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"A Parametric Study of Point-Image-Quality Assessment of an Optical System with an Amplitude Apodiser"

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Abstract

In this paper, a parametric study of some of the most important corollaries of the Green's function of an optical system has been carried out. The parameters chosen for this study are Full-Width at half-Maximum Negative amplitude, second order movement and Ripple Factor. The results obtained have been discussed with the help of graphs and tables.

Key words: Mathematical optics, Fourier optics, Apodization etc,

1. Introduction

Wetherell[1] has suggested several image quality evaluation parameters which are based on the PSF of the optical system. Papoulis[2] has also listed a number of parameters, which are useful for characterizing an optical system in terms of its PSF. Earlier, Hessel[3] had pointed out that while exploring the possibility of using apodisation filters for a particular purpose, the other side effects of apodisation should not be ignored. In fact in many common apodisation processes, there are always some detrimental effects as the main-lobe components are considerably broadened and the system resolution is highly decreased. Therefore, in order to make a comprehensive study of the performance of various apodisation filters, it is necessary to make a detailed study of the various parameters as suggested by Papoulis and Wetherill. Keeping this in mind, we have studied in this paper the above-mentioned characteristics associated with the PSF of an optical system with our chosen apodisation filters.

2. Full-Width at Half-Maximum (FWHM):

A direct corollary of the PSF, the FWHM is defined as twice the distance of the point, where the PSF drops to 50% of its peak relative intensity, from the center of the diffraction pattern. Normally the diffraction-limited PSF of a two-dimensional optical system with circular symmetry will be the Bessel-squared type. But these diffraction-limited PSF commonly known as the Airy pattern, can be considerably modified by including the effects of various amplitude and phase filters, aberrations, image motion, atmospheric turbulence and other factors external to the optical system. Consequent upon a large amount of modification brought about by one or several of these factors, a major deviation of the normal PSF from the Bessel-squared type will not only be detrimental from the image quality point of view, but it will also make it practically impossible for a direct application of the well-established Rayleigh criterion for the estimation of the limits of resolution for a pair of points or line objects. For example in many cases of practical interest, the first minimum in the PSF may not reach the zero level at all. In such cases, knowledge of FWHM will be immensely useful in correctly estimating the amount of departure from the classical Rayleigh-limit of resolution for a real optical system causing a large amount of degradation of the normal PSF due to various causes mentioned above.

Table 1

Values of FWHM for various β

β	FWHM
0	1.002
0.1	0.7441
0.2	0.5795
0.3	0.4269
0.4	0.3259
0.5	0.2508
0.6	0.191
0.7	0.1468
0.8	0.1112
0.9	0.0872
1	0.066

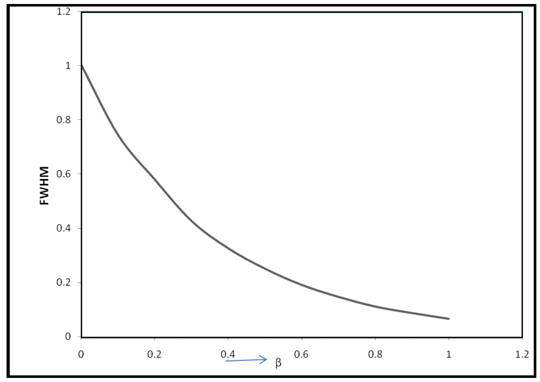

Fig. 1. Variation of FWHM with β

In the table.1, we have presented the results for FWHM for various values of the apodisation parameter β. The FWHM values become less and insensitive to apodisation with our filters when the of apodisation parameter is high, viz. β>0.5. The study of FWHM is most useful when the image situation is much more complicated in the presence of external factors such as image motion or atmospheric turbulence when it correlates directly with factors degrading the normal diffraction-limited PSF. It is also very useful where the large amounts of aberrations are present in the particular image forming system. In all such cases the PSF is not only modified to a complicated shape, but it is also observed that the first minimum does not reach the zero value of the intensity.

The application of Rayleigh criterion for determining the limit of resolution for two points or line objects do not hold good in this cases. If the performance of this systems has to be assessed in terms of its PSF, the only other alternative is study the FWHM of the system.

It can thus be concluded that the FWHM is not useful in comparing diffraction limited and near-diffraction optical systems, but wherever the shape of the PSF is significantly difference from the Bessel-Squared form. The knowledge of FWHM is absolutely essential for assessing the system performance in terms of its PSF. As the number of such systems with complicated PSF is increasing in modern optical engineering problems, it is anticipated that FWHM will soon become a more frequently used system-assessment parameter like its spectroscopic equivalent **Half-Power Diameter (HPD).**

3. First point of inflection :

Papoulis[2] has included the "first point of inflection" as another PSF based corollary for the assessment of image quality. At the **first point of inflection,** the following condition has to be satisfied

$$\frac{d^2}{dz^2}\left[I(y,z)\right]_{z=\delta}=0 \tag{1}$$

Where d^2/dz^2 is the second derivative of the point spread function and δ is the first point of inflection satisfying the condition (1). As this parameter is conceptually similar to the FWHM, its dependence on the value of β and y is also similar to that for FWHM. In other words we are led to conclude that when either one of this parameters viz., FWHM and the first point of inflection is considered for evaluating the system performance in terms of its PSF, the other one need not be considered.

4. Strehl ratio (SR):

Strehl[4] had suggested the use of the relative intensity of the diffraction pattern as a measure of the image quality. Strehl originally gave the name "**Definition shelligkeit**" for this parameter which is now known as the **Strehl definition, Strehl Ratio or Strehl intensity**. In its original nomenclature, the term **'definition'** was used to mean '**distinctness of an outline or a detail'** in the image. The Strehl (SR) has been defined as the ratio of the central intensity of the PSF of the system with the non-uniform pupil function to that with the uniform pupil function for diffraction limited system.

$$SR=\frac{[B_I(0,0)]_\beta}{[B_I(0,0)]_A} \tag{2}$$

Where the subscripts β and A stand for apodised and Airy pupils respectively. For the evaluation of the expression (2) Kusava[5] has obtained an alternative simplified expression in terms of pupil function f(r) of the system, viz.,

$$SR=\frac{\left|A_I(0,0)\right|^2{}_\beta}{\left|A_I(0,0)\right|^2{}_A}$$

$$=4\int_0^1 f(r)dr$$

$$=4\int_0^1\frac{(1+\beta Cos\pi r^2)}{(1+\beta)}rdr \tag{3}$$

In an ideal case, when an image of a point object is formed by an optical system, the image should stand out as a singular point with undiminished of the central intensity. When the value of the central intensity is reduced, it means that it stands out less prominent in its neighborhoods. Hence the higher is the value of the SR, the better is the performance of the apodising filter. For the performance analysis of optical systems, Marechal[6] has stipulated the criterion known after his name. **The Marechal criterion** requires the SR value to be equal or to exceed 80% for the performance analysis of the diffraction-limited systems. However it can be extended to cover the cases with image motion aperture obstructions and various forms of amplitude transmission variations by using the form of the Strehl definition as given in the expression (3).

The SR values for various values β have been presented in table 2. It is found that the SR values steadily decreases as the values of β are increased.

Table 2.
Values of SR for various β

0.0	2.0000
0.1	.7273
0.2	1.5000
0.3	1.3077
0.4	1.1429
0.5	1.0000
0.6	0.8750
0.7	0.7647
0.8	0.6667
0.9	0.5789
1.0	0.5000

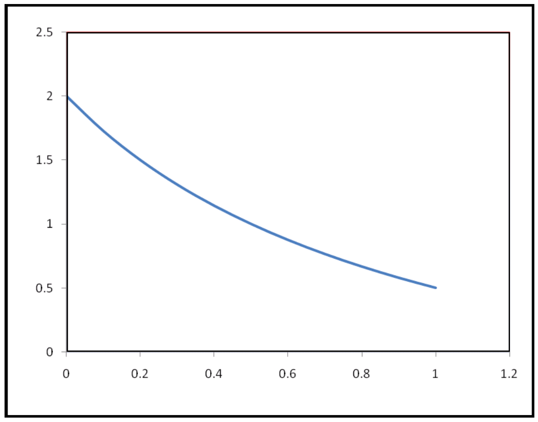

Fig. 2. variation of SR with β

5. Pass-Flux Ratio :

The pass-flux ratio or the total transmission factor τ as it is called sometimes is an important parameter from the photometric point of view as it gives an indication of the total amount of light flux that passes into the image space. The total transmission factor τ for non-Airy pupils was originally defined as

$$\tau = \frac{\int_0^\infty \left|A_I(0,z)\right|^2_\beta zdz}{\int_0^\infty \left|A_I(0,z)\right|^2_A zdz} \tag{4}$$

Also referred to as the pass-flux ratio obtained an expression for τ in terms of the pupil function of the system as

$$\tau = 2\int_0^1 \left|f(r)\right|^2 rdr \tag{5}$$

This for co-sinusoidal filters reduces to

$$\tau = 2\int_0^1 [\frac{1+\beta Cos\pi r^2}{1+\beta}]^2 rdr \quad (6)$$

A simple interpretation of τ is obtained as follows. If the total light flux received in the Gaussian focal plane is denoted by TLF, then it can be written as using the expression (5)

$$TLF = \tau\int_0^1 |A_I(0,z)|^2 zdz$$

$$= 4\tau\int_0^\infty [\int_0^1 J_0(zr)rdr]^2 zdz$$

$$= 4\tau\int_0^1 J_1^2(z)z^{-1}dz = 4\tau(1/2) = 2\tau \quad (7)$$

The value of the pass-flux ratio ô for various values of the apodisation parameter β have been shown in the table 3, and the same results have been presented graphically in the Fig. 3,

Table 3.
Values of PFR for various β

β	PFR
0.0	1.000
0.1	0.747
0.2	0.565
0.3	0.432
0.4	0.333
0.5	0.259
0.6	0.203
0.7	0.160
0.8	0.128
0.9	0.102
1.0	0.083

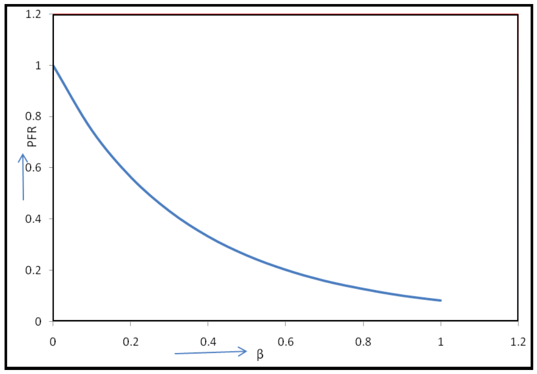

Fig. 3. Variation of PFR with β

6. Maximum negative amplitude :

The maximum value of the negative amplitude in the PSF controls the amount of ‘**edge ringing**’ or the so-called **Gibbs Oscillations** in the coherent imagery of sharply defined edge objects. It has been observed that a filter which gives rise to small or almost zero negative amplitude response in the PSF, suppresses the edge-ringing phenomenon to a considerably large extent. For a co-sinusoidal amplitude filter values of maximum negative amplitude for various values of β have been indicated in the table 4.

Table 4. Values of MNA for various β

β	MNA
0	0.0828
0.1	0.0838
0.2	0.0848
0.3	0.0859
0.4	0.0869
0.5	0.0879
0.6	0.089
0.7	0.09
0.8	0.091
0.9	0.0921
1	0.0931

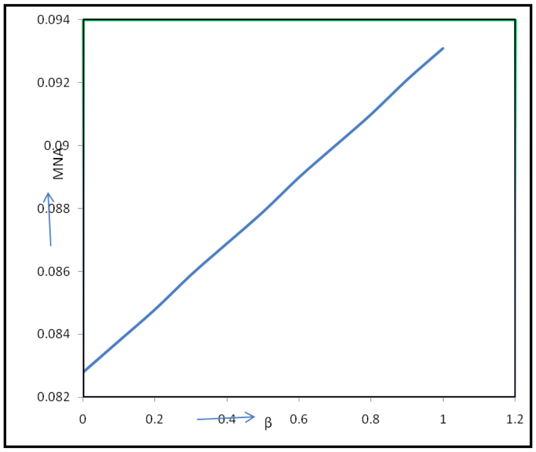

Fig. 4. curve showing the variation of MNA with β

7. Second order moment :

This parameter for the image quality evaluation has been defined by Asakura[7] as

$$\Delta = \int_0^\infty \frac{|A_I(0,z)|^2{}_\beta}{|A_I(0,z)|^2{}_A} z^3 dz \qquad (8)$$

For our co-sinusoidal filters, the above expression becomes

$$\Delta = \frac{\int_0^1 [\frac{d}{dr}\{\frac{1+\beta Cos\pi r^2}{1+\beta}\}]^2 rdr}{[\int_0^1 \{\frac{1+\beta Cos\pi r^2}{1+\beta}\} rdr]^2} \qquad (9)$$

It is observed from Fig. 5, smaller is the value of the second Order Moment, and higher is the light flux concentration in and around the central region of the diffraction pattern. The computed values of the second order moment of various values of the apodisation parameter have been tabulated in the table 5. There is a remarkable departure from normal behavior for β=1.0, when the Second Order Moment progressively increases to very high values at an extra-ordinary faster rate.

Table 5. Values of SOM for various β

Δ	SOM
0	0
0.1	0.007291
0.2	0.032493
0.3	0.081964
0.4	0.164497
0.5	0.292439
0.6	0.483419
0.7	0.763109
0.8	1.16975
0.9	1.76188
1	2.63195

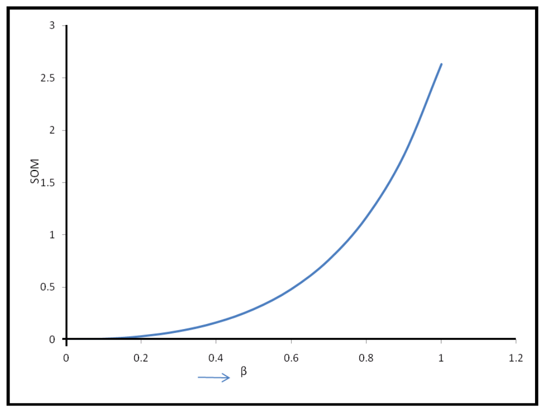

Fig. 5. Variation of SOM with β

8. Ripple factor :

The term '**Ripple Factor' (RF)** has been borrowed from the domain of digital signal processing where it implies

$$RF = \frac{[Maximum\ side - lobe\ amplitude]x100\%}{Main - lobe\ amplitude} \quad (10)$$

The most important objective of employing an amplitude apodisation filter in optical systems is to suppress, to the extent possible the secondary side-lobes in a normal diffraction pattern. It is therefore, very important that the performance of an apodiser should be assessed in terms of the relative peak intensity of the first secondary maximum just outside the central Airy spot. In that concest an estimate of the RF for the various values of the apodisation parameter should be considered as a very useful figure of merit for the evaluation of image quality of an optical system operating with a given specific class of amplitude apodisation filters. With the above consideration in mind we have tabulated the values of RF in table 6.

Table 6. Values of RF for various y and β

y	β=1	β=0.9	β=0.8	β=0.7	β=0.6	β=0.5	β=0.4	β=0.3	β=0.2	β=0.1	β=0
0	0.32	0.371	0.441	0.548	0.711	0.88	1	1.163	1.25	1.609	1.7
0.314	5.8	5.987	6.607	7.632	8.409	9.4	10	10.923	11.6	12.761	13.213
0.628	6	6.207	6.97	7.895	8.716	9.6	10.316	11.077	11.733	22.093	13.353
0.942	6.143	6.618	7.154	8.098	9.131	9.869	10.538	11.281	12.297	12.791	14.127
1.256	6.439	6.927	7.62	8.563	9.537	10.348	11.045	11.965	12.714	13.512	14.213
1.57	6.95	7.617	8.271	9.366	10.029	10.785	11.693	12.577	13.446	14.185	14.83
1.884	7.835	8.348	9.272	10.009	10.846	11.835	12.683	13.481	14.28	14.95	15.596
2.198	2.521	2.755	3.327	3.787	11.917	12.785	13.771	14.477	15.198	15.794	16.375
2.512	2.648	3.02	3.465	4.058	4.731	14.008	14.666	15.55	16.189	16.704	17.214
2.826	2.792	3.207	3.739	4.389	5.048	5.603	15.877	16.686	17.237	17.89	18.299
3.14	3.08	3.413	3.922	4.631	5.272	5.942	6.398	18.168	18.565	19.116	19.65
3.454	3.263	3.779	4.319	5.043	5.662	6.321	6.889	7.269	20.396	20.83	21.252
3.768	1.425	1.702	4.803	5.506	6.099	6.743	7.294	7.784	22.304	22.605	22.902
4.082	1.549	1.84	5.162	5.841	6.585	7.21	7.625	8.208	8.656	24.433	24.589
4.396	1.621	1.937	2.482	6.395	7.123	7.724	8.227	8.87	9.346	9.846	26.633
4.71	1.798	2.136	2.683	3.15	7.748	8.561	9.064	9.705	10.133	10.594	11.132
5.024	1.05	2.362	2.911	3.371	8.545	9.348	9.806	10.422	11	11.587	11.913
5.338	2.122	2.506	3.046	3.626	4.201	10.201	10.858	11.424	11.953	12.497	12.932
5.652	0	2.782	3.32	3.95	4.666	11.122	12.036	12.534	13.248	13.711	14.058
5.966	0	1.461	3.645	4.314	5.038	5.636	13.014	13.763	14.425	14.805	15.3
6.28	0	0	4.051	4.722	5.453	6.044	6.689	15.124	15.714	16.291	16.671

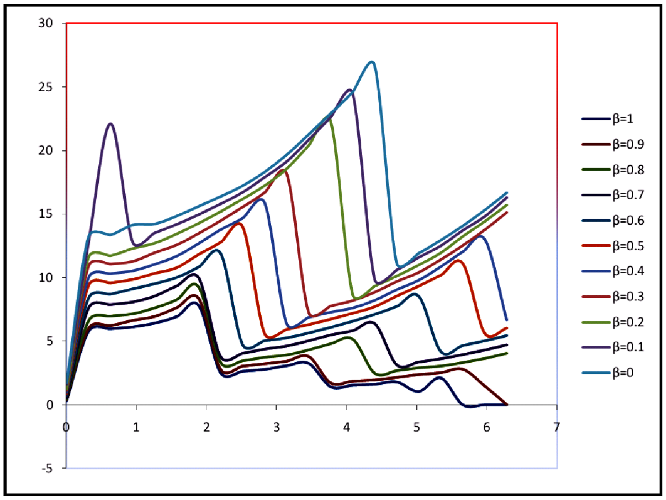

Fig. 6. variation of RF with β

It is observed from the table that for a particular value of y, the value of RF decreases as the value of β is decrease from β=0 to 0.7. Beyond this value of β=0.7 the RF value is again found to increase indicating thereby the optimum value of the apodisation parameter β for the purpose of suppressing the first secondary maximum to the extent possible, must be in the neighborhood of 0.7. A very insignificant minor departure from the above value of β may be noticeable in a very small number of cases of high value of the defocusing parameter y. It is therefore concluded that information of RF value is very helpful in understanding the amount of separation of the first diffraction maximum.

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