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Poiseuille Flow of Conducting Jeffrey Fluid Between Parallel Plates When one of the Walls is Provided with Porous Lining

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Abstract

MHD flow of a Jeffrey fluid in a parallel plate channel is investigated when the walls are provided with non-erodible porous lining. The flow in the free flow region is governed by Jeffrey model and the flow in the porous region is described by Darcy law. The influence of the thickness of porous lining on the velocity field in the channel has been studied. The results are depicted graphically and discussed for various relevant parameters. It is observed that the velocity increases with the increase in Jeffrey parameter whereas the opposite behavior is noticed due to the increase in magnetic parameter M. It is also found that the skin friction is higher for non-Newtonian Jeffrey fluid when compared with Newtonian fluid.

Key words: Parallel Plate channel, Jeffrey fluid, porous lining, MHD.

I. Introduction

Interest in the flows of non-Newtonian fluids has increased due to the applications in science and engineering including thermal oil recovery, food and slurry transportation, polymer and food processing etc. A variety of non-Newtonian fluid models have been proposed in the literature keeping in view of their several rheological features. There is one subclass of non-Newtonian fluids known as Jeffrey fluids which have been attracted much by the researchers in view of their simplicity. This fluid model is capable of describing the characteristics of relaxation and retardation times. It has been accepted that majority of the physiological fluids behave like a non-Newtonian fluids. Hayat *et al.*[1] have analyzed the influence of an endoscope on the peristaltic flow of a Jeffrey fluid under the effective of magnetic field in a tube. Peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube was discussed by Hayat and Ali[2].

MHD flow between parallel plates is a classical problem that occurs in MHD power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Especially the flow of non-Newtonian fluids in channels is encountered in various engineering applications. For example, injection molding of plastic parts involves the flow of polymers inside channels. During the last few years the industrial importance of non-Newtonian fluids is widely known. Such fluids in the presence of a magnetic field have applications in the electromagnetic propulsion, the flow of nuclear fuel slurries and the flows of liquid state metals and alloys. The effect of magnetic field on viscous fluid has also been reported for treatment of the following pathologies: Gastroenric pathologies, rheumatisms, constipation and hypertension that can be treated by placing one electrode either on the back or on the stomach and the other on the sole of the foot and this location will induce a better blood circulation. Sarparkaya[3] has presented the first study for MHD Bingham plastic and power law fluids. Effect of magnetic field on pulsatile flow of blood in a porous channel was investigated by Bhuyan and Hazarika[4]. Misra *et al*.[5] have investigated a mathematical modeling of blood flow in porous vessel having double stenosis in the presence of an external magnetic field.

Ahmed and Sajid[6] investigate the combined effects of magnetic field and slip boundary conditions on the thin film flow of a Jeffrey fluid on a vertically moving belt. The flow of non-Newtonian fluids through a porous medium under different conditions was studied by Chamkha *et al*.[7]. Krishna Gopal Singha[8] investigated analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field. Sreenadh *et al*.[9] investigated the MHD free convective flow of a Jeffrey fluid between coaxial cylinders. Recently, Santhosh and Radhakrishnamacharya[10] studied a two fluid model for the flow of Jeffrey fluid in the presence of a magnetic field through a porous medium in tubes of small diameters. Nallapu *et al*.[11] are studied the effects of porous medium on a two-fluid model for the flow of Jeffrey fluid in tubes of small diameters. It is assumed that the core region consists of Jeffrey fluid and Newtonian fluid in the peripheral region. Kumaraswamy Naidu *et al*.[12] discussed the effect of the thickness of the porous material on the parallel plate channel flow of Jeffrey fluid when the walls are provided with non-erodible porous lining.

The object of this paper is to develop a theoretical model for analyzing the conducting Jeffrey fluid flow in a parallel plate channel, when one of the parallel walls is provided with non-erodible porous lining. The influence of the non dimensional parameters representing the thickness and the permeability of the porous medium on the velocity field in the channel has been studied. The results are depicted graphically and discussed for various relevant parameters.

II. Mathematical Formulation :

We consider the rectilinear flow of a conducting Jeffrey fluid through a channel formed by two rigid impermeable parallel walls at $y=0$ and $y=h$ as shown in Fig. 1. The lower wall is lined with a homogenous and isotropic porous material of thickness $h'(\neq 0)$ and thus dividing the flow region into two zones. Zone 1 denotes the region of the free flow between the upper impermeable wall and the nominal surface $y=h'$ and Zone 2 represents the region of flow through the porous material. A uniform transverse magnitic field of strength B_o is applied perpendicular to the plates. The Zone 1 is described by Jeffrey model whereas the flow in Zone 2 is governed by Darcy law.

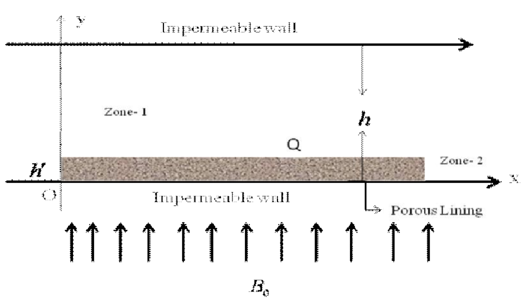

Fig.1. Physical model

The flow which is caused by a uniform pressure gradient in the longitudinal direction in both the zones is assumed to be fully developed and the fluid properties are all assumed to be constant. Then the flow in Zone 1 is governed by the equation

$$\frac{\mu}{1+\lambda_1}\frac{d^2u}{dy^2}-\sigma B_0^2 u=\frac{dp}{dx} \tag{1}$$

and that in Zone 2 by the Darcy law

$$Q=\frac{-k(1+\lambda_1)}{\mu}\left[\frac{dp}{dx}+\sigma B_0^2 Q\right] \tag{2}$$

where u is the velocity, λ_1 is the Jeffrey parameter, P is the pressure, μ is the viscosity, Q is the Darcy velocity, k is absolute permeability of the material, B_0 is the magnetic field.

The boundary conditions are

$$u=0 \text{ at } y=h \tag{3}$$

and the BJ boundary condition (Beavers and Joseph[13]) is

$$\frac{du}{dy}=\frac{\alpha}{\sqrt{k}}(u_B-Q) \text{ at } y=h' \tag{4}$$

We introduce the following non-dimensional quantities:

$$v=\frac{u}{\bar{u}},\, v_B=\frac{u_B}{\bar{u}},\ \eta=\frac{y}{h},\ \xi=\frac{x}{h}\ \pi=\frac{p}{\frac{1}{2}\rho\bar{u}^{2}},$$

$$R=\frac{\rho h\bar{u}}{\mu},\ P=\frac{-R}{2}\frac{\partial\pi}{\partial\xi},\ \sigma=\frac{h}{\sqrt{k}},\ Q'=\frac{Q}{\bar{u}},$$

$$\varepsilon=\frac{h'}{h},\ M^2=\frac{\sigma B_0^2h^2}{\mu} \tag{5}$$

where ρ is the density, R is the Reynolds number, $\bar{u}$ is the average velocity in the channel, ε is the thickness of the porous material, M is the magnetic parameter.

In view of (5), equations (1)-(4) reduce to

$$\frac{d^2v}{d\eta^2}-(1+\lambda_1)M^2v=-(1+\lambda_1)P \tag{6}$$

$$Q^{'} = \frac{P(1+\lambda_1)}{\sigma^2 + (1+\lambda_1)M^2} \tag{7}$$

$$v = 0 \text{ at } \eta = 1 \tag{8}$$

$$\frac{dv}{d\eta} = \alpha\sigma(v_B - Q^{'}) \text{ at } \eta = \varepsilon \tag{9}$$

III. Solution of the problem :

Solution of (6) satisfying (8) and (9) is

$$v = A_{12}e^{a\eta} + A_{11}e^{-a\eta} + \frac{(1+\lambda_1)}{a^2}P \tag{10}$$

and

$$v_B = \frac{A_3(B_1-1)e^{-a\varepsilon} + A_1 - B_2}{1+\left(\frac{\alpha\sigma(B_1-1)e^{-a\varepsilon}}{a}\right)} \tag{11}$$

where

$$a^2 = (1+\lambda_1)P, A_1 = \frac{(1+\lambda_1)}{a^2}P,\ A_2 = \frac{\alpha\sigma v_B}{a},$$

$$A_3 = \frac{\alpha\sigma P(1+\lambda_1)}{a[\sigma^2 + M^2(1+\lambda_1)]}, A_4 = (A_2 - A_3)e^{-a\varepsilon},$$

$$A_5 = e^{-2a\varepsilon},\ A_6 = e^a,\ A_7 = e^{2a},\ A_8 = A_1A_6,$$

$$A_9 = A_4A_7, A_{10} = A_5A_7,\ A_{11} = \frac{A_8 + A_9}{1+A_{10}},$$

$$A_{12} = A_4 + A_5A_{11},\ A_{13} = \frac{A_8}{1+A_{10}},\ A_{14} = A_5 - 1,$$

$$A_{15} = A_{13}A_{14},\ A_{16} = \frac{A_7}{1+A_{10}},\ A_{17} = 1 - A_{16}A_{14},$$

$$A_{18} = \frac{A_{15}}{A_{17}},\ A_{19} = A_{18}e^{a\varepsilon},\ A_{20} = A_{19} + A_3,$$

$$B_1 = \frac{A_7(A_5 + e^{-a\varepsilon})}{1+A_{10}},\ B_2 = \frac{A_1A_6}{1+A_{10}}(A_5 + e^{-a\varepsilon})$$

IV. Mass Flow Rate :

To find the quantitative effect of slip on the flow, we calculate the non-dimensional flow rate

$$m = m_1 + m_2 \tag{12}$$

where

$$m_1 = \int_{\varepsilon}^{1} v\,d\eta = E_4 - E_1e^{a\varepsilon} + E_2e^{-a\varepsilon} - A_1\varepsilon \tag{13}$$

and $$m_2 = Q^{'}\varepsilon = \frac{P(1+\lambda_1)\varepsilon}{\sigma^2 + (1+\lambda_1)M^2} \tag{14}$$

where

$$E_1 = \frac{A_{12}}{a},\ E_2 = \frac{A_{11}}{a},\ E_3 = e^{-a},$$
$$E_4 = E_1A_6 - E_2E_3 + A_1$$

V. Skin Friction

We calculate the absolute value of skin friction as

$$|\tau| = \left(\frac{\partial v}{\partial \eta}\right)_{\eta=1} = a(A_{12}e^{a} - A_{11}e^{-a}) \tag{15}$$

Table 1. Skin friction $|\tau|$ at $y = 1$ for different values of Jeffrey parameter λ_1

λ_1	0	0.2	0.4	0.6	0.8
$\|\tau\|$	0.0040	0.0041	0.0042	0.0043	0.0044

Table 2. Skin friction $|\tau|$ at $y=1$ for different values of Magnetic parameter M

M	1	1.5	2	2.5	3
$\lvert\tau\rvert$	0.0041	0.0020	0.0012	0.0008	0.0005

VI. Results and Discussions

The numerical values of velocity are computed from equation (10) and are depicted in Figures 2 to 7 for flow in a channel with one side porous lining. We observe from Fig. 2 that the velocity decreases with the increase in the thickness of porous layer ε. We observe the same phenomenon for velocity from Fig. 3 and 4 with an increase in slip parameter α and permeability parameter σ. From Fig. 5 we observe that the velocity increases with the increase in the pressure gradient P. From Fig. 6 we observe that the velocity increases with the increase in Jeffrey parameter λ_1. From Fig. 7 we observe that the velocity decreases with the increase in Magnetic parameter M. From Figure 8 we observed that the mass flow rate (m) covering one side porous lining decreases with the increase in the permeability parameter σ. From Figures 9 we observe that the mass flow rate (m) covering one side porous lining decreases with the increase in the magnetic parameter M. From Figures 10 we observe that the mass flow rate (m) covering one side porous lining increases with the increase in the magnetic parameter M From Figures 11 we observe that the mass flow rate (m) covering one side porous lining increases with the increase in the Jeffrey parameter λ_1. From Figure 12 we observed that the mass flow rate (m) covering one side porous lining decreases with the increase in the permeability parameter σ.

The numerical values of the magnitude of skin friction at the wall $y=1$ is computed from equation (15) for different values of Jeffrey parameter λ_1 and magnetic parameter M and is presented in Table 1 and Table 2. It is observed that the skin friction increases with increase in Jeffrey parameter λ_1 and decreases with increase in magnetic parameter M. Higher skin friction is observed in the non-Newtonian Jeffrey fluid when compared with a Newtonian fluid.

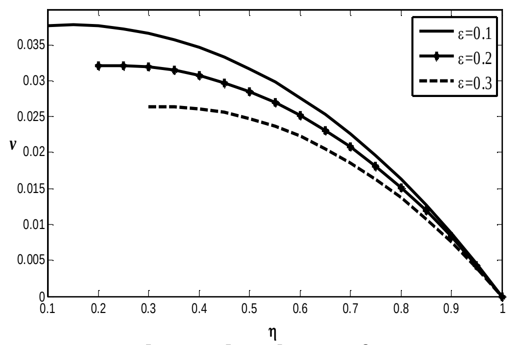

Fig. 2. Velocity distribution for various values of ε for fixed

P=0.1, λ_1 =0.5, α =0.1, σ=2, M=1

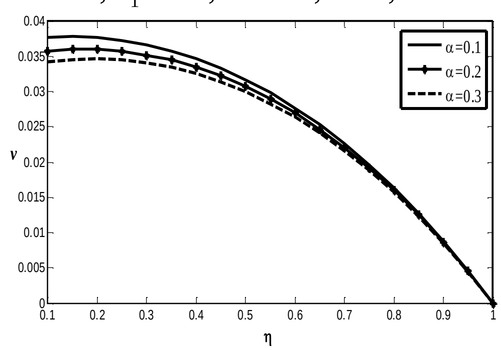

Fig. 3. Velocity distribution for various values of α for fixed

P=0.1, λ_1 =0.5, ε=0.1, σ=2, M=1

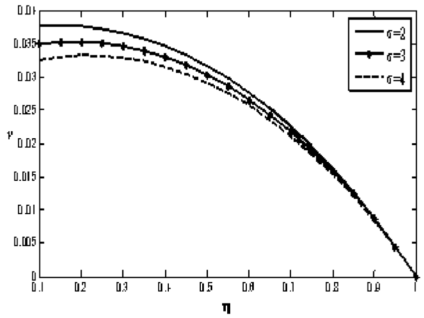

Fig. 4. Velocity distribution for various Values σ for fixed

P=0.1, λ_1=0.5, ε =0.1, α=0.1, M=1

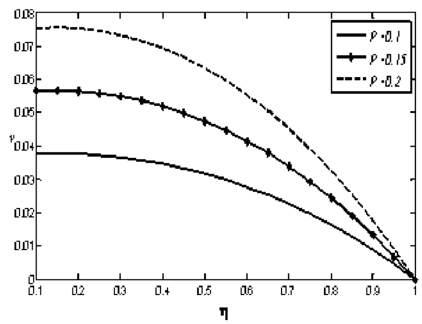

Fig. 5. Velocity distribution for various values of P for fixed

σ=2, λ_1=0.5, ε=0.1, α=0.1, M=1

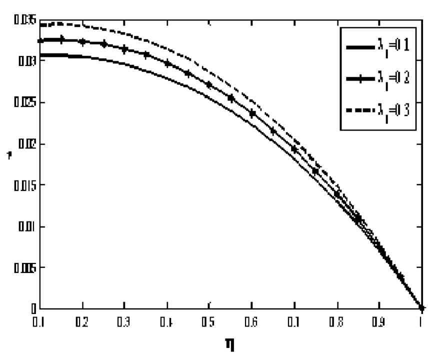

Fig. 6. Velocity distribution for various values of λ_1 for fixed

σ =2, P=0.1, α=0.1, M =1

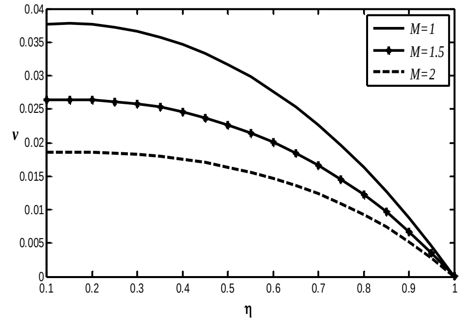

Fig. 7. Velocity distribution for various values of M for fixed

σ=2, P=0.1, λ_1=0.5, α=0.1

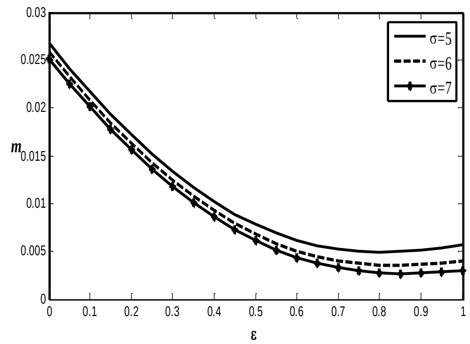

Fig. 8. Variation of m with ε for fixed

λ_1=0.1, P=0.1, α=0.1, M =1

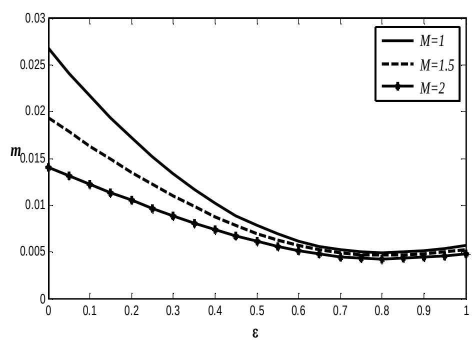

Fig. 9. Variation of m with ε for fixed

σ = 5, λ_1=0.1, P=0.1, α=0.1

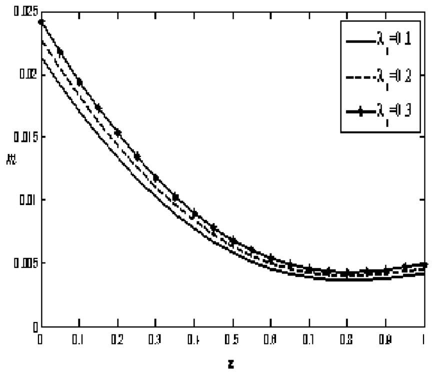

Fig. 10. Variation of with for fixed

$\sigma = 5, P$=0.1, α=0.1, M =1

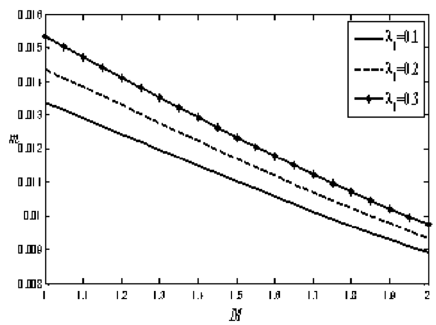

Fig. 11. Variation of m with Magnetic parameter M for fixed

$\sigma = 3, P$=0.1, ε =0.1, α=0.1

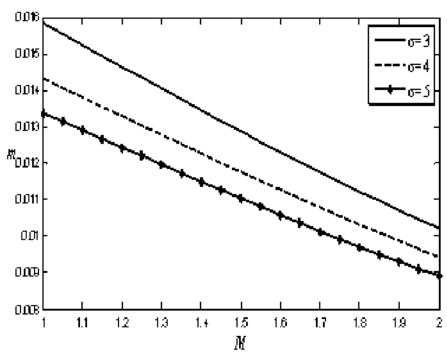

Fig. 12. Variation of m with Magnetic parameter M for fixed

P=0.1, ε =0.1, α=0.1, λ_1 =0.1

VII. Conclusion

The influence of the thickness of porous lining on the velocity field in the channel has been studied. It is observed that the velocity increases with the increase in Jeffrey parameter whereas the opposite behavior is noticed due to the increase in magnetic parameter. It is also found that the skin friction is higher for non-Newtonian Jeffrey fluid when compared with Newtonian fluid.

VIII. Acknowledgement

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