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Analysis of General-Input Bulk Queue with Vacations $GI/M^{(a,b)}/1$

SANDEEP DIXIT

Department of Mathematics V.S.S.D. College, Kanpur (India)

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Abstract

In this section we consider a bulk service $GI/M^{(a,b)}/1$ Queue with multiple exponential Vacations. customers are served in batches according to the bulk service rule, in which at least a customers are needed to start a service and the maximum capacity of each service is b. When the server finishes a service and finds fewer than a customers in the system, he takes a vacation. When the server returns from a vacation and finds fewer than a customers in the system, the server immediately takes another vacation and continuous in this manner until the server finds at least a customers in the system.

Key words: Bulk queue, General Input, Single Server.

Introduction

This paper consider a bulk service queue with single server. The bulk service queue used in the various areas such as traffics, transportation, manufacturing, and other stochastic systems. Here we study if then arrivals customers are fewer than the limit a server takes a vacation. In such cases we find the steady state differential equation if then server has no vocations[1-4].

Then the Interarrival time $\{T_n, n \geq 1\}$ are i.i.d. random variable with the General distribution function G (t), the density function a(t), the mean λ^{-1} and the LST $a^*(s)$. The service time and the vacation times are independent of the arrival process and are i.i.d. exponential random variable with rates μ and θ, respectively. Let

$$\ell = \lambda(b\mu)^{-1} < 1.$$

To obtain the distribution of the queue length at arrivals instants and at arbitrary instants simultaneously. We use the supplementary variable method that has been used in other bulk queue models.

Now we use the residual interarrival time as the supplementary variable. At an arbitrary instant, the steady state of the system can be described by the following random variables:

$$\delta = \begin{cases} 0 & \text{if the server is on vacation} \\ j & \text{if the server is busy with j customers in a batch,, } a \le j \le b. \end{cases}$$

$$\text{Lq} = \text{the number of customers in the queue}$$

$$\hat{A} = \text{the residual interarrival time}$$

Define

$$F_{i0}(x)dx = P(Lq = i, x < \hat{A} \le x + dx, \delta = 0), \quad i \ge 0,$$

$$F_{ij}(x)\,dx = P\,(Lq = i, x < \hat{A} \le x + dx, \delta = j), \quad i \ge 0, a \le j \le b,$$

and the LSTs

$$F_{ij}^{*}(t) = \int_0^\infty e^{-tx} F_{ij}(x)dx$$

the steady-state system, we obtain the following differential difference equations:

$$-\frac{dF_{00}(x)}{dx} = \sum_{n=a}^{b} \mu F_{0n}(x) \tag{1}$$

$$-\frac{dF_{i0}(x)}{dx} = \sum_{n=a}^{b} \mu F_{in}(x) + a(x)F_{i-1,0}(0), \quad i < a, \tag{2}$$

$$\frac{dF_{i0}(x)}{dx} = -\theta F_{i0}(x) + a(x)F_{i-1,0}(0), \quad i \ge a, \tag{3}$$

$$-\frac{dF_{0j}(x)}{dx} = -\mu F_{0j}(x) + \theta F_{j0}(t) + \sum_{n=a}^{b} \mu F_{jn}(x), \quad a \le j \le b, \tag{4}$$

$$-\frac{dF_{ij}(x)}{dx} = -\mu F_{ij}(x) + a(x)F_{i-1,j}(0), \quad i \ge 1, a \le j \le b-1, \tag{5}$$

$$-\frac{dF_{ib}(x)}{dx} = -\mu F_{ib}(x) + \theta F_{i+b,0}(x) + \sum_{n=a}^{b} \mu F_{i+b,n}(x) + a(x) F_{i-1,b}, \; i \geq 1. \quad (6)$$

Taking the LST on both sides of the equations of (6), we have

$$tF^*_{00}(t) = F_{00}(0) - \sum_{n=a}^{b} \mu F^*_{0n}(t), \quad (7)$$

$$tF^*_{i0}(t) = F_{i0}(0) - a^*(t) F_{i-1,0}(0) - \sum_{n=a}^{b} \mu F^*_{in}(t), \qquad i < a, \quad (8)$$

$$(t - \theta) F^*_{i0}(t) = F_{i0}(0) - a^*(t) F_{i-1,0}(0), \qquad i \geq a, \quad (9)$$

$$(t - \mu) F^*_{0j}(t) + \sum_{n=a}^{b} \mu F^*_{jn}(t) + \theta F^*_{j0}(t) = F_{0j}(0), \qquad a \leq j \leq b, \quad (10)$$

$$(t - \mu) F^*_{ij}(t) = F_{ij}(0) - a^*(t) F^*_{i-1,j}(0), \qquad i \geq 1, a \leq j \leq b-1, \quad (11)$$

$$(t - \mu) F^*_{ib}(t) + \sum_{n=a}^{b} \mu F^*_{i+b,n}(t) + \theta F^*_{i+b,0}(t) = F_{ib}(0) - a^*(t) F_{i-1,b}(0), \quad i > 1 \quad (12)$$

To find the general solution of the difference equation that occur in the supplementary variable method, it is well known that the polynomial of the right shift operator can be used (see Gross and Harris (1985). A brief summary of the operational calculus is presented here for the convenience of reference[2-4].

For a sequence $\{x_n\}$ of complex numbers, the right shift operator, denoted by D, is defined of $Dx_n = x_{n+1}$ for n. If $f(z) = \alpha_0 + \alpha_1 z + \ldots\ldots + a_k z^k$ is a polynomial with complex coefficients α_i, then f $(D) = \alpha_0 + \alpha_1 D + \ldots\ldots\ldots\alpha_k D^K)$ is defined by

$$f(D).x_n = \alpha_0 x_n + \alpha_1 x_n + 1 + \ldots\ldots + \alpha_k x_{n+k}.$$

For a geometric sequence $\{w^n\}$ and $f(z) = \sum_{k=0}^{\infty} \alpha_k z^k$, it is natural to define $f(D) = \sum_{k=0}^{\infty} \alpha_k D^k$ by

$$f(D).w^n = \left(\sum_{k=0}^{\infty} \alpha_k D^K\right).w^n = f(w).w^n.$$

For instance, since $\exp(z) = \sum_{n=0}^{\infty} \frac{1}{k!} z^k$, it follow that

$$\exp(D).\,w^n = e^w.w^n$$

and for the LST of a function $a^*(t) = \int_0^\infty e^{-sx}a(x)dx$, we have $a^*(D).\,w^n.w^n$. If $f(D).x_n = w^n$.

Conclusion

In the last we discuss then steady-state differential difference equation and find then general solution of the difference equation and also discuss the right shift operator to the sequence of complex numbers[1-4].

Reference

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