Ultra Scientist Vol. 24(1)A, 63-66 (2012).

A Solution of Longitudinal dispersion of miscible fluid flow through porous media by Bender-Schmidt method

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(Acceptance Date 24th November, 2011)

Abstract

The present paper discusses the solution of longitudinal dispersion of miscible fluid flow in porous media by numerical method.

1 Introduction

The present paper discusses the solution of longitudinal dispersion of miscible fluid flow in porous media by numerical method.

The one dimensional miscible fluid flow through porous media has great importance in agriculture, water-salubility problems, chemical engineering problems, biomathematics and in oil recovery processes.

The phenomenon of longitudinal dispersion which is the process by which miscible fluids in laminar flow mix in the direction of the flow. Here we have considered the cross sectional flow velocity component u along x-axis as directly propositional to the concentration of fluid at distance x for any time t.

The mathematical formulation yields to non linear partial differential equation in Burger's equation[1] and it has been reduced to heat equation by using Hopf-Cole transformation[4,6]. We have assumed velocity component in x-direction of miscible fluid flow at any time is directly proportional to the concentration of the fluid at that time[7]. Many researchers have discussed this phenomenon from different aspects:- for example Bear[2], Greekarn[4], Patel[7].

Finally, it is solution of Burger's equation (6) which represent concentration of one dimensional dispersion of miscible fluid flow through porous media at any longitudinal distance x for any time $t \geq 0$.

2. *Mathematical formulation of the problem:*

According to Darcy's law, the equation of continuity for the mixture, in the case of compressible fluids are given by

$$\frac{\partial \rho}{\partial t^*} + \nabla.\left(\rho \bar{v}\right) = 0 \tag{1}$$

Where ρ is the density for mixture and $\bar{v}$ is the pore seepage velocity.

The equation of diffusion for a fluid flow through a homogeneous porous medium, without increasing or decreasing the dispersing material is given by

$$\frac{\partial C}{\partial t^*} + \nabla.\left(C\bar{v}\right) = \nabla.\left[\rho \bar{D} \nabla\left(\frac{C}{\rho}\right)\right] \tag{2}$$

Where C is the concentration of the fluid A into the other host fluid B, D is the tensar co-efficient of dispersion with nine component D_{ij}.

In a laminar flow through homogeneous porous medium at a constant temperature ρ is constant. Then

$$\nabla.\bar{v} = 0 \tag{3}$$

And equation (2) becomes

$$\frac{\partial C}{\partial t^*} + \bar{v}.\nabla C = \nabla.\left(\bar{D}\nabla C\right) \tag{4}$$

When the seepage velocity is along the x-axis, the non-zero components are $D_{11}=D_L$ (co efficient of longitudinal dispersion) and $D_{22}=D_T$ (co efficient of transverse dispersion) and other D_{ij} are zero.

In this case the equation (4) becomes

$$\frac{\partial C}{\partial t^*} + u\frac{\partial C}{\partial x^*} = D_L \frac{\partial^2 C}{\partial x^{*2}} \tag{5}$$

Where u is the component of velocity along x –axis which is time dependent as well as concentration along x-axis in $x \geq 0$ direction & $D_L > 0$ and it is the cross sectional flow velocity in porous medium[8,9].

$$u = \frac{C\left(x^*, t^*\right)}{C_0}, \text{ for } x^* > 0,$$

The boundary conditions in longitudinal direction are:

$$C\ (0, t^*) = C_0, \quad (\ > 0) \tag{6}$$

$$C\ (x^*, 0) = C_1, \quad \text{for } \ > 0. \tag{7}$$

Where C_1 is the initial concentration of the tracer (one fluid) & C_0 is the concentration of the tracer (of the same fluid) at $x^* = 0$.

Hence (5) becomes,

$$\frac{\partial C}{\partial t^*} + \frac{C}{C_0}\frac{\partial C}{\partial x^*} = D_L \frac{\partial^2 C}{\partial x^{*2}} \tag{8}$$

consider the variables

$$x = C_0 \frac{x^*}{L}, \quad t = \frac{t^*}{L}, \quad 0 \leq x^* \leq \frac{L}{B}, \quad t^* \geq 0$$

The equation (8) becomes

$$\frac{\partial C}{\partial t} + C\frac{\partial C}{\partial x} = \varepsilon \frac{\partial^2 C}{\partial x^2}, \text{ where } \varepsilon = \frac{C_0^{\ 2} D_L}{L} \tag{9}$$

$$C\left(0, t\right) = C_1, \quad (t > 0)$$

$$C\left(x, 0\right) = C_0 \ \textit{for } x > 0,$$

Choose the transformation[3,5],

$$C=\psi_x, \psi=-2\varepsilon \log \xi \tag{10}$$

Which reduces (9) to the diffusion type heat equation

$$C_t^{\bullet}=\varepsilon C_{xx}^{\bullet} \tag{11}$$

3 *The solution of the problem :*

Fundamentals of the technique :

Bender-Schmidt method has been used to find the solution of non linear diffusive Burger's equation. It uses the finite difference representation of diffusion type heat equation and solves the set of simultaneous algebraic equations to get the values of unknown temperatures at the grid points.

In this method the central difference approximation is used for second order space derivative at central point of two steps of space. Two point forward difference approximations is used at instant of time t_j

The second order space derivative at position x_i time t_j can be obtained by three point central difference formula

$$\left.\frac{\partial^2 C^*}{\partial x^2}\right|_{x_i, t_j}=\frac{C_{i+\sigma j}^* - 2C_{ij}^* + C_{i-1j}^*}{h_I^2} \tag{12}$$

The first order time derivative at position x_i time t_j can be obtained by two point forward difference formula

$$\left.\frac{\partial C^*}{\partial t}\right|_{x_i, t_j}=\frac{C_{ij+1}^* - C_{i,j}^*}{h_I} \tag{13}$$

Using the above two equation in diffusion equation its finite difference form is obtained as

$$\frac{DC_{i+1,j}^* + C_{i-1,j}^* - 2C_{i,j}^*}{h_x^2}=\frac{C_{i,j+1}^* - C_{i,j}^*}{h_t} \tag{14}$$

$$C_{i,j+1}^*=\frac{Dh_t}{h_x^2}\left(C_{i+1,j}^* + C_{i-1,j}^* - 2C_{i,j}^{\ *}\right)+C_{i,j}^* \tag{15}$$

Or $C_{i,j+1}^*=C_{i,j}^*\left(1-\frac{2Dh_t}{h_x^2}\right)+\frac{Dh_t}{h_x^2}\left(C_{i+1,j}^*+C_{i-1,j}^*\right)$

$$= C_{i,j}^*(1-2r)+r\left(C_{i+1,j}^* + C_{i-1,j}^*\right) \quad where \quad r=\frac{Dh_t}{h_x^2} \tag{16}$$

If we choose step size h_x and h_t such that 1-2r = 0 or r = ½, then the finite difference equation representing heat conduction equation will become

$$C_{i,j+1}^*=\frac{1}{2}\left(C_{i+1,j}^* + C_{i-1,j}^*\right)$$

Which represents the concentration of dispersion phenomenon of one dimensional miscible fluid flow through porous media for any distance x for any t > 0.

4. Concluding remark

Here the solution of the longitudinal dispersion phenomenon has been obtained by the Bender-Schmidt technique.

It may be concluded that the concentration in this longitudinal phenomenon will increase at different level for fix time t. *i.e.* concentration will be increased when will take different approximation at different level.

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