one of $\{v_1, v_2, v_3\}$ and v_4. In this case $\{u_i, v_2, v_3, v_4\}$ is a γ_{ip} set of G so that $\gamma_{ip} = 4$ and $n = 6$. Hence $K = K_2 = <u_1, u_2>$. Let u_1 be adjacent to v_1 and v_4. If $deg(v_1) = 2 = deg(v_2)$, $deg(v_4) = 1 = deg(v_3)$ then $G \cong G_3$. Let u_1 be adjacent to v_1 and v_4 and u_2 be adjacent to v_2. If $deg(v_1)=2$, $deg(v_2)= 3$, $deg(v_3) = 1 = deg(v_4)$, then $G \cong C_4(P_2, 0, P_2, 0)$. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 and u_j for $i \neq j$ is adjacent to v_4. In this case $\{u_i, u_j, v_2, v_3\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and $n = 6$ and hence $K = K_2 = u_1u_2$. Let u_1 be adjacent to v_1 and v_3 and u_2 be adjacent to v_4. If $deg(v_1) = 2$, $deg(v_2) = deg(v_3) = 2$, $deg(v_4) = 1$, then $G \cong C_4(P_3)$. Since G is connected, there exists a vertex u_i in K_{n-4}, which is adjacent to v_2 and u_j for $i \neq j$ is adjacent to v_4. In this case $\{u_j, u_k, v_2, v_3\}$ for some u_j for $i \neq j \neq k$ in K_{n-4} is a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and $n = 6$ and hence $K = K_2 = <u_1, u_2>$. Let u_1 be adjacent to v_2 and u_2 be adjacent to v_4. If $deg(v_1) = 1$, $deg(v_4) = deg(v_3) = 1$, $deg(v_2) = 3$, then $G \cong G_1$.

Subcase vii : $<S> = K_3 \cup K_1$.

Let v_1, v_2, v_3 be the vertices of K_3 and v_4 be the vertices of K_1. Since G is connected, there exists a vertex u_i in K_{n-4} is adjacent to any one of $\{v_1, v_2, v_3\}$ and v_4. In this case

$\{u_i, v_2\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 4$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_2 and u_j for $i \neq j$ is adjacent to v_4. In this case $\{u_j, u_k, v_1, v_2\}$ for some u_j for $i \neq j \neq k$ in K_{n-4} is a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and $n = 6$ and hence $K = K_2$, which is a contradiction. Hence no graph exists.

Subcase viii: $<S> = K_4 - \{e\}$

Let v_1, v_2, v_3, v_4 be the vertices of K_4. Let $\{e\}$ be any one the edge inside the cycle C_4. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1 of degree 3. In this case $\{u_i, v_1\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 4$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_2 of degree 2. In this case $\{u_i, u_j, v_1, v_3\}$ is a γ_{ip} set of G. So that $\gamma_{ip} = 4$ and $n = 6$, which is a contradiction. Hence no graph exists.

Subcase ix: $<S> = C_3(1, 0, 0)$.

Let v_1, v_2, v_3 be the vertices of C_3 and v_4 is adjacent to v_1. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_2 and u_j for $i \neq j$ and u_k for $i \neq j \neq k$. In this case $\{u_j, u_k, v_1, v_2\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and $n = 6$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1. In this case $\{u_i, v_1\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 4$, which is a contradiction. Hence no graph exists. Since G isconnected, there exists a vertex u_i in K_{n-4} which is adjacent to v_4 and u_j for $i \neq j$. In this case $\{u_i, u_j, v_1, v_2\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and $n = 6$, which is a contradiction. Hence no graph exists.

Subcase x: $<S> = K_{1,3}$.

Let v_1 be the root vertex and v_2, v_3, v_4 are adjacent to v_1. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to v_1. In this case $\{u_i, v_1\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 4$, which is a contradiction. Hence no graph exists. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to any one of $\{v_2, v_3, v_4\}$. Let u_i be adjacent to v_2. In this case $\{u_j, u_k, v_1, v_2\}$is a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and $n = 6$, and hence $K = K_2 = <u_1, u_2>$. Let u_1 be adjacent to v_2. If $\deg(v_1) = 3$, $\deg(v_2) = 2$, $\deg(v_3) = 1 = \deg(v_4)$, then G G_1. Let u_1 be adjacent to v_2 and v_3. If $\deg(v_1) = 3$, $\deg(v_2) = 2 = \deg(v_3)$, $\deg(v_4) = 1$, then $G \cong C_4(P_2, 0, P_2, 0)$. Let u_1 be adjacent to v_2 and v_4. If $\deg(v_1) = 3$, $\deg(v_2) = 2 = \deg(v_4)$, $\deg(v_3) = 1$, then $G \cong C_4(P_2, 0, P_2, 0)$.

Subcase xi: $<S> = C_4$.

In this case it can be verified that no graph exists.

If G does not contain a clique K on n-4 vertices, then it can be verified that no new graph exits.

Case iii. $\gamma_{ip} = n - 3$ and $\chi = n - 3$.

Since $\chi = n - 3$, G contains a clique K on n - 3 vertices or does not contain a clique K on n - 3 vertices. Let G contains a clique K on $n - 3$. Let $S = V(G) - V(K) = \{v_1, v_2, v_3\}$. Then the induced subgraph <S> has the following possible cases. $<S> = K_3$ $\overline{K}_3$, P_3, $K_2 \cup K_1$.

Subcase i: $<S> = K_3 = <v_1, v_2, v_3>$.

Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to any one of $\{v_1, v_2, v_3\}$. Let u_i be adjacent to v_1, then $\{u_i, v_1\}$is an γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 5$, which is a contradiction. Hence no graph exists.

Subcase ii: $<S> = \overline{K}_3 = <v_1, v_2, v_3>$.

Since G is connected, one of the vertices of K_{n-3} say u_i is adjacent to all the vertices of S (or) u_i be adjacent to v_1 ,v_2 and u_j be adjacent to v_3 ($i \neq j$) (or) u_i be adjacent v_1 and u_j be adjacent to v_2 and u_k be adjacent to v_3 ($i \neq j \neq k$). If u_i for some i is adjacent to all the vertices of S, then $\{u_i, u_j\}$ for some u_j for ($i \neq j$) in K_{n-3} is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 5$ and hence $K = K_2 = <u_1, u_2>$. If u_1 is adjacent to v_1, v_2 and v_3, then $G \cong (K_{1,4})$. If u_i is adjacent to v_1 and u_j for $i \neq j$ is adjacent to v_2 and v_3, then $\{u_i,u_j\}$ is an γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 5$. Hence $K = K_2 = <u_1,u_2>$. If u_1 is adjacent to v_1 and v_2 and u_2 is adjacent to v_3, then $G \cong S^*(K_{1,3})$. Since G is connected, If u_i is adjacent to v_1, u_j for $i \neq j$ in K_{n-3} is adjacent to v_2 and u_k for $i \neq j \neq k$ in K_{n-3}, is adjacent to v_3. Then γ_{ip} set does not exist.

Subcase iii: $<P_3> = v_1v_2v_3$.

Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to v_1 (or equivalently v_3) or v_2. If u_i is adjacent to v_2, then $\{u_i, v_2\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 5$. Hence $K = K_2 = <u_1,u_2>$. If u_1 is adjacent to v_2, then $G \cong S^*(K_{1,3})$. If u_1 is adjacent to v_2 and u_2 is adjacent to v_3. If $\deg(v_1) = 1$, $\deg(v_2) = 3$, $\deg(v_3) = 2$, then $G \cong C_4(P_2)$. If u_1 is adjacent to v_2 and u_2 is adjacent to v_1 and v_3. If $\deg(v_1) = 3$, $\deg(v_2) = 3$, $\deg(v_3) = 2$, then $G \cong G_2$. Since G is conneted, there exists a vertex $u_i$in K_{n-3} which is adjacent to v_1. Then $\{u_i, u_j, v_2, v_3\}$ and u_j for $i \neq j$ is a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and $n = 7$ and hence $K = K_4 = <u_1,u_2,u_3,u_4>$. Let u_1 be adjacent to v_1. If $\deg(v_1) = 2 = \deg(v_2)$, $\deg(v_3) = 1$, then $G \cong K_4(P_4)$. Let u_1 be adjacent to v_1 and u_3 be adjacent to v_1. If $\deg(v_1) = 3$, $\deg(v_2) = 2$, $\deg(v_3) = 1$, then $G \cong G_3$.

Subcase iv: $<S>=K_2 \cup K_1$.

Let v_1, v_2 be the vertices of K_2 and v_3 be the isolated vertex. Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to any one of $\{v_1, v_2\}$ and $\{v_3\}$ (or) u_i is adjacent to any one of $\{v_1, v_2\}$ and u_j for $i \neq j$ is adjacent to v_3. In this case $\{v_1, v_2, v_3, u_j\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 4$ and $n = 7$ and hence $K=K_4=<u_1,u_2,u_3,u_4>$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_3. If $\deg(v_1) = 2$, $\deg(v_2) = 1=\deg(v_3)$, then $G \cong K_4(P_3, P_2, 0, 0)$. Let u_1 be adjacent to v_1 and u_3 be adjacent to v_1 and u_2 be adjacent to v_3. If $\deg(v_1) = 3$, $\deg(v_2) = 1 = \deg(v_3)$, then $G \cong G_4$. If a vertex

u_i in K_{n-3} is adjacent to v_1 and v_3 then $\{u_i, v_1\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 5$ and hence $K = K_2 = <u_1, u_2>$. Let u_1 be adjacent to v_1 and v_3. If $\deg(v_1) = 2$, $\deg(v_2) = 1 = \deg(v_3)$ then $G \cong S^*(K_{1,3})$.

If G does not contain a clique K on n - 3 vertices, then it can be verified that no new graph exists.

Case v: $\gamma_{ip} = n - 4$ and $\chi = n - 2$.

Since $\chi = n - 2$, G contains a clique K on n - 2 vertices or does not contain a clique K on n - 2 vertices. Let G contains a clique K on n - 2 vertices. If G contains a clique K on n - 2 vertices. Let $S = V(G) - V(K) = \{V_1, V_2\}$. Then $<S> = K_2$, $\overline{K}_2$.

Subcase i: $<S> = K_2$.

Since G is connected, there exists a vertex u_i in K_{n-2} is adjacent to any one of $\{v_1, v_2\}$ then $\{u_i, v_1\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 6$ and hence $K = K_4 = <u_1, u_2, u_3, u_4>$. Let u_1 be adjacent to v_1. If $\deg(v_1) = 2$, $\deg(v_2) = 1$, then $G \cong K_4(P_3)$. Let u_1 be adjacent to v_1 and u_3 be adjacent to v_1. If $\deg(v_1) = 3$, $\deg(v_2) = 1$, then $G \cong G_5$. Let u_1 be adjacent to v_1 and u_3 be adjacent to v_1 and u_4 be adjacent to v_1. If $\deg(v_1) = 4$, $\deg(v_2) = 1$ then $G \cong G_6$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2. If $\deg(v_1) = 2 = \deg(v_2)$, then $G \cong G_7$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_1. If $\deg(v_1) = 3$, $\deg(v_2) = 2$, then $G \cong G_8$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_2 and u_4 be adjacent to v_2. If $\deg(v_1) = 2$, $\deg(v_2) = 4$, then $G \cong G_9$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2. If $\deg(v_1) = 3$, $\deg(v_2) = 2$, $G \cong G_{10}$.

Subcase ii: Let $<S> = \overline{K}_2$.

Since G is connected, v_1 and v_2 are adjacent to a common vertex say u_i of K_{n-2} (or) v_1 is adjacent to u_i for some i and v_2 is adjacent to u_j for some $i \neq j$ in K_{n-2}. In both cases $\{u_i, u_j\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and $n = 6$ and hence $K = K_4 = <u_1, u_2, u_3, u_4>$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2. If $\deg(v_1) = 1 = \deg(v_2)$, then $G \cong K_4(P_2, P_2, 0, 0)$. Let u_1 be adjacent to v_1 and v_2. If $\deg(v_1) = 1 = \deg(v_2)$, then $G \cong K_4(2P_2, 0, 0, 0)$. Let u_1 be adjacent to v_1 and v_2 and u_2 be adjacent to v_2. If $\deg(v_1) = 1$, $\deg(v_2) = 2$, then $G \cong G_{11}$. Let u_1 be adjacent to v_1 and v_2 and u_2 be adjacent to v_1 and v_2. If $\deg(v_1) = 2 = \deg(v_2)$, then $G \cong G_{12}$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_1 and u_4 be adjacent to v_2. If $\deg(v_1) = 2 = \deg(v_2)$, then $G \cong G_{13}$. Let u_1 be adjacent to v_1 and v_2, u_2 is adjacent to v_2 and u_3 be adjacent to v_1. If $\deg(v_1) = 2$, $\deg(v_2) = 2$, then $G \cong G_{14}$. Let u_1 be adjacent to v_1 and v_2 and u_2 be adjacent to v_2 and u_4 be adjacent to v_2. If $\deg(v_1) = 1$, $\deg(v_2) = 3$, then $G \cong G_{15}$. Let u_1 be adjacent to v_1 and v_2 and u_2 be adjacent tov$_1$ and v_2 and u_3 be adjacent to v_1 and u_4 be adjacent to v_2. If $\deg(v_1) = 3$, $\deg(v_2)=3$, then $G \cong G_{16}$.

If G does not contain a clique K on n-2 vertices, then it can be verified that no new graph exits.

Case v: $\gamma_{ip} = n - 5$ and $\chi = n - 1$.

Since $\chi = n - 1$, G contains a clique K on n - 1 vertices. Let v_1 be the vertex not on K_{n-1}. Since G is connected, there exists a vertex v_1 is adjacent to one vertex u_i of K_{n-1}. In this case $\{u_i,v_1\}$ is a γ_{ip} set of G, so that $\gamma_{ip} = 2$ and n = 7 and hence $K = K_6 = \langle u_1,u_2,u_3,u_4,u_5,u_6 \rangle$. Let u_1 be adjacent to v_1. If $\deg(v_1) = 1$, then $G \cong K_6(1)$. Let u_1 be adjacent to v_1 and u_6 be adjacent to v_1. If $\deg(v_1) = 2$, then $G \cong K_6(2)$. Let u_1 be adjacent to v_1 and u_6 be adjacent to v_1 and u_5 be adjacent to v_1. If $\deg(v_1) = 3$, then $G \cong K_6(3)$. Let u_1 be adjacent to v_1 and u_6 be adjacent to v_1 and u_5 be adjacent to v_1 and u_4 be adjacent to v_1. If $\deg(v_1) = 4$, then $G \cong K_6(4)$. Let u_1 be adjacent to v_1 and u_6 be adjacent to v_1 and u_5 be adjacent to v_1 and u_4 be adjacent to v_1 and u_3 be adjacent to v_1. If $\deg(v_1) = 5$, then $G \cong K_6(5)$.

Case vi. $\gamma_{ip} = n - 6$ and $\chi = n$.

Since $\chi = n$, $G = K_n$. But for K_n, $\gamma_{ip} = 2$, so that n = 8. Hence $G \cong K_8$.

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