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Certain simultaneous five tuple series equations involving laguerre polynomials

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Abstract

Lowndes[4,3] have obtained the solution of some dual series equations involving laguerre polynoimials and then solved triple series equations involving laguerre polynomials. Singh, Rokne and Dhaliwal[9] obtained closed form solution of triple series equations involving Laguerre polynomials and Srivastava[10] have also obtained the solutions of certain dual series equations involving Laguerre polynomials. In the present paper, an exact solution has been obtained for the simultaneous five tuple series equations involving Laguerre polynomials by Noble's[8] modified multiplying factor technique.

Key words: 45F 10 Five tuple Series Equations, 33C45 Laguerre polynomials, 33D45 Basic Orthogonal Polynomials and Functions, 42C05 Orthogonal Functions and Polynomials, General Theory, 26A33 Fractional Derivatives and Integrals, 33B15 Beta Function, 34BXX Boundary Value Problem.

1. Introduction

In the present paper, an exact solution of the simultaneous five tuple series equations has been given:

$$\sum_{n=o}^{\infty}\sum_{j=1}^{s} a_{ij}\,\frac{A_{nj}}{\Gamma(\alpha+ni+p+1)}\,L_{ni+p}^{(\alpha)}(x)=u_i(x),\quad 0\le x<a \tag{1.1}$$

$$\sum_{n=o}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)} L_{ni+p}^{(\alpha+\beta-1)}(x) = v_i(x),\ a < x < b \tag{1.2}$$

$$\sum_{n=o}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)} L_{ni+p}^{(\alpha+\beta-1)}(x) = w_i(x),\quad b < x < c \tag{1.3}$$

$$\sum_{n=o}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)} L_{ni+p}^{(\alpha+\beta-1)}(x) = y_i(x),\quad c < x < d \tag{1.4}$$

$$\sum_{n=o}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)} L_{ni+p}^{(\sigma)}(x) = z_i(x),\ d < x < \infty \tag{1.5}$$

i = 1, 2, 3, ..., s

where, $\alpha+\beta+1>\beta>1-m$, $\sigma+1>\alpha+\beta>0$, m is a positive integer, p is an arbitrary non-negative integer, a_{ij}, b_{ij}, c_{ij}, and d_{ij} are known constants;, $u_i(x)$, $v_i(x)$, $w_i(x)$ and $z_i(x)$ are prescribed functions and

$$L_n^{(\alpha)}(x) = \sum_{k=0}^{n} \binom{n+\alpha}{n-k} \frac{(-x)^k}{k},\quad n = 0,1,2,...... \tag{1.6}$$

is the Laguerre polynomial[1-7] of order α and degree n in x.

2. Priliminary Results :

The following results are required in our investigation:

(i) The orthogonality property of the Laguerre polynomials is given by Erdelyi:

$$\int_0^{\infty} e^{-x} x^{\alpha} L_m^{(\alpha)}(x) L_n^{(\alpha)}(x)dx = \frac{\Gamma(\alpha+n+1)}{n!}\delta_{m,n},\ \alpha > -1; \tag{2.1}$$

where $\delta_{m,n}$ is the kronecker delta.

(ii) Formula (27), pp. 190 in the form:

$$\frac{d^m}{dx^m}\left\{x^{\alpha+m}L_n^{(\alpha+m)}(x)\right\}=\frac{\Gamma(\alpha+m+n+1)}{\Gamma(\alpha+n+1)}x^{\alpha}L_n^{(\alpha)}(x) \tag{2.2}$$

(iii) The following forms[1-11] of the known results of Erdelyi {pp. 191(30)} and {pp. 405(20)}:

$$\int_0^{\xi} x^{\alpha}(\xi-x)^{\beta-1}L_n^{(\alpha)}(x)dx=\frac{\Gamma(\alpha+n+1)\Gamma\beta}{\Gamma(\alpha+\beta+n+1)}\xi^{\alpha+\beta}L_n^{(\alpha+\beta)}(\xi) \tag{2.3}$$

where, $\alpha>-1,\ \beta>0$ and

$$\int_{\xi}^{\infty} e^{-x}(x-\xi)^{\beta-1}L_n^{(\alpha)}(x)dx=\Gamma(\beta)e^{-\xi}L_n^{(\alpha-\beta)}(\xi) \tag{2.4}$$

where $\alpha+1>\beta>0$.

3. Solution of quadruple series equations :

Multiplying equation [3.3] by $x^{\alpha}(\xi-x)^{\beta+m-2}$, where m is a positive integer, and equation [3.3] by $e^{-x}(x-h)^{s-a-b}$, integrating them with respect to x over the intervals $(0,\xi)$ and (ξ,∞) respectively, we find on using [3.3] and [3.3], that

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}}{\Gamma(\alpha+\beta+m+ni+p)}L_{ni+p}^{(\alpha+\beta+m-1)}(\xi)=\frac{\xi^{-\alpha-\beta-m+1}}{\Gamma(\beta+m-1)}\int_0^{\xi} x^{\alpha}(\xi-x)^{\beta+m-2}u_i(x)dx \tag{3.1}$$

where, $0<\xi<a,\ \alpha>-1,\ \beta+m>1,$ $i=1,2,3,\dots\dots\dots,s;$ and

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} bij\frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)}L_{ni+p}^{(\alpha+\beta-1)}(\xi)=\frac{e^{\xi}}{\Gamma(\sigma-\alpha-\beta+1)}\int_{\xi}^{\infty} e^{-x}(x-\xi)^{\sigma-\alpha-\beta}z_i(x)dx \tag{3.2}$$

where, $c<\xi<\infty,\ \sigma+1>\alpha+\beta>0,$ $i=1,2,3,\dots\dots,s$.

Now multiplying equation [3.3] by $\xi^{\alpha+\beta+m-1}$, differentiating both sides m times with respect to ξ, and using the formula [3.3], we thus find[7-11]

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)}L_{ni+p}^{(\alpha+\beta-1)}(\xi)=\sum_{j=1}^{s} e_{ij}\frac{\xi^{-\alpha-\beta+1}}{\Gamma(\beta+m-1)}\frac{d^m}{d\xi^m}\int_0^{\xi} x^{\alpha}(\xi-x)^{\beta+m-2}u_i(\xi)dx \tag{3.3}$$

where, e_{ij} are the element of the matrix $\left[b_{ij}\right]\left[a_{ij}\right]^{-1}$ and $0<\xi<a,\ \alpha>-1,$

$\beta + m > 1,\ i = 1,2,3,......,s$.

From [3.3]
we have

$\sum_{n=1}^{\infty} \sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)} L_{ni+p}^{(\alpha+\beta-1)}$

$(x) = \sum_{j=1}^{s} e_{ij}\, w_i\,(x)$ (3.4)

Where, e_{ij} are the elements of the matrix $[b_{ij}]\,[a_{ij}]^{-1}$ and $b < x < c$, $\alpha + \beta > 0$, $i = 1,2,\ldots\ldots\ldots\ldots\ldots,s$.

we have $\sum_{n=1}^{\infty} \sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(\alpha+\beta+ni+p)}\ L_{ni+p}^{(\alpha+\beta-1)}$

$(x) = \sum_{j=1}^{s} f_{ij}\, y_i(x)$ (3.5)

Where, f_{ij} are the elements of the matrix $[b_{ij}]\,[a_{ij}]^{-1}$ and $c < x < d$, $\alpha + \beta > 0$, $i = 1,2,\ldots\ldots\ldots\ldots\ldots,s$.

Now, the left hand sides of the equations [3.3], [1.2] , [3.4] , [3.5]and [3.2] are identical and hence on using the orthogonality relation [3.3], we obtain the solution of series equations [3.3], [1.2], [3.4], [35] and [3.2] in the form:

$$A_{nj} = \sum_{j=1}^{s} d_{ij} \left[\begin{array}{c} \sum_{j=1}^{s} e_{ij} \frac{(ni+p)!}{\Gamma(\beta+m-1)} \int_0^a e^{-\xi} L_{ni+p}^{(\alpha+\beta-1)}(\xi)\, U_i\,(\xi) d\xi \\ + (ni+p)! \int_a^b \xi^{\alpha+\beta-1} e^{-\xi} L_{ni+p}^{(\alpha+\beta-1)}(\xi)\, V_i(\xi) d\xi \\ + (ni+p)! \int_b^c \xi^{\alpha+\beta-1} e^{-\xi} L_{ni+p}^{(\alpha+\beta-1)}(\xi) W_i(\xi) d\xi \\ + \sum_{j=1}^{s} f_{ij}\ (ni+p)! \int_c^d \xi^{\alpha+\beta-1} e^{-\xi} L_{ni+p}^{(\alpha+\beta-1)}(\xi) Y_i(\xi) d\xi \\ + \frac{(ni+p)!}{\Gamma(\sigma-\alpha-\beta+1)} \int_c^{\infty} \xi^{\alpha+\beta-1} L_{ni+p}^{(\alpha+\beta-1)}(\xi) Z_i(\xi) d\xi \end{array} \right]$$

where, $n,\ \ p = \{0,1,2,..........\},\ \ j = 1,$ $2,3,.........,s;\ d_{ij}$ are the element of the matrix $[b_{ij}]^{-1}$, (3.6)

and $U_i(\xi) = \dfrac{d^m}{d\xi^m} \int_0^{\xi}\ x^{\alpha}(\xi - x)^{\beta+m-2} u_i(x) dx$ (3.7)

$V_i(x) = v_i\,(x)$ (3.8)

$W_i\,(x) = w\,(x)$ (3.9)

$Y_i(x) = y_i(x)$ (3.10)

$Z_i(\xi) = \int_{\xi}^{\infty}\ e^{-x}(x-\xi)^{\sigma-\alpha-\beta} z_i(x) dx$ (3.11)

provided that $\alpha + \beta + 1 > \beta > 1 - m$ and $\sigma + 1 > \alpha + \beta > 0,\ m$ being a positive integer. Where $i = 1,2,\ldots\ldots\ldots\ldots, s$.

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