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Magnetized Bianchi type III anti-stiff fluid consmological models with time dependent Λ and variable magnetic permeability

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Abstract

The behavior of cosmological constant in homogeneous Bianchi type-III universe is investigated for magnetized anti-stiff perfect fluid distribution. The magnetic permeability is assumed as a variable quantity. The magnetic field is due to an electric current produce along x- axis, thus F_{23} is the only non vanishing component of electromagnetic field tensor. For the complete determination of the model we assume a condition between metric potential A = C^n. Physical and Geometrical features of model in presence and absence of magnetic field are also discussed.

Key words: Bianchi type III, electromagnetic field, anti-stiff perfect fluid, Variable Λ, Magnetic Permeability.

Introduction

In the recent years there has been a lot of interest in the study of large scale structure of the universe because of the fact that the origin of the structure in the universe is one of the greatest cosmological mysteries even today.

Spatially homogeneous and anisotropic cosmological models play a significant role in description of the large scale behavior. The choice of anisotropic model in the Einstein system of field equation permits us to obtain cosmological model more general than FRW models. The Einstein field equations are a coupled system of highly non-linear differential equation and we seek physical solution to the field equation for applications in cosmology and astrophysics.

Tikekar and Patel[1] have investigated some exact solution of massive string for Bianchi type-III space time in presence and absence of magnetic field. Bali and Dave[2] investigated the Bianchi type-III string cosmological model with bulk viscosity. Recently Bali and Pradhan[3] investigated the Bianchi type-III string cosmological model with time dependent bulk viscosity.

The cosmological problem within the frame work of general relativity consists of finding a model of the physical universe which correctly predicts the result of astronomical observations and which is determined by those physical laws that describe the behavior of matter on scales up to those of clusters of galaxies. The simplest models of the expanding universe are those which are spatially homogeneous and isotropic at each instance of time. The Bianchi cosmologies which are spatially homogeneous and anisotropic play an important role in theoretical cosmology for simplification and description of large scale behavior of actual universe. One of the most interesting puzzles concerning our current understanding of the physical world is the tiny value of the cosmological constant $\Lambda<10^{-120}M^2{}_p$. This problem is reviewed by Weinberg[4] some time ago. Several authors have tried independently to account for the present value of Λ considering it as a variable rather than a constant. Ratra and Peebles[5] discussed in detail the cosmological model and cosmology with time varying constant. Sahni and Starobinsky[6], Dolgov[7] proved that in absence of any interaction with the matter or radiation, the cosmological constant remain a constant, however, in presence of any interaction with the matter and radiation, a solution of Einstein's equation and the assumed equation of covariant conservation of stress-energy with time varying cosmological constant could be found. Some of the researchers[8-10] have proposed several cosmological studies in which Λ-term decays with time. The most significant study done by Chen and Wu[11] has been further modified by several authors[12-14].

Several researchers like Zeldovich[15], Bertolami[16], 0zer and Taha[17], Lorenz[18], Friemann and Waga[19] and Pradhan *et. al.*[20] investigated more significant cosmological model with cosmological constant. Linde[21] has suggested that Λ is a function of temperature and is related to the spontaneous symmetry breaking process. Therefore, it could be a function of time in a spatially homogenous expanding universe[22].

The occurrence of magnetic field on galactic scale is well-established fact today and their importance for a variety of astrophysical phenomena is generally acknowledged, as pointed out by Zeldovich *et al.*[23]. Harrison[24] has suggested that magnetic field could have a cosmological origin. As a natural consequence we should include magnetic field in energy-momentum tensor of early universe.

The large scale distribution of galaxies in our universe shows that the matter distribution can satisfactorily be described by perfect fluid. Cosmological model with perfect fluid is studied by numbers of researchers in various contexts. Singh and Kumar[25] have investigated some spatially homogeneous and anisotropic Bianchi type-I perfect fluid cosmological model with variable cosmological constant. Pradhan *et.al.*[26] discussed some homogeneous cosmological model with electromagnetic field in presence of perfect fluid with variable Λ. Singh[27] has obtained spatially homogeneous LRS Bianchi type-V cosmological model with perfect fluid in general relativity. Singh[28] has investigated Bianchi type-V cosmological model with a specific Hubble parameter in presence of perfect fluid. Tiwari[29] has discussed Bianchi type-III cosmological model filled with perfect fluid in presence of cosmological constant. Bianchi Cosmologies with variable cosmological term in presence of perfect fluid have studied by numbers of authors viz. Chakravarty and Biswas[30], Adhav *et.al*.[31]. Tabensky and Taub[32] have investigated Plane symmetric self-gravitating fluids with pressure equal to energy density.

In most of investigations, the magnetic permeability where it is considered, assumed as a constant quantity. Bali[33] has investigated cosmological model with variable magnetic permeability.

In this chapter, we investigated Bianchi type-III cosmological models with time dependent cosmological term and variable magnetic permeability. To get deterministic model, we have assumed that the fluid is anti-stiff and F_{23} is the only non- vanishing component of electromagnetic field tensor. The physical and geometrical implications of models are also discussed.

The metric and field equations :

We consider homogeneous Bianchi type-III metric in the form of

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2dz^2 \tag{1}$$

where A, B *and* C are functions of t alone. In this paper we have considered distribution of matter to consist of anti-stiff perfect fluid and magnetic field.

The energy-momentum tensor of the composite field is assumed to be the sum of the corresponding energy-momentum tensor. Thus

$$T_i^j = (p+\rho)v_i v^j + pg_i^j + E_i^j \tag{2}$$

here ρ is energy density, p is cosmological pressure and v^j the flow vector satisfying the relation

$$g_{ij}v^i v^j = -1 \tag{3}$$

Also E_i^j is the electromagnetic field given by Lichnerowicz[34] as

$$E_i^j = \mu\left[|h|^2\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right] \tag{4}$$

Were μ is the magnetic permeability and h_i is the magnetic flux vector define by

$$h_i = \frac{1}{\mu}{}^*F_{ji}v^j \tag{5}$$

where $^*F_{ij}$ is the dual electromagnetic field tensor define by synge[35] as

$${}^*F_{ji} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{jikl}F^{kl} \tag{6}$$

F_{ij} is electromagnetic field tensor and ε_{ijkl} levi-cevita tensor density.

Here, the co-moving coordinates are

taken to be $v^1 = v^2 = v^3 = 0$ *and* $v^4 = 1$. We take the incident magnetic field to be in the direction of the x-axis so that $h_1 \neq 0$, $h_2 = 0 = h_3 = h_4$. On the assumption of infinite conductivity of the fluid, we get $F_{14} = 0 = F_{24} = F_{34}$. The only non –vanishing component of F_{ij} is F_{23}. The Maxwell's equations

$F_{ij;k} + F_{jk;i} + F_{kj;i} = 0$ *and* $F^{ij}_{;k} = 0$ are satisfying by

$$F_{23} = K \text{ (constant)} \tag{7}$$

where the semicolon represent a covariant differentiation.

From equation (5) and (6) we find that

$$h_1 = \frac{A K e^{\alpha x}}{\mu BC} \tag{8}$$

The non vanishing components of electro-magnetic field tensor are

$$E_1^1 = -\frac{K^2 e^{2\alpha x}}{2\mu B^2 C^2}, \quad E_2^2 = \frac{K^2 e^{2\alpha x}}{2\mu B^2 C^2} = E_3^3$$

$$\text{and } E_4^4 = -\frac{K^2 e^{2\alpha x}}{2\mu B^2 C^2} \tag{9}$$

The Einstein's field equations ($G = c = 1$) with time dependent cosmological term read as

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -8\pi T_i^j \tag{10}$$

Einstein field equations (10) for the line element (1) and matter distribution (2) together with (9) has been set up as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \Lambda = -8\pi\left(p - \frac{K^2 e^{2\alpha x}}{2\mu B^2 C^2}\right) \tag{11}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \Lambda = -8\pi\left(p + \frac{K^2 e^{2\alpha x}}{2\mu B^2 C^2}\right) \tag{12}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\alpha^2}{A^2} + \Lambda = -8\pi\left(p + \frac{K^2 e^{2\alpha x}}{2\mu B^2 C^2}\right) \tag{13}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{\alpha^2}{A^2} + \Lambda = 8\pi\left(\rho + \frac{K^2 e^{2\alpha x}}{2\mu B^2 C^2}\right) \tag{14}$$

$$\alpha\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0 \tag{15}$$

The Suffix 4 by the symbol A, B *and* C denotes differentiation with respect to t. Here we assume that the magnetic permeability is a variable quantity and such as

$$\mu = e^{2\alpha x} \tag{16}$$

Thus $\mu \to 1$ when $x \to 0$.

Solution of field equations :

Equation (15) leads to

$B = \lambda A$, where λ is constant of integration. (17)

Equations (11) – (14) with help of (17) and (17) take the form

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \Lambda = -8\pi\left(p - \frac{K^2}{2\lambda^2 A^2 C^2}\right) \tag{18}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\Lambda=-8\pi\left(p+\frac{K^2}{2\lambda^2A^2C^2}\right) \quad (19)$$

$$2\left(\frac{A_{44}}{A}\right)+\left(\frac{A_4}{A}\right)^2-\frac{\alpha^2}{A^2}+\Lambda=-8\pi\left(p+\frac{K^2}{2\lambda^2A^2C^2}\right) \quad (20)$$

$$\left(\frac{A_4}{A}\right)^2+2\left(\frac{A_4C_4}{AC}\right)-\frac{\alpha^2}{A^2}+\Lambda=8\pi\left(\rho+\frac{K^2}{2\lambda^2A^2C^2}\right) \quad (21)$$

Since the fluid is anti-stiff perfect fluid, we get

$$p+\rho=0 \quad (22)$$

Equations (18) to (21) are four equations in five unknowns $A, C, \Lambda, p \, and \, \rho$, therefore, for the complete determination of model we use an extra condition, we assume that the expansion (θ) in the model is proportional to the shear (σ), which is physically plausible condition. This condition leads to

$$A=C^n \quad (23)$$

Equations (20), (21) and (22) lead to

$$\left(\frac{A_{44}}{A}\right)-\left(\frac{A_4C_4}{AC}\right)=-\frac{4\pi k^2}{\lambda^2A^2C^2} \quad (24)$$

Equations (23) and (24) lead to

$$\frac{C_{44}}{C}+(n-2)\frac{C_4^2}{C^2}=-\frac{4\pi k^2}{\lambda^2nC^{2n+2}} \quad (25)$$

Equation (25) leads to

$$CC_{44}+(n-2)C_4^2=-\frac{4\pi k^2}{\lambda^2nC^{2n}} \quad (26)$$

Now on putting $C_4=f(C)$ in equation (26), we get

$$Cff'+(n-2)f^2=-\frac{4\pi k^2}{\lambda^2nC^{2n}} \quad (27)$$

Equation (27) leads to

$$\frac{df^2}{dC}+\frac{(2n-4)}{C}f^2=-\frac{8\pi k^2}{\lambda^2nC^{2n+1}} \quad (28)$$

Equation (28) leads to

$$f^2=\frac{2\pi k^2}{\lambda^2nC^{2n}}+\frac{L}{C^{2n-4}} \quad (29)$$

where L is constant of integration.

Equation (29) leads to

$$\int\frac{dC}{\sqrt{\dfrac{2\pi k^2}{\lambda^2nC^{2n}}+\dfrac{L}{C^{2n-4}}}}=t+M \quad (30)$$

where M is constant of integration .Value of C can be determined by eq. (30).

After a suitable transformation of co-ordinates metric (1) reduces in form of

$$ds^2=\frac{-dT^2}{\left[\dfrac{2\pi k^2}{\lambda^2nT^{2n}}+\dfrac{L}{T^{2n-4}}\right]}+T^{2n}dX^2+\lambda^2T^{2n}e^{-2\alpha X}dY^2+T^2dZ^2 \quad (31)$$

where $C=T, x=X, y=Y, z=Z$.

Some physical and geometrical features :

The pressure and density for the model are given by

$$8\pi p = \frac{2\pi k^2\left(1-n-n^2\right)}{\lambda^2 nT^{2n+2}} - \frac{(n+2)L}{T^{2n-2}} - \Lambda \quad (32)$$

$$8\pi\rho = \frac{2\pi nk^2}{\lambda^2 T^{2n+2}} + \frac{n(n+2)L}{T^{2n-2}} - \frac{\alpha^2}{T^{2n}} + \Lambda \quad (33)$$

The scalar of expansion for the flow vector v^i is given by

$$\theta = (2n+1)\sqrt{\lambda\frac{2\pi k^2}{\ell^2 nT^{2n+2}} + \frac{L}{T^{2n-2}}} \quad (34)$$

The scalar of shear for the flow vector v^i is given by

$$\sigma^2 = \frac{1}{3}(n-1)^2\left[\lambda\frac{2\pi k^2}{\ell^2 nT^{2n+2}} + \frac{L}{T^{2n-2}}\right] \quad (35)$$

The cosmological parameter Λ is determined if fluid is known to obey an equation of state of the form

$$p = \gamma\rho, \quad \text{where } 0 \le \gamma \le 1 \quad (36)$$

Equations (32), (33) and (34) lead to

$$\Lambda(1+\gamma) = \frac{2\pi k^2\left[(1-n) - n^2(1+\gamma)\right]}{\lambda^2 nT^{2n+2}} - \frac{(n+2)(1+n\gamma)L}{T^{2n-2}} + \frac{\alpha^2\gamma}{T^{2n}} \quad (37)$$

Solution in absence of magnetic field :

In the absence of magnetic field the geometry of space time is given by

$$ds^2 = \frac{-dT^2}{\left[\frac{L}{T^{2n-4}}\right]} + T^{2n}dX^2 + \ell^2 T^{2n}e^{-2\alpha X}dY^2 + T^2 dZ^2 \quad (38)$$

The pressure and density of the model (38) are given by

$$8\pi p = -\frac{(n+2)L}{T^{2n-2}} - \Lambda \quad (39)$$

$$8\pi\rho = \frac{n(n+2)L}{T^{2n-2}} - \frac{\alpha^2}{T^{2n}} + \Lambda \quad (40)$$

The cosmological constant for the model (38) is given by

$$\Lambda(1+\gamma) = -\frac{(n+2)(1+n\gamma)L}{T^{2n-2}} + \frac{\alpha^2\gamma}{T^{2n}} \quad (41)$$

Conclusion

We have obtained a new class of anisotropic cosmological models including an electromagnetic perfect fluid as the source. In general the models represent expanding, shearing and non rotating universe.

When n>1 the models start with big bang at T= 0 and the expansion in the model decreases as time increases. The expansion stops as $T\to\infty$. We further observe that expansion in the model stops when n = -1/2. As $T\to\infty$, $p\to 0$ and $\rho\to 0$ provided n >1. Since $T\to\infty$ $\frac{\sigma}{\theta} \neq 0$ hence the model doesn't approach isotropy for large value of T. However the model isotropizes for n = 1. The cosmological constant in both models is found decreasing function of time for n > 1. A point type singularity[36] is observed for n>0 as $T \to 0$, $g_{11} \to 0$, $g_{22} \to 0$, $g_{33} \to 0$.

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