Ultra Scientist Vol. 24(2)A, 379-390 (2012).

On fuzzy feebly open sets and fuzzy feebly closed sets

T. THANGAM[1], K. BAGEERATHI[2] and P. THANGAVELU[3]

[1,2]Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur-628215 (INDIA)
[3]Department of Mathematics, Karunya University, Coimbatore (INDIA)
e-mail: sivarathi_2006@yahoo.in, thangammathavan@yahoo.com
e-mail: ptvelu12@gmail.com , thangavelu@karunya.edu

(Acceptance Date 29th July, 2012)

Abstract

In this paper we introduce the concept of fuzzy $\mathcal{C}$–feebly open and fuzzy $\mathcal{C}$–feebly closed sets by using arbitrary complement function $\mathcal{C}$ and by using fuzzy $\mathcal{C}$ - semi closure operators of a fuzzy topological space where $\mathcal{C}:[0, 1]\rightarrow [0, 1]$ is a function and investigate some of their basic properties of a fuzzy topological space.

Key words: Fuzzy $\mathcal{C}$ - feebly open, fuzzy $\mathcal{C}$ - feebly closed sets, fuzzy $\mathcal{C}$-quasi-coincident, fuzzy $\mathcal{C}$ -q-neighborhood and fuzzy topology.

MSC 2010: 54A40, 3E72.

1. Introduction

The concept of complement function is used to define a fuzzy closed subset of a fuzzy topological space. That is a fuzzy subset λ is fuzzy closed if the standard complement $1-\lambda = \lambda'$ is fuzzy open. Here the standard complement is obtained by using the function $\mathcal{C} : [0, 1]\rightarrow [0, 1]$ defined by $\mathcal{C}(x) = 1-x$, for all $x \in [0, 1]$. Several fuzzy topologists used this type of complement while extending the concepts in general topological spaces to fuzzy topological spaces. But there are other complements in the fuzzy literature[8]. This motivated the second and third authors to introduce the concepts of fuzzy $\mathcal{C}$ -closed sets and fuzzy $\mathcal{C}$-semi closed sets in fuzzy topological spaces, where $\mathcal{C} : [0, 1]\rightarrow [0, 1]$ is an arbitrary complement function.

Skin[10] defined the notion of fuzzy feebly open and fuzzy feebly closed sets in fuzzy topological spaces and studied their properties. In this paper, we generalize the concept of fuzzy feebly open and fuzzy feebly closed sets by using the arbitrary complement function $\mathcal{C}$, instead of the usual fuzzy complement function, and by using fuzzy $\mathcal{C}$ -semi closure instead of fuzzy semi closure.

For the basic concepts and notations, one can refer Chang[7]. The concepts that are needed in this paper are discussed in the second section. The section three is dealt with the concept fuzzy **C** -quasi-coincident. The concepts of fuzzy**C** - feebly open and**C** - feebly closed sets in fuzzy topological spaces and studied their properties in the fourth and fifth section respectively.

2. Preliminaries :

Throughout this paper (X,τ) denotes a fuzzy topological space in the sense of Chang. Let **C** : $[0, 1]\rightarrow[0, 1]$ be a complement function. If λ is a fuzzy subset of (X,τ) then the complement C λ of a fuzzy subset λ is defined by C $\lambda(x)$ = C $(\lambda(x))$ for all $x\in X$. A complement function C is said to satisfy

(i) the boundary condition if **C** (0) = 1 and **C** (1) = 0,

(ii) monotonic condition if $x \leq y \Rightarrow$ **C** $(x) \geq$ **C** (y), for all $x, y \in [0, 1]$,

(iii) involutive condition if **C** (**C** (x)) = x, for all $x \in[0, 1]$.

The properties of fuzzy complement function C and C λ are given in George Klir[10] and Bageerathi *et al.*[2]. The following lemma will be useful in sequel.

Definition 2.1 [Definition 3.1,[2]]

Let (X,τ) be a fuzzy topological space and **C** be a complement function. Then a fuzzy subset λ of X is fuzzy **C** -closed in (X,τ) if **C** λ is fuzzy open in (X,τ).

Lemma 2.2 [Proposition 3.2,[2]]

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the involutive condition. Then a fuzzy subset λ of X is fuzzy open in (X,τ) if **C** λ is a fuzzy **C** -closed subset of (X,τ).

Definition 2.3 [Definition 4.1,[2]]

Let (X, τ) be a fuzzy topological space. Then for a fuzzy subset λ of X, the fuzzy**C** -closure of is defined as the intersection of all fuzzy **C** -closed sets μ containing λ. The fuzzy **C** -closure of λ is denoted by $Cl_{\mathbf{C}}\lambda$ that is equal to $\wedge\{\mu: \mu \geq \lambda,$ **C** $\mu\in\tau\}$.

Lemma 2.4 [Lemma 4.2,[2]]

If the complement function **C** satisfies the monotonic and involutive conditions, then for any fuzzy subset λ of X, (i) **C** (*Int* λ) = $Cl_{\mathbf{C}}$(**C** λ) and (ii) **C** $(Cl_{\mathbf{C}}\lambda)$ = *Int* (**C** λ).

Definition 2.5[3]

A fuzzy topological space (X,τ) is **C** -product related to another fuzzy topological space (Y,σ) if for any fuzzy subset ν of X and ζ of Y, whenever **C** $\lambda \not\geq \nu$ and **C** $\mu \not\geq \zeta$ imply**C** $\lambda\times 1 \vee 1 \times$ **C** $\mu \geq \nu \times \zeta$, where $\lambda \in \tau$ and $\mu \in\sigma$, there exist $\lambda_1 \in\tau$ and $\mu_1\in\sigma$ such that **C** $\lambda_1 \geq \nu$ or **C** $\mu_1 \geq \zeta$ and **C** $\lambda_1\times 1 \vee 1 \times$ **C** μ_1 = **C** $\lambda \times 1 \vee 1 \times$ **C** μ.

Definition 2.6[11]

A fuzzy point with support $x\in X$ and the value r $(0 < r \leq 1)$ at $x\in X$ will be denoted by x_r and for fuzzy subset λ, $x_r \in \lambda$ if and only if $\lambda(x) \geq r$.

Definition 2.7[11]

For any two fuzzy subsets λ and μ, we shall write $\lambda q \mu$ to mean that λ is quasi-coincident (q- coincident, for short) with μ if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$.

Definition 2.8 :

Let λ and μ be any two fuzzy subsets of a fuzzy topological space. Then λ is q -neighborhood with μ (q-nbd, for short) if there exists a fuzzy open set δ with $\lambda q \delta \leq \mu$.

Definition 2.9 :

Let λ be a fuzzy subset of a fuzzy topological space X then λ is said to be fuzzy **C** - feebly open if there exists a fuzzy open set δ in X such that $\delta \leq \lambda \leq sCl_{C} \delta$.

Definition 2.10

Let λ be a fuzzy subset of a fuzzy topological space X then λ is said to be fuzzy feebly open if its complement is fuzzy feebly closed.

*3. Fuzzy**C** -quasi-coincident*

Definition 3.1

A fuzzy point x_r is said to be fuzzy **C** -quasi-coincident (q_C -coincident, for short) with λ denoted by $x_r q_C \lambda$ if $r >$ **C** λ, where **C** : $[0, 1] \rightarrow [0, 1]$ is a arbitrary complement function.

Definition 3.2

Let λ and μ be any two fuzzy subsets of a fuzzy topological spaces. Then λ is said to be fuzzy q_C -coincident with μ ($\lambda q_C \mu$, for short) iff there exists $x \in X$ such that $\lambda(x) >$ **C** $\mu(x)$ and μ is said to be a fuzzy **C** -quasi - neighborhood (q_C - nbd, for short) of λ if there is a fuzzy open set δ with $\lambda q_C \delta \leq \mu$.

Proposition 3.3

Let (X, τ) be a fuzzy topological space and **C** : $[0, 1] \rightarrow [0, 1]$ be a complement function that satisfies the monotonic and involutive conditions. Then for a fuzzy λ of a fuzzy topological space X, $sCl_C \lambda$ is the union of all fuzzy points x_r such that every fuzzy **C** - semi open set δ with $x_r q_C \delta$ is fuzzy q_C – coincident with λ.

Proof :

Let $x_r \in sCl_C \lambda$. Suppose there is a fuzzy **C** - semi open set δ such that $x_r q_C \delta$ and $_C$.λ That implies that **C** $\delta \geq \lambda$, where **C** δ is fuzzy **C** - semi closed. Since **C** satisfies the monotonic and involutive conditions, by using Proposition 3.6 in [6], **C** $\delta \geq sCl_C \lambda$. By using Definition 2.6, $x_r \notin$ **C** δ implies that $x_r \notin sCl_C \lambda$. This is a contradiction to our assumption. Therefore for every**C** - semi open with $x_r q_C \delta$ is q_C -coincident with λ.

Conversely, for every**C** - semi open δ with $x_r q_C \delta$ is q_C -coincident with λ. Suppose $x_r \notin sCl_C \lambda$. Then there is a fuzzy **C** - semi closed set $G \geq \lambda$ with $x_r \notin G$. Since **C** satisfies involutive condition, **C** G is fuzzy **C** - semi open set with $x_r q_C$ (**C** G) and $\lambda \not{q}_C$ (**C** G). That is $\lambda(x) >$**C** (**C** G)=G. This is a contradiction to the assumption. Therefore $x_r \in sCl_C \lambda$.

Proposition 3.4

Let (X, τ) be a fuzzy topological space and **C** : $[0, 1] \rightarrow [0, 1]$ be a complement function that satisfies the monotonic and involutive conditions. Let λ and μ be fuzzy subsets of a fuzzy topological space X. Then

(i) If $\lambda \wedge \mu = 0$ then $\lambda \not{q}_{\mathbf{C}}\ \mu$

(ii) $\lambda \leq \mu \Leftrightarrow x_r\ q_{\mathbf{C}}\ \mu$ for each $x_r\ q_{\mathbf{C}}\ \lambda$

(iii) $\lambda \not{q}_{\mathbf{C}}\ \mu \Leftrightarrow \lambda \leq \mathbf{C}\ \mu$

(iv) $x_r\ q_{\mathbf{C}}\ (V_{\propto 0 \in \Delta} \lambda_{\propto}) \Leftrightarrow$ there is $\propto_0 \in \Delta$ such that $x_r\ q_{\mathbf{C}}\ \lambda_{\propto 0}$

Proof

Let $(\lambda \wedge \mu)(x) = 0$. Then min $\{\lambda(x), (x)\} = 0$. This implies that $\lambda(x) = 0$ and $\mu(x) \leq 1$ (or) $\mu(x) = 0$ and $\lambda(x) \leq 1$. Since $\mathbf{C}$ satisfies the monotonic and involutive conditions, $\mathbf{C}\ \mu \geq \mathbf{C}\ (1) = \lambda$ (or)
$\mathbf{C}\ \lambda \geq \mathbf{C}\ (1) = \mu$.That implies $\lambda \leq \mathbf{C}\ \mu$. That shows $\lambda_{\mathbf{C}}\ \mu$. This proves (i).

Let $\lambda \leq \mu$. Then $x_r\ q_{\mathbf{C}}\ \lambda$ implies that $\mathbf{C}\ \lambda(x) < r$ and $\lambda \leq \mu$ implies that $\mathbf{C}\ \lambda \geq \mathbf{C}\ \mu$ that gives $\mathbf{C}\ \mu < r$. Therefore $x_r\ q_{\mathbf{C}}\ \mu$. Now $x_r\ q_{\mathbf{C}}\ \lambda$ implies that $x_r\ q_{\mathbf{C}}\ \mu$. So, $\mathbf{C}\ \mu < r$.

Suppose $\lambda(x) > \mu(x)$. Since $\mathbf{C}$ satisfies monotonic condition, $\mathbf{C}\ \lambda < r$ does not implies $\mathbf{C}\ \mu < r$. This is a contradiction. Therefore $\lambda(x) \leq \mu(x)$. This proves (ii).

By using Definition 3.2, $\lambda \not{q}_{\mathbf{C}}\ \mu$ if and only if for each $x \in X$, $\lambda(x) \leq \mathbf{C}\ \mu(x)$. That is $\lambda \leq \mathbf{C}\ \mu$. This proves (iii).

Now $x_r\ q_{\mathbf{C}}\ (V_{\propto \in \Delta} \lambda_{\propto})$ if and only if $\mathbf{C}\ (V_{\propto \in \Delta} \lambda_{\propto})(x) < r$, for some $\alpha_o \in \Delta$. Since $\mathbf{C}$ satisfies monotonic and involutive conditions, $\wedge_{\propto \in \Delta} \mathbf{C}\ \lambda_{\propto} < r$, for every $\propto_o \in \Delta$. By using Definition 3.1, $x_r\ q_{\mathbf{C}}\ \lambda_{\propto o.}$

Proposition 3.5 :

Let (X, τ) be a fuzzy topological space and $\mathbf{C} : [0, 1] \rightarrow [0, 1]$ be a complement function that satisfies monotonic and involutive conditions. Let λ be a fuzzy subset of a fuzzy topological space X. Then

(i) $Int\ Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \lambda = Int\ Cl_{\mathbf{C}}\ \lambda$ and $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ Int \lambda = Cl_{\mathbf{C}}\ Int \lambda$

(ii) $\mathbf{C}\ (Int\ Cl_{\mathbf{C}}\ \lambda) = Cl_{\mathbf{C}}\ Int\ \mathbf{C}\ \lambda$ and $\mathbf{C}\ (Cl_{\mathbf{C}}\ Int \lambda) = Int\ Cl_{\mathbf{C}}\ \mathbf{C}\ \lambda$

Proof

We know that $Int\ Cl_{\mathbf{C}}\ \lambda \leq Cl_{\mathbf{C}}\ \lambda$. Since $\mathbf{C}$ satisfies the monotonic and involutive conditions, by using Theorem 4.3, 4.4 in [2], $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \lambda \leq Cl_{\mathbf{C}}\ (Cl_{\mathbf{C}}\ \lambda) = Cl_{\mathbf{C}}\ \lambda$. This implies that $Int(Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}} \lambda) \leq Int(Cl_{\mathbf{C}} \lambda)$. Since $Int\ Cl_{\mathbf{C}}\ \lambda$ is fuzzy open and $Int\ Cl_{\mathbf{C}}\ \lambda$ $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \lambda$, $Int\ Cl_{\mathbf{C}}\ \lambda = Int(Int\ Cl_{\mathbf{C}} \lambda)$ $Int(Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \lambda)$. From the above $Int(Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \lambda) = Int\ Cl_{\mathbf{C}}\ \lambda$. This proves (i). (ii) follows from Theorem 4.3 and 4.4 in[2].

Proposition 3.6

Let (X, τ) be a fuzzy topological space and $\mathbf{C}$ be a complement function that satisfies the monotonic and involutive conditions.

(a) Let x_r and λ be a fuzzy point, a fuzzy subset, resp., of a fuzzy topological space X. Then $x_r \in \lambda$ if and only if x_r is not $q_{\mathbf{C}}$ -coincident with $\mathbf{C}\ \lambda$.

(b) Let λ and μ be any two fuzzy open subsets of a fuzzy topological space X with $\lambda \not{q}_{\mathbf{C}} \mu$. Then $\lambda \not{q}_{\mathbf{C}} Cl_{\mathbf{C}} \mu$ and $Cl_{\mathbf{C}} \lambda \not{q}_{\mathbf{C}} \mu$.

Proof

Let $x_r \in \lambda$. Then $x_r \in \lambda$ if and only if $\lambda(x) \geq r$. Since the complement function $\mathbf{C}$ satisfies the monotonic condition, we get $\mathbf{C}(\mathbf{C}\lambda(x)) \geq r$. By using Definition 3.1, $x_r \not{q}_{\mathbf{C}} \mathbf{C}\lambda$. This proves (a).

Suppose $\lambda \not{q}_{\mathbf{C}} \mu$. This implies that $\lambda(x) \leq \mathbf{C}\mu(x)$ for all x. Let $x_r \in \lambda(x)$ implies $\lambda(x) \geq r$. Taking complement on both sides implies $\mathbf{C}\lambda(x) < \mathbf{C} r$. Since $\mathbf{C}\lambda$ is fuzzy $\mathbf{C}$-closed, $Cl_{\mathbf{C}}\mathbf{C}\lambda(x) < \mathbf{C} r$. Since $\mathbf{C}$ satisfies the monotonic and involutive conditions, $\mathbf{C}(Cl_{\mathbf{C}}\mu(x)) \geq r$. This implies that $x_r \in \mathbf{C}(Cl_{\mathbf{C}}\mu)$. That shows $\lambda(x) \leq \mathbf{C}(Cl_{\mathbf{C}}\mu(x))$. From the above conclusion, $\lambda \not{q}_{\mathbf{C}} Cl_{\mathbf{C}} \mu$.

Let $x_r \in Cl_{\mathbf{C}} \lambda$. Then by using Definition 2.11, $Cl_{\mathbf{C}} \lambda(x) \geq r$. Since $\lambda \not{q}_{\mathbf{C}} \mu$, we have $Cl_{\mathbf{C}}\lambda(x) \leq \mathbf{C}\mu(x)$. This implies that $\mathbf{C}\mu(x) \geq r$. It follows that $Cl_{\mathbf{C}}\lambda \leq \mathbf{C}\mu$, this shows $Cl_{\mathbf{C}} \lambda \not{q}_{\mathbf{C}} \mu$.

Proposition 3.7

Let (X, τ) be a fuzzy topological space and $\mathbf{C}$ be a complement function that satisfies the monotonic and involutive conditions. Let λ be a fuzzy subset of a fuzzy topological space (X, τ). Then $Int\, Cl_{\mathbf{C}} \lambda \leq sCl_{\mathbf{C}} \lambda$.

Proof

Let $x_r \in Int\, Cl_{\mathbf{C}} \lambda$. Then by using Definition 2.6, $r \leq Int\, Cl_{\mathbf{C}} \lambda(x)$. This can be written as $r \leq Cl_{\mathbf{C}}\lambda(x)$. This implies that $x_r \in sCl_{\mathbf{C}} \lambda$. This shows that $x_r \in sCl_{\mathbf{C}}\lambda$.

Theorem 3.8

Let (X, τ) be a fuzzy topological space and $\mathbf{C} : [0, 1] \to [0, 1]$ be a complement function that satisfies monotonic and involutive conditions. If a fuzzy subset λ is fuzzy open, then $Int\, Cl_{\mathbf{C}} \lambda = sCl_{\mathbf{C}} \lambda$.

Proof

By using Proposition 3.7, it suffices to show that $sCl_{\mathbf{C}} \lambda \leq Int\, Cl_{\mathbf{C}} \lambda$. Let $x_r \notin Int\, Cl_{\mathbf{C}} \lambda$. Then $x_r\, q_{\mathbf{C}}\, \mathbf{C}(Int\, Cl_{\mathbf{C}} \lambda)$. By using Lemma 2.4, $x_r\, q_{\mathbf{C}}\, (Cl_{\mathbf{C}}\, Int\, \mathbf{C}\lambda)$. Since $\mathbf{C}$ satisfies the monotonic and involutive conditions, by using Proposition 3.5, $Cl_{\mathbf{C}} Int \mathbf{C}\lambda = Cl_{\mathbf{C}} Int\, Cl_{\mathbf{C}} Int\, \mathbf{C}$. This can be written as $Cl_{\mathbf{C}}\, Int\, \mathbf{C}\lambda \leq Cl_{\mathbf{C}}\, Int(Cl_{\mathbf{C}}\, Int\, \mathbf{C}\lambda)$. By using Proposition 3.2 in [4], $Cl_{\mathbf{C}}\, Int\, \mathbf{C}\lambda$ is fuzzy $\mathbf{C}$-semi open. By using Proposition 3.6, $\lambda \not{q}_{\mathbf{C}} Cl_{\mathbf{C}}\, Int\, \mathbf{C}\lambda$, that implies $x_r \notin sCl_{\mathbf{C}} \lambda$. That shows $sCl_{\mathbf{C}} \lambda \leq Int\, Cl_{\mathbf{C}} \lambda$. Therefore $Int\, Cl_{\mathbf{C}} \lambda = sCl_{\mathbf{C}} \lambda$.

4. Fuzzy $\mathbf{C}$ – feebly open sets.

In this section we introduce the concept of $\mathbf{C}$ - feebly open sets in a fuzzy topological space with respect to a complement function $\mathbf{C} : [0, 1] \to [0, 1]$.

Definition 4.1

Let λ be a fuzzy subset of a fuzzy topological space X then λ is said to be fuzzy $\mathbf{C}$ - feebly open if there exists a fuzzy open set δ in X such that $\delta \leq \lambda \leq sCl_{\mathbf{C}}\, \delta$.

The class of all fuzzy feebly open sets coincides with the class of all fuzzy $\mathbf{C}$ - feebly open sets if the standard complement function coincides with the arbitrary complement function.

Proposition 4.2

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involutive conditions. Then a fuzzy subset λ of a fuzzy topological space (X,τ) is fuzzy **C** - feebly open if and only if $\lambda \leq Int\ Cl_{C}\ Int\ \lambda$.

Proof.

Let λ be a fuzzy **C** -feebly open set of a fuzzy topological space X. Then by using Definition 4.1, there is fuzzy open set δ such that $\delta \leq \lambda \leq sCl_{C}\ \delta$. By using Theorem 3.8, δ $\lambda \leq Int\ Cl_{C}\ \delta$, $\delta = Int\ \delta \leq Int\ \lambda$, it follows that $Cl_{C}\ \delta$ $Cl_{C}\ Int\lambda$. This implies that $Int\ Cl_{C}\ \delta \leq Int\ Cl_{C}\ Int\lambda$. From the above $\lambda \leq Int\ Cl_{C}\ \delta \leq Int\ Cl_{C}\ Int\lambda$. That shows λ $Int\ Cl_{C}\ Int\lambda$.

Conversely we assume that $\lambda \leq Int\ Cl_{C}\ \mathrm{Int}\lambda$. Now $Int\ \lambda \leq \lambda$ that implies $Int\lambda \leq Int\ Cl_{C}\ (Int\lambda)$. Since **C** satisfies the monotonic and involutive conditions, by using Proposition 3.7, $Int\ Cl_{C}\ Int\ \lambda \leq sCl_{C}\ (Int\lambda)$. This implies that λ is fuzzy **C** -feebly open set in X.

It is obvious that every fuzzy open set is fuzzy **C** -feebly open and every fuzzy **C** - feebly open set is fuzzy **C** - semi open but the separate converses may not be true as shown by the following example.

Example 4.3

Let X = {a, b} and τ = **{**0, {$a_{.3}$, $b_{.8}$}, {$a_{.2}$, $b_{.5}$}, {$a_{.7}$, $b_{.05}$}, {$a_{.3}$, $b_{.5}$}, {$a_{.3}$, $b_{.05}$}, {$a_{.2}$, $b_{.05}$}, {$a_{.7}$, $b_{.8}$}, {$a_{.7}$, $b_{.5}$}, 1**}**. Then (X, τ) is a fuzzy topological space. Let $\mathbf{C}(x) = \dfrac{1-x}{1+2x}$, $0 \leq x \leq 1$. The family of all fuzzy **C** -closed sets **C** (τ) = **{**0,{$a_{.4375}$, $b_{.077}$}, {$a_{.57}$, $b_{.25}$}, {$a_{.125}$, $b_{.86}$}, {$a_{.4375}$, $b_{.25}$}, {$a_{.4375}$, $b_{.86}$}, {$a_{.57}$, $b_{.86}$}, {$a_{.125}$, $b_{.077}$}, {$a_{.125}$, $b_{.25}$}, 1**}**.

Let λ={$a_{.2}$, $b_{.7}$}. Then $Int\lambda$ = {$a_{.2}$, $b_{.5}$}, $Cl_{C}\ Int\ \lambda$ = {$a_{.4375}$, $b_{.86}$}. $Int\ Cl_{C}Int\ \lambda$ = {$a_{.3}$, $b_{.8}$}. Thus we see that $\lambda < Int\ Cl_{C}Int\ \lambda$. By using Proposition 4.2, λ is fuzzy **C** - feebly open set but not fuzzy open.

Let μ={$a_{.4}$, $b_{.5}$}. Then $Int\mu$ = {$a_{.3}$, $b_{.5}$}, $Cl_{C}\ Int\ \mu$ = {$a_{.4375}$, $b_{.86}$}. Thus we see that $\mu < Cl_{C}\ Int\ \mu$. By using Proposition 3.2 in [4], μ is fuzzy **C** - semi open. But $Int\ Cl_{C}\ Int\ \mu$ = {$a_{.3}$, $b_{.8}$}. Here μ $Int\ Cl_{C}\ Int\ \mu$. By using Proposition 4.2, μ is not fuzzy **C** - feebly open.

The following example shows that the intersectionion of any two fuzzy **C** - feebly open sets is not fuzzy **C** - feebly open.

Example 4.4

Let X = {a, b, c} and τ = **{**0, {$a_{.6}$, $b_{.3}$}, {$b_{.4}$, $c_{.7}$}, {$a_{.2}$, $c_{.5}$}, {$b_{.3}$}, {$a_{.6}$, $b_{.4}$, $c_{.7}$}, {$a_{.2}$, $b_{.4}$, $c_{.7}$}, {$c_{.5}$}, {$a_{.2}$}, {$a_{.6}$, $b_{.3}$, $c_{.5}$}, {$a_{.2}$, $b_{.3}$}, {$a_{.2}$, $b_{.3}$, $c_{.5}$}, {$b_{.3}$, $c_{.5}$}, 1**}**. Let $\mathbf{C}(x) = \dfrac{1-x}{1+2x}$, $0 \leq x \leq 1$, be a complement function. The family of all fuzzy **C**-closed sets C(τ) = **{**0, {$a_{.18}$, $b_{.44}$, c_{1}}, {a_{1}, $b_{.33}$, $c_{.125}$}, {$a_{.57}$, b_{1}, $c_{.25}$}, {a_{1}, $b_{.4375}$, c_{1}}, {$a_{.18}$, $b_{.33}$, $c_{.125}$}, {$a_{.57}$, $b_{.33}$, $c_{.125}$}, {a_{1}, b_{1}, $c_{.25}$}, {$a_{.57}$, b_{1}, c_{1}}, {$a_{.18}$, $b_{.4375}$, $c_{.25}$}, {$a_{.57}$, $b_{.4375}$, c_{1}}, {$a_{.57}$, $b_{.4375}$, $c_{.25}$}, {a_{1}, $b_{.4375}$, $c_{.25}$}, 1**}**.

Let λ = {$a_{.2}$, $b_{.2}$} and μ = {$b_{.4}$, $c_{.5}$} it can be computed that $Int\ \lambda$ = {$a_{.2}$}, $Cl_{C}\ Int\lambda$ = {$a_{.57}$, $b_{.33}$, $c_{.125}$} and $Int\ Cl_{C}Int\ \lambda$ =

$\{a_{.2}, b_{.3}\}$. That shows $\lambda \leq Int\ Cl_{\mathbf{C}}\ Int\lambda$.

Also $Int\,\mu = \{b_{.3}, c_{.5}\}$, $Cl_{C} Int\ \mu = \{a_{.18}, b_{.4375}, c_{1}\}$ and $Int\ Cl_{C} Int\ \mu = \{b_{.4}, c_{.7}\}$. This shows that $\mu \leq Int\ Cl_{C} Int\ \mu$. Now $\lambda \wedge \mu = \{b_{.2}\}$ it can be find that $Int\ Cl_{C} Int\ (\lambda \wedge \mu) = 0$, we see that $(\lambda \wedge \mu)\ \ Int\ Cl_{C} Int\ (\lambda \wedge \mu)$. From the above and by using Proposition 4.2, we see that $\lambda \wedge \mu$ is not fuzzy **C** - feebly open even if λ and μ are fuzzy **C** - feebly open.

B. Skin[9] established that any union of fuzzy feebly open sets is a fuzzy feebly open set. Further the following example shows that the union of any two fuzzy**C** - feebly open sets is not fuzzy**C** - feebly open.

Example 4.5

Let $X = \{a, b, c\}$ and $\tau = \{0, \{a_{.6}, b_{.3}\}, \{b_{.4}, c_{.7}\}, \{a_{.2}, c_{.5}\}, \{b_{.3}\}, \{a_{.6}, b_{.4}, c_{.7}\}, \{a_{.2}, b_{.4}, c_{.7}\}, \{c_{.5}\}, \{a_{.2}\}, \{a_{.6}, b_{.3}, c_{.5}\}, \{a_{.2}, b_{.3}\}, \{a_{.2}, b_{.3}, c_{.5}\}, \{b_{.3}, c_{.5}\}, 1\}$. Let **C** (x) $= \dfrac{2x}{1+x}$, $0 \leq x \leq 1$, be a complement function.

The family of all fuzzy **C** -closed sets

$C(\tau) = \{0, \{b_{.57}, c_{.75}\}, \{a_{.75}, b_{.46}\}, \{a_{.33}, c_{.75}\}, \{b_{.46}\}, \{a_{.75}, b_{.57}, c_{.75}\}, \{a_{.33}, b_{.57}, c_{.75}\}, \{c_{.75}\}, \{a_{.33}\}, \{a_{.75}, b_{.46}, c_{.75}\}, \{a_{.33}, b_{.46}\}, \{a_{.33}, b_{.46}, c_{.75}\}, \{b_{.46}, c_{.75}\}, 1\}$. Let $\lambda = \{b_{.3}, c_{.7}\}$ and $\mu = \{a_{.6}, b_{.2}\}$ it can be computed that $Int\,\lambda = \{b_{.3}, c_{.5}\}$, $Cl_{\mathbf{C}}\ Int\lambda = \{b_{.46}, c_{.75}\}$ and $Int\ Cl_{\mathbf{C}}\ Int\ \lambda = \{b_{.4}, c_{.7}\}$. That shows $\lambda \leq Int\ Cl_{\mathbf{C}}\ Int\lambda$.

Also $Int\,\mu = \{a_{.6}, b_{.3}\}$, $Cl_{\mathbf{C}} Int\,\mu = \{a_{.75}, b_{.46}\}$ and $Int\ Cl_{\mathbf{C}} Int\ \mu = \{a_{.6}, b_{.3}\}$. This shows that $\mu \leq Int\ Cl_{\mathbf{C}} Int\ \mu$. Now $\lambda \vee \mu = \{a_{.6}, b_{.3}, c_{.7}\}$ it can be find that $Int\ Cl_{\mathbf{C}} Int\ (\lambda \vee \mu) = \{a_{.6}, b_{.3}\}$, we see that $(\lambda \vee \mu)\ \ Int\ Cl_{\mathbf{C}}\ Int\ (\lambda \vee \mu)$. From the above and by using Proposition 4.2, we see that $\lambda \vee \mu$ is not fuzzy **C** - feebly open even if λ and μ are fuzzy **C** - feebly open.

Remark 4.6

From Example 4.4, it is to observe that the intersection of any two fuzzy **C** - feebly open sets is not fuzzy **C** - feebly open, even if the complement function **C** satisfies the monotonic and involutive conditions.

Also, in view of the next proposition, if the complement function**C** satisfies the monotonic and involutive conditions, then the arbitrary union of fuzzy **C** - feebly open sets is fuzzy **C** - feebly open.

Proposition 4.7

Let (X, τ) be a fuzzy topological space and **C** be a complement function that satisfies the monotonic and involutive conditions. Then the arbitrary union of fuzzy **C** -feebly open sets is fuzzy **C** -feebly open.

Proof.

Let Δ be a collection of fuzzy **C** - feebly open sets of a fuzzy topological space (X, τ). Then by using Proposition 4.2, for each $\lambda \in \Delta$, $\lambda \leq Int\ Cl_{\mathbf{C}} Int\ \lambda$. Thus $\bigvee_{\lambda\in\Delta} \lambda \leq \bigvee_{\lambda\in\Delta} Int\ Cl_{\mathbf{C}}\ Int\ \lambda$. Since **C** satisfies the monotonic and involutive conditions, by using Theorem 4.4 in[2], we have $\bigvee_{\lambda\in\Delta} \lambda \leq Int\ Cl_{\mathbf{C}}\ Int\ \bigvee_{\lambda\in\Delta} \lambda$. Again by using Proposition 4.11, $\bigvee_{\lambda\in\Delta} \lambda_{\alpha}$ is fuzzy **C** -feebly open.

Theorem 4.8

Let (X, τ) and (Y, σ) be any two fuzzy topological spaces such that X is **C** -product related to Y. Then the product $\lambda_1\times\lambda_2$ of a fuzzy **C** -feebly open set λ_1 of X and a fuzzy **C** -feebly open set λ_2 of Y is a fuzzy **C** -feebly open set of the fuzzy product space X $\times$ Y.

Proof.

Let λ_1 be a fuzzy**C** -feebly open subset of X and λ_2 be a fuzzy **C** -feebly open subset of Y. Then by using Proposition 4.2, we have $\lambda_1 \leq Int\ Cl_{C} Int\ \lambda_1$ and $\lambda_2 \leq Int\ Cl_{C} Int\ \lambda_2$. By using Theorem 2.19 in [3], implies that $\lambda_1 \times \lambda_2 \leq Int\ Cl_{C} Int\ (\lambda_1 \times \lambda_2)$. By using Proposition 4.2, $\lambda_1 \times \lambda_2$ is a fuzzy **C** -feebly open set of the fuzzy product space X $\times$ Y.

5. Fuzzy**C** –feebly closed sets

In this section we introduce the concept of **C -** feebly closed sets in a fuzzy topological space with respect to a complement function **C** : [0, 1]$\rightarrow$ [0, 1].

Definition 5.1

Let λ be a fuzzy subset of a fuzzy topological space X then λ is said to be fuzzy **C** - feebly closed if there exists a fuzzy **C** - closed set δ in X such that $sInt_{C}\ \delta \leq \lambda \leq \delta$.

The class of all fuzzy feebly closed sets coincides with the class of all fuzzy **C** - feebly closed sets if the standard complement function coincides with the arbitrary complement function.

Proposition 5.2

Let (X, τ) be a fuzzy topological space and let **C** be a complement function that satisfies the monotonic and involutive conditions. Then a fuzzy subset λ of a fuzzy topological space (X,τ) is fuzzy **C**- feebly closed if $Cl_{C}\ Int\ Cl_{C}\ \lambda \leq \lambda$.

Proof.

Let λ be a fuzzy **C** -feebly closed set of a fuzzy topological space X. Then by using Definition 5.1, there is fuzzy **C** -closed set δ in X such that $\delta \geq \lambda \geq sInt_{C}\ \delta$. Since **C** satisfies the monotonic and involutive conditions, by using Theorem 3.8, it follows that $\delta \geq \lambda \geq$ **C** (sCl_{C} C δ). By using Theorem 3.5, $\delta \geq \lambda \geq Cl_{C} Int\ \delta$. Since δ is fuzzy **C** -closed, $\delta = Cl_{C}\delta \geq Cl_{C}\lambda$, it follows that $Int\,\delta \geq Int\ Cl_{C}\ \lambda$. This implies that $Cl_{C} Int\ \delta \geq Cl_{C}\ Int\ Cl_{C}\ \lambda$. From the above $\lambda \geq Cl_{C} Int\ \delta \geq Cl_{C} Int\ Cl_{C}\ \lambda$. That shows $Cl_{C}\ Int\ Cl_{C}\ \lambda \leq \lambda$.

Conversely we assume that $\lambda \geq Cl_{C}\ Int\ Cl_{C}\ \lambda$. Now $Int\ \lambda \leq Cl_{C}\ \lambda$ that implies $Cl_{C}\ \lambda \geq \lambda \geq$ **C** ($Int\ Cl_{C}$C ($Cl_{C}\ \lambda$)). Since **C** satisfies the monotonic and involutive conditions, by using Theorem 3.8, $Cl_{C}\ \lambda \geq \lambda \geq$ **C** (sCl_{C}C ($Cl_{C}\ \lambda$)) = $sInt_{C}$ ($Cl_{C}\ \lambda$). Therefore, by using Theorem 3.8, λ is fuzzy **C** -feebly closed set in X.

The standard complement of fuzzy feebly open set is fuzzy feebly closed. But the analogous result is not true for fuzzy **C** - feebly -open. If the complement function **C** satisfies the monotonic and involutive conditions, then the arbitrary complement of fuzzy **C** - feebly open is fuzzy **C** - feebly closed as shown in the next proposition.

Proposition 5.3

Let λ be a fuzzy subset of a fuzzy topological space (X,τ) and **C** be a complement function that satisfies the monotonic and involutive conditions. Then λ is fuzzy **C** - feebly closed if and only if **C** λ is fuzzy **C** - feebly open.

Proof.

Let λ be fuzzy **C** - feebly closed. Then by using Proposition 5.2, $Cl_C\ Int\ Cl_C\ \lambda \leq \lambda$. Taking complement on both sides, we get **C** $(Cl_C\ Int\ Cl_C\ \lambda) \geq$ **C** λ. Since **C** satisfies the monotonic and involutive conditions, by using Lemma 2.4 (i) and (ii), we have $Int\ Cl_C\ Int$ **C** $\lambda \geq$ **C** λ. By using Proposition 4.2, **C** λ is fuzzy **C** - feebly ope

Conversely, **C** λ is fuzzy **C** - feebly open. By using Proposition 4.2, **C** λ $Int\ Cl_C\ Int$ **C** λ. Taking complement on both sides, we get **C** (**C** λ) $\geq$ **C** ($Int\ Cl_C\ Int$ **C** λ). Since **C** satisfies the monotonic and involutive conditions, by using Lemma 2.4 (i) and (ii), we have $Cl_C\ Int\ Cl_C\ \lambda \leq \lambda$. By using Proposition 5.2, λ is fuzzy **C** - feebly closed.

The following example shows that if the conditions monotonic and involutive cannot be dropped from the hypothesis of Proposition 5.3.

Example 5.4

Let $X = \{a, b\}$ and $\tau = \{0, \{a_{.3}, b_{.8}\}, \{a_{.2}, b_{.5}\}, \{a_{.7}, b_{.05}\}, \{a_{.3}, b_{.5}\}, \{a_{.3}, b_{.05}\}, \{a_{.2}, b_{.05}\}, \{a_{.7}, b_{.8}\}, \{a_{.7}, b_{.5}\}, 1\}$. Let **C** $(x) = \sqrt{x}$, $0 \leq x \leq 1$, be a complement function. We see that the complement function **C** does not satisfy the monotonic and involutive conditions. The family of all fuzzy **C** -closed sets is **C** $(\tau) = \{0, \{a_{.548}, b_{.894}\}, \{a_{.447}, b_{.707}\}, \{a_{.837}, b_{.223}\}, \{a_{.548}, b_{.707}\}, \{a_{.548}, b_{.223}\}, \{a_{.447}, b_{.223}\}, \{a_{.837}, b_{.894}\}, \{a_{.837}, b_{.707}\}, 1\}$.

Let $\lambda = \{a_{.548}, b_{.223}\}$. Then $Cl_C\ \lambda = \{a_{.548}, b_{.223}\}$, $Int\ Cl_C\ \lambda = \{a_{.3}, b_{.05}\}$ and $Cl_C\ Int\ Cl_C\ \lambda = \{a_{.447}, b_{.223}\}$. This shows that $Cl_C\ Int\ Cl_C\ \lambda \leq \lambda$. By using Definition 5.2, λ is fuzzy **C** - feebly closed.

But **C** $\lambda = \{a_{.74}, b_{.472}\}$. Then Int **C** $\lambda = \{a_{.7}, b_{.05}\}$, $Cl_C\ Int$ **C** $\lambda = \{a_{.837}, b_{.223}\}$ and $Int\ Cl_C\ Int$ **C** $\lambda = \{a_{.7}, b_{.05}\}$. This shows that **C** $\lambda \not\leq Int\ Cl_C\ Int$ **C** λ. By using Proposition 4.2, **C** λ is not fuzzy **C** - feebly open.

The following example shows that, if the complement function **C** does not satisfy the involutive condition then the conclusion of Proposition 5.3 is not true.

Example 5.5

Let $X = \{a, b\}$ and $\tau = \{0, \{a_{.3}, b_{.8}\}, \{a_{.2}, b_{.5}\}, \{a_{.7}, b_{.05}\}, \{a_{.3}, b_{.5}\}, \{a_{.3}, b_{.05}\}, \{a_{.2}, b_{.05}\}, \{a_{.7}, b_{.8}\}, \{a_{.7}, b_{.5}\}, 1\}$. Let $C(x) = \dfrac{1-x^3}{(1+x)^3}$, $0 \leq x \leq 1$, be a complement function. This complement function C does not satisfy the involutive condition. The family of all fuzzy **C** -closed sets **C** $(\tau) = \{0, \{a_{.443}, b_{.084}\}, \{a_{.574}, b_{.254}\}, \{a_{.134}, b_{.864}\}, \{a_{.443}, b_{.259}\}, \{a_{.574}, b_{.864}\}, \{a_{.134}, b_{.084}\}, \{a_{.134}, b_{.084}\}, \{a_{.134}, b_{.259}\}, 1\}$. Let $\mu = \{a_{.443}, b_{.259}\}$ Then $Cl_C\ \mu = \{a_{.443}, b_{.259}\}$, $Int\ Cl_C\ \mu = \{a_{.3}, b_{.05}\}$ and $Cl_C\ Int\ Cl_C\mu = \{a_{.443}, b_{.084}\}$. This shows that $Cl_C\ Int\ Cl_C\mu \leq \mu$. By using Proposition 5.2, μ is fuzzy

$\mathbf{C}$- feebly closed. But $\mathbf{C}\mu = \{a_{.304}, b_{.492}\}$. Then *Int* $\mathbf{C}\,\mu = \{a_{.3}, b_{.05}\}$, $Cl_{\mathbf{C}}\, Int\, \mathbf{C}\mu = \{a_{.443}, b_{.084}\}$ and $Int\, Cl_{\mathbf{C}}\, Int\, \mathbf{C}\mu = \{a_{.3}, b_{.05}\}$. This shows that $\mathbf{C}\mu \;\; Int\, Cl_{\mathbf{C}}\, Int\, \mathbf{C}\,\mu$. By using Proposition 4.2, $\mathbf{C}\mu$ is not fuzzy $\mathbf{C}$- feebly open.

It is clear that every fuzzy $\mathbf{C}$ -closed is a fuzzy $\mathbf{C}$- feebly closed set. But the converse is not true as shown in the following example.

Example 5.6

Let $X = \{a, b, c\}$ and $\tau = \{0, \{a_{.6}, b_{.3}\}, \{b_{.4}, c_{.7}\}, \{a_{.2}, c_{.5}\}, \{b_{.3}\}, \{a_{.6}, b_{.4}, c_{.7}\}, \{a_{.2}, b_{.4}, c_{.7}\}, \{c_{.5}\}, \{a_{.2}\}, \{a_{.6}, b_{.3}, c_{.5}\}, \{a_{.2}, b_{.3}\}, \{a_{.2}, b_{.3}, c_{.5}\}, \{b_{.3}, c_{.5}\}, 1\}$. Then (X, τ) is a fuzzy topological space. Let $\mathbf{C}(x) = \dfrac{1-x}{1+3x}$, $0 \le x \le 1$, be a complement function. The family of all fuzzy $\mathbf{C}$-closed sets $\mathbf{C}(\tau) = \{0, \{a_1, b_{.27}, c_{.097}\}, \{a_{.143}, b_{.368}, c_1\}, \{a_{.5}, b_1, c_{.2}\}, \{a_1, b_{.368}, c_1\}, \{a_{.143}, b_{.273}, c_{.097}\}, \{a_{.5}, b_{.273}, c_{.097}\}, \{a_1, b_1, c_{.2}\}, \{a_{.5}, b_1, c_1\}, \{a_{.143}, b_{.368}, c_{.2}\}, \{a_{.5}, b_{.368}, c_1\}, \{a_{.5}, b_{.368}, c_{.2}\}, \{a_1, b_{.368}, c_{.2}\}, 1\}$. Let $\lambda = \{a_{.5}, b_{.5}, c_{.2}\}$, it can be computed that $Cl_{\mathbf{C}}\lambda = \{a_{.5}, b_1, c_{.2}\}$,

$Int\, Cl_{\mathbf{C}}\, \lambda = \{a_{.2}, b_{.3}, c_{.5}\}$ and $Cl_{\mathbf{C}}\, Int\, Cl_{\mathbf{C}}\, \lambda = \{a_{.5}, b_{.368}, c_{.2}\}$. This shows that $Cl_{\mathbf{C}}\, Int\, Cl_{\mathbf{C}}\, \lambda \le \lambda$. By using Proposition 5.2, λ is fuzzy $\mathbf{C}$ - feebly closed. Also λ is not fuzzy $\mathbf{C}$ -closed.

The following example show that the union of fuzzy $\mathbf{C}$ - feebly closed sets is not fuzzy $\mathbf{C}$ - feebly closed.

Example 5.7

From the above Example 5.6, let $X = \{a, b, c\}$ and $\tau = \{0, \{a_{.6}, b_{.3}\}, \{b_{.4}, c_{.7}\}, \{a_{.2}, c_{.5}\}, \{b_{.3}\}, \{a_{.6}, b_{.4}, c_{.7}\}, \{a_{.2}, b_{.4}, c_{.7}\}, \{c_{.5}\}, \{a_{.2}\}, \{a_{.6}, b_{.3}, c_{.5}\}, \{a_{.2}, b_{.3}\}, \{a_{.2}, b_{.3}, c_{.5}\}, \{b_{.3}, c_{.5}\}, 1\}$. Let $\lambda = \{a_{.5}, b_{.5}, c_{.2}\}$ and $\mu = \{a_{.5}, b_{.368}, c_1\}$. Then it can be computed that $Cl_{\mathbf{C}}\lambda = \{a_{.5}, b_1, c_{.2}\}$, $Int\, Cl_{\mathbf{C}}\lambda = \{a_{.2}, b_{.3}, c_{.5}\}$ and $Cl_{\mathbf{C}}\, Int\, Cl_{\mathbf{C}}\, \lambda = \{a_{.5}, b_{.368}, c_{.2}\}$. That shows $Cl_{\mathbf{C}}\, Int\, Cl_{\mathbf{C}}\lambda \le \lambda$.In the same way, $Cl_{\mathbf{C}}\, \mu = \{a_{.5}, b_{.368}, c_1\}$, $Int\, Cl_{\mathbf{C}}\, \mu = \{a_{.2}, b_{.3}, c_{.5}\}$ and $Cl_{\mathbf{C}}\, Int\, Cl_{\mathbf{C}}\mu = \{a_{.5}, b_{.368}, c_1\}$. This shows that $Cl_{\mathbf{C}}\, Int\, Cl_{\mathbf{C}}\mu \;\; \mu$. By using Proposition 5.2, λ and μ are fuzzy $\mathbf{C}$ - feebly -closed. Now let $\lambda\vee = \{a_{.5}, b_{.5}, c_1\}$. Then $Cl_{\mathbf{C}}(\lambda\vee\mu) = \{a_{.5}, b_1, c_1\}$, $Int\, Cl_{\mathbf{C}}(\lambda\vee\mu) = \{a_{.2}, b_{.4}, c_{.7}\}$ and $Cl_{\mathbf{C}}\, Int\, Cl_{\mathbf{C}}(\lambda\vee\mu) = \{a_{.5}, b_1, c_1\}$. This shows that $Cl_{\mathbf{C}}\, Int\, Cl_{\mathbf{C}}\, (\lambda\vee\mu)\; \lambda\vee\mu$. By using Proposition 5.2, $\lambda \vee \mu$ is not fuzzy $\mathbf{C}$ - feebly closed.

Furthermore, the following example shows that the intersection of fuzzy $\mathbf{C}$- feebly closed sets is not fuzzy $\mathbf{C}$- feebly closed.

Example 5.8

Let $X = \{a, b\}$ and $\tau = \{0, \{a_{.3}, b_{.8}\}, \{a_{.2}, b_{.5}\}, \{a_{.7}, b_{.05}\}, \{a_{.3}, b_{.5}\}, \{a_{.3}, b_{.05}\}, \{a_{.2}, b_{.05}\}, \{a_{.7}, b_{.8}\}, \{a_{.7}, b_{.5}\}, 1\}$. Let $\mathbf{C}(x) = \dfrac{2x}{1+x}$, $0 \le x \le 1$ be the complement function. Then the family of all fuzzy $\mathbf{C}$ -closed sets $\mathbf{C}(\tau) = \{0, \{a_{.462}, b_{.889}\}, \{a_{.33}, b_{.667}\}, \{a_{.824}, b_{.095}\}, \{a_{.462}, b_{.667}\}, \{a_{.462}, b_{.095}\}, \{a_{.33}, b_{.095}\}, \{a_{.824}, b_{.889}\}, \{a_{.824}, b_{.667}\}, 1\}$. Let $\lambda = \{a_{.4}, b_{.667}\}$ and $\mu = \{a_{.5}, b_{.3}\}$. Then it can be computed that $Cl_{\mathbf{C}}\, \lambda = \{a_{.462}, b_{.667}\}$, $Int\, Cl_{\mathbf{C}}\, \lambda$

$= \{a_{.3}, b_{.5}\}$ and $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \lambda = \{a_{.33}, b_{.667}\}$. That shows $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \lambda \leq$. In the same way, $Cl_{\mathbf{C}}\ \mu = \{a_{.824}, b_{.667}\}$, $Int\ Cl_{\mathbf{C}}\ \mu = \{a_{.7}, b_{.5}\}$ and $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \mu = \{a_{.824}, b_{.667}\}$. This shows that $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}\ \mu \leq \mu$. By using Proposition 3.19, λ and μ is fuzzy $\mathbf{C}$ - feebly closed.

Now $\lambda \wedge \mu = \{a_{.4}, b_{.3}\}$. Then $Cl_{\mathbf{C}}(\lambda\wedge\mu) = \{a_{.462}, b_{.667}\}$, $Int\ Cl_{\mathbf{C}}(\lambda\wedge\mu)=\{a_{.3}, b_{.5}\}$ and $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}(\lambda\wedge\mu) = \{a_{.462}, b_{.667}\}$. This shows that $Cl_{\mathbf{C}}\ Int\ Cl_{\mathbf{C}}(\lambda\wedge \mu)\ \ \lambda\wedge \mu$. By using Proposition 5.2, $\lambda\wedge\mu$ is not fuzzy $\mathbf{C}$- feebly closed.

Remark 5.9

Further, the Example 3.24 shows that the union of any two fuzzy $\mathbf{C}$ - feebly closed sets need not be fuzzy $\mathbf{C}$ - feebly closed, even though the complement function satisfies the monotonic and involutive conditions.

If the complement functions $\mathbf{C}$ satisfies the monotonic and involutive conditions, then arbitrary intersection of fuzzy $\mathbf{C}$ - feebly closed sets is fuzzy $\mathbf{C}$ - feebly closed as shown in the following proposition.

Proposition 5.10

Let (X, τ) be a fuzzy topological space and $\mathbf{C}$ be a complement function that satisfies the monotonic and involutive conditions. Then the arbitrary intersection of fuzzy $\mathbf{C}$ - feebly closed sets is fuzzy $\mathbf{C}$ - feebly closed.

Proof.

Let Δ be a collection of fuzzy $\mathbf{C}$- feebly closed sets of a fuzzy topological space (X, τ). Then by using Proposition 5.2, $\mathbf{C}\ \Delta$ be a collection of fuzzy $\mathbf{C}$- feebly open sets. By using Proposition 4.2, for each $\lambda\in\Delta$, $\mathbf{C}\ \lambda\leq Int\ Cl_{\mathbf{C}}\ Int\ \mathbf{C}\ \lambda$. Also by Proposition 4.7, we have arbitrary union of fuzzy $\mathbf{C}$ - feebly open sets are fuzzy $\mathbf{C}$ - feebly open. Thus we have $\mathbf{C}\ \lambda \leq Int\ Cl_{\mathbf{C}}\ Int\ \mathbf{C}\ \lambda$. Since $\mathbf{C}$ satisfies the monotonic and involutive conditions, by using Lemma 2.9 in [2], we have $\mathbf{C}(\lambda) \leq Int\ Cl_{\mathbf{C}}\ Int\ \mathbf{C}(\lambda)$. This shows that, $\mathbf{C}(\lambda)$ is fuzzy $\mathbf{C}$- feebly open. Then by using Proposition 5.2, we have λ is fuzzy $\mathbf{C}$ - feebly closed.

References

1. K.K. Azad, On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, *J. Math. Anal. Appl. 82(1)*, 14-32 (1981).
2. K. Bageerathi, G. Sutha, P. Thangavelu, A generalization of fuzzy closed sets, *International Journal of fuzzy systems and rough systems, 4(1),* 1-5 (2011).
3. K. Bageerathi, P. Thangavelu, A generalization of fuzzy Boundary, *International Journal of Mathematical Archive – 1(3),* 73-80 (2010).
4. K. Bageerathi, P. Thangavelu, On generalization of nearly fuzzy open sets, Proceedings of the International Seminar On New Trends in Applications of Mathematics (ISNTAM 2011) Bharata Mata College, Kerala, Jan 31, Feb 1-2(2011). **(To appear).**
5. K. Bageerathi, P. Thangavelu, On nearly fuzzy $\mathbf{C}$-closed sets in fuzzy topological spaces, *Advances in Theoretical Mathematics and Applied Sciences –6(5),* 617-633 (2011).

6. K. Bageerathi, P. Thangavelu, A generalization of fuzzy semi-boundary, *International Journal of Ultra Scientist of Physical Sciences,* 23(3)A, Dec, 855-868 (2011).
7. C. L. Chang, Fuzzy topological space, *J. Math. Anal. Appl. 24,* 182-190 (1968).
8. George J. Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic Theory and Applications, *Prentice – Hall, Inc,* (2005).
9. K. Katsaras and D.B. Liu, Fuzzy vector spaces and fuzzy topological vector spaces, *J. Math. Anal. Appl. 8 (3),* 459-470 (1978).
10. B. Sikn, On fuzzy FC-Compactness, Comm. Korean Math. Soc. *13,* 137-150 (1998).