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Bianchi type I inflationary cosmological model with cosmological constant Λ

RAJ BALI* and SWATI

Department of Mathematics
University of Rajasthan, Jaipur – 302004 (INDIA)
*E-mail : balir5@yahoo.co.in

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Abstract

Inflationary cosmological model with cosmological constant Λ in the structure of Bianchi Type I space-time is investigated. To get the deterministic model of the universe, it has been assumed that $ABC=e^{3Ht}$ and $V(\phi)$ = constant, where A,B,C are metric potentials, H the Hubble constant, V the flat potential, ϕ the Higg's field. It is found that inflationary scenario exists in Bianchi Type I space-time, the Higg's field decreases with time, the cosmological constant decreases with time and Λ tends to constant quantity when $t \to \infty$. Since shear $\sigma \neq 0$ for large values of t, hence anisotropy is maintained for large values of t. However, when a=0, d = 0 then the model isotropizes. The physical aspects of the model in context to inflationary scenario are also discussed.

Key words: Bianchi I, Inflationary, Cosmological, Cosmological Constant (Λ)

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1. Introduction

The inflationary universe is a cosmological model in which there is a period during the very early universe when volume of space expands exponentially. Inflationary period arises when matter is described by particle physics (instead of a ideal gas) in the early universe. These models provide a mechanism which produces energy density fluctuations which lead to galaxies and other structure in the universe. Inflationary models provide a natural explanation of the flatness of universe and of its homogeneity at large scales and primordial monopole problem of grand unified field theories[1]. Homogeneity follows from the observed isotropy of the microwave background of radiation to less than 1 part in 10^4. The flatness is quantified by comparing the present energy density (ρ) with critical energy density ρ_c. The ratio ρ/ρ_c for flat universe is most likely between 0.1 and 1 as per astronomical observations.

The inflationary hypothesis was originally proposed by American Physicist Guth[2] who named it inflation. It was also proposed by Sato[3] around the same period. Several authors *viz*. Albrecht and Steinhardt[4], Wald[5], Barrow[6], Ellis and Madsen[7] have studied different aspects of Scalar field in the evolution of universe in FRW space-time. Using the concept of Higg's field (ϕ) with potential $V(\phi)$, inflation will take place if $V(\phi)$ has a flat region and ϕ field evolves slowly but the universe expands in an exponential way due to vacuum field energy (Stein-Schabes[8]). Rothman and Ellis[9] have pointed out that we can have the solution of the isotropy problem if we work with anisotropic metrics. Bali and Jain[10], Rahman *et al.*[11], Reddy *et al.*[12] have studied the role of self-interacting scalar fields in inflationary cosmology. In cosmology, cosmological constant problem is one of the outstanding problem. (Weinberg[13]). Bachall *et al.*[14] in their investigations have pointed out that there is enormous discrepancy between the value of vacuum energy density (Λ) as predicted by Quantum field theory of standard model and observed value of Λ. Linde[15] in his investigation has shown for spatially homogeneous universe that Λ in function of temperature and is related to spontaneous symmetry breaking process. Hence Λ is function of time. Cermeli and Kuzmenko[16] have shown that the cosmological relativistic theory predicts $\Lambda = 1.934 \times 10^{-35} s^{-2}$ which is in agreement with supernova cosmological project (Perlmutter *et al.*[17] and Riess *et al.*[18]. Recently, Bali[19] have investigated inflationary scenario in Bianchi Type I space-time considering scale factor $R^3 \sim e^{3Ht}$ without cosmological constant (Λ) where H is the Hubble constant.

In this paper we have discussed the inflationary scenario in Bianchi Type I space-time using the condition $R^3 \sim e^{3Ht}$ as considered by Bali[19] with cosmological constant (Λ). It is found that inflationary scenario exists in Bianchi type I space-time with cosmological constant (Λ). The Higg's field (ϕ) decreases with time, the cosmological constant decreases with time and Λ tends to a constant when $t \to 0$ and $t \to \infty$. Since shear (σ) $\neq 0$ for large values of t, hence anisotropy is maintained throughout. However, if a = 0, d = 0 then the model isotropizes for large values of t.

2. Metric and Field Equation :

We consider Bianchi Type-I in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \qquad (2.1)$$

Where A, B and C are metric potential and the function of 't' alone.

Einstein field equation is given by –

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -8\pi T_i^j \qquad (2.2)$$

(in geometrized unit G=1=c)

with

$$T_{ij} = \partial_i\phi\,\partial_j\phi - \left[\frac{1}{2}\partial_\sigma\phi\partial^\sigma\phi + V(\phi)\right]g_{ij} \qquad (2.3a)$$

and

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\,\partial^\mu\phi\right] = -\frac{dV}{d\phi} \qquad (2.3b)$$

where

$$\partial_i\phi = \frac{\partial\phi}{\partial x^i},\ \partial^\sigma\phi = g^{\sigma\mu}\partial_\mu\phi = g^{\sigma\mu}\frac{\partial\phi}{\partial x^\mu}$$

Einstein field equation (2.2) together with (2.3a) for the metric (2.1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \Lambda = -8\pi\left(\frac{\dot\phi^2}{2} - V(\phi)\right) \qquad (2.4)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \Lambda = -8\pi\left(\frac{\dot\phi^2}{2} - V(\phi)\right) \qquad (2.5)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \Lambda = -8\pi\left(\frac{\dot\phi^2}{2} - V(\phi)\right) \qquad (2.6)$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} + \Lambda = 8\pi\left(\frac{\dot\phi^2}{2} - V(\phi)\right) \qquad (2.7)$$

Equation (2.3b) leads to –

$$\ddot\phi + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\dot\phi = \frac{dV}{d\phi} \qquad (2.8)$$

3. Solution of Field Equations :

We are interested in inflationary solution so flat region is considered where potential V(ϕ) is constant i.e.

$$V(\phi) = \text{constant} = k \text{ (say)} \qquad (3.1)$$

Now equations (2.8) and (3.1) lead to

$$\dot\phi = \frac{\ell}{ABC} \qquad (3.2)$$

where ‘*l*’ is constant of integration.

From equations (2.4) and (2.5), we get

$$\frac{A_{44}}{A} + \frac{A_4C_4}{AC} - \frac{B_{44}}{B} - \frac{B_4C_4}{BC} = 0 \qquad (3.3)$$

Now equations (2.5) and (2.6) lead to

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4B_4}{AB} - \frac{A_4C_4}{AC} = 0 \qquad (3.4)$$

From equations (2.6) and (2.7), we get

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{2A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} + 2\Lambda = 16\pi\kappa \qquad (3.5)$$

as V(ϕ) = constant.

To find the solution, we assume the condition

$$ABC = e^{3Ht} \qquad (3.6)$$

As used by Bali[19].

Equation (3.6) leads to

$$\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = 3H \qquad (3.7)$$

Equations (3.3) and (3.4) lead to

$$\frac{A_{44}}{A} - \frac{C_{44}}{C} + \frac{A_4B_4}{AB} - \frac{B_4C_4}{BC} = 0 \qquad (3.8)$$

From equations (3.3) and (3.5), we have

$$2\frac{A_{44}}{A} + 2\frac{A_4C_4}{AC} + 2\frac{A_4B_4}{AB} + 2\Lambda = 16\pi\kappa \quad (3.9)$$

Equations (3.8) and (3.9) lead to

$$2\frac{C_{44}}{C} + 2\frac{A_4C_4}{AC} + 2\frac{B_4C_4}{BC} + 2\Lambda = 16\pi\kappa \quad (3.10)$$

From equation (3.7), we have, $\frac{B_4}{B} = 3H - \frac{A_4}{A} - \frac{C_4}{C}$ Substituting the above value in equation (3.10), we get

$$\frac{C_{44}}{C} - \frac{C_4^2}{C^2} + 3H\frac{C_4}{C} = 8\pi\kappa - \Lambda$$

$$= \alpha,\ 8\pi\kappa - \Lambda = \alpha\ (\text{say}) \quad (3.11)$$

To find the solution of (3.11), we suppose $C = e^{\lambda}$. Thus equation (3.11) leads to

$$\lambda_{44} + 3H\,\lambda_4 = \alpha \quad (3.12)$$

Equation (3.12) leads to

$$\lambda_4 = \frac{\alpha}{3H} + \beta e^{-3Ht} \quad (3.13)$$

where β is constant of integration.

Equation (3.13) leads to

$$\lambda = \frac{\alpha t}{3H} - \frac{\beta}{3H} e^{-3Ht} + \gamma \quad (3.14)$$

γ being constant of integration.

Thus

$$C = e^{\lambda} = \exp\left(\frac{\alpha t}{3H} - \frac{\beta}{3H} e^{-3Ht} + \gamma\right) \quad (3.15)$$

From equation (3.3), we get,

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_4}{C}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0 \quad (3.16)$$

which leads to –

$$\frac{\left(BA_4 - AB_4\right)_4}{\left(BA_4 - AB_4\right)} = -\frac{C_4}{C} \quad (3.17)$$

Equation (3.17) leads to

$$B^2\left(\frac{A}{B}\right)_4 = \frac{\ell_1}{C} \quad (3.18)$$

l_1 being constant of integration.

To find the solution, we suppose

$$AB = \mu \quad (3.19)$$

and

$$A / B = \nu \quad (3.20)$$

From above two equations, we get –

$$\frac{A_4}{A} + \frac{B_4}{B} = \frac{\mu_4}{\mu} \quad (3.21)$$

$$\frac{A_4}{A} - \frac{B_4}{B} = \frac{\nu_4}{\nu} \quad (3.22)$$

and

$$B^2 = \frac{\mu}{\nu} \text{ and } A^2 = \mu\nu \quad (3.23)$$

Using (3.21) in (3.7), we get –

$$\frac{\mu_4}{\mu} = 3H - \frac{C_4}{C} \quad (3.24)$$

which leads to

$$\mu = \frac{\ell_2}{C} e^{3Ht} \quad (3.25)$$

l_2 is constant of integration

From equations (3.18) and (3.25), we get –

$$\frac{\nu_4}{\nu} = \frac{\ell_1}{C\mu} \qquad (3.26)$$

which leads to

$$\frac{\nu_4}{\nu} = L\, e^{-3Ht} \qquad (3.27)$$

where $L = l_1 / l_2$. Equation (3.27) leads to

$$\nu = L_1 \exp\left(-\frac{L}{3H}e^{-3Ht}\right)$$

where L_1 is constant of integration.

Thus,

$$B^2 = \frac{\mu}{\nu}$$

which leads to

$$B^2 = \frac{\ell_2}{L_1} e^{3Ht} \exp\left[e^{-3Ht}\left(\frac{\beta}{3H} + \frac{L}{3H}\right) - \frac{\alpha t}{3H} - \gamma\right] \qquad (3.28)$$

and

$$A^2 = \mu\nu = \ell_2 L_1 e^{3Ht} \exp\left[-\frac{\alpha t}{3H} + e^{-3Ht}\left(\frac{\beta}{3H} - \frac{L}{3H}\right) - \gamma\right] \qquad (3.29)$$

Therefore, the metric (2.1) reduces to the form

$$ds^2 = -dt^2 + e^{3Ht}\exp\left\{-\frac{\alpha t}{3H} + e^{-3Ht}\left(\frac{\beta}{3H} - \frac{L}{3H}\right) + \gamma\right\}dx^2 + e^{3Ht}\exp\left\{e^{-3Ht}\left(\frac{\beta}{3H} + \frac{L}{3H}\right) - \frac{\alpha t}{3H} - \gamma\right\}dy^2 + \exp\left(\frac{\alpha t}{3H} - \frac{\beta}{3H}\, e^{-3Ht} + \gamma\right)dz^2 \qquad (3.30)$$

The spatial volume R^3 is given by

$$R^3 = e^{3Ht} \qquad (3.31)$$

Equation (2.7) leads to

$$\Lambda = (4\pi\ell^2 - d_3)e^{-6Ht} - d_2\, e^{-3Ht} + (8\pi k - d_1) \qquad (3.32)$$

where d_1, d_2 and d_3 are constant.

The shear (σ) is given by

$$\sigma = \frac{1}{\sqrt{2}}[(2a^2 + d^2) + (b^2 + c^2 + 9\beta^2)e^{-6Ht} + (2ab - 2ac + 6d\beta)e^{-3Ht}]^{1/2}$$

where

$$a = \frac{3H}{2} - \frac{\alpha}{2H}$$

$$b = \frac{3L}{2} - \frac{3\beta}{2}$$

$$c = \frac{3L}{2} + \frac{3\beta}{2}$$

$$d = \frac{\alpha}{H} - 3H$$

and

$$\sigma^2 = \frac{1}{2}[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2]$$

4. Conclusions

The spatial volume (R^3) increases with time. Hence inflationary scenario exists in Bianchi Type I space-time with cosmological constant. The Higg's field (ϕ) decreases with time and ϕ vanishes when $t \rightarrow \infty$. Since shear $\sigma \neq 0$, when $t \rightarrow \infty$, hence anisotrophy is maintained throughout. However, when a = 0, d = 0 then the model isotropizes.The cosmological constant (Λ) decreases with time and tends to a constant quantity when $t \rightarrow 0$ and $t \rightarrow \infty$ i.e. the model reduces to General Relativity of Einstein who introduced cosmological constant (Λ) to find the solution for static universes.

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