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Fuzzy relational maps between two disjoint units of association

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Abstract

The paper is addressed to the introduction of a new notion of fuzzy relational maps by dividing the casual association of fuzzy cognitive maps of Taber[1] into two disjoint units respectively named as domain and range space, the number of elements in these spaces being not equal. We also determine the methods of determination of hidden pattern and combined fuzzy relational maps.

Keywords : Fuzzy relational maps, relational matrix, hidden pattern.

1. Introduction

Taber, W.R.[1] constructed fuzzy cognitive maps where he promoted the correlations between casual associations among concurrently active units, But in our fuzzy relational maps, we divide the very casual associations into two disjoint units such as relation between a doctor and a patient, a teacher and a student and so on. These two disjoint units will be domain and range space with no intermediate existing relation to each. The number of elements in domain need not be equal to that of range.

Thus a fuzzy relational map is a function $f\ : \mathbf{R}^n \longrightarrow \mathbf{V}^m\ (n \neq m)$ where $\mathbf{R}^n$ is a real vector space of dimension **n** whereas range $\mathbf{V}^m$ is a vector space of dimension **m** whose elements are real vectors.

We denote by **V** the set of all nodes $V_1, V_2, \ldots\ldots V_m$ of the range space $\mathbf{V}^m$, where $\mathbf{V_1 = \{(V_1, V_2, \ldots\ldots\ldots V_m) : V_i = 0 \text{ or } 1,\ i = 1,2,\text{----}m\}}$

If $\mathbf{V_i = 0}$, it means that the node V_1 is on State and $\mathbf{V_i} = \mathbf{1}$ indicates the node in off state. Likewise **R** denotes the nodes $\mathbf{R_1, R_2, \ldots\ldots R_n}$ of the domain space $\mathbf{R}^n$, where $\mathbf{R_1 = \{(r_1, r_2, \ldots\ldots r_n) : r_i = 0 \text{ or } 1, i = 1, 2, \ldots\ldots\ldots n\}}$

If $\mathbf{r_i = 1}$, it means that the node $\mathbf{R_1}$ is in on state and $\mathbf{r_i = 0}$ means it is in off state.

2. *Fuzzy relational maps :*

A fuzzy relational map is a map from

R to **V** with concepts like events as nodes and casualties as edges. It represents causal relations between spaces **R and V.**

The directed edge from $\mathbf{R_1}$ to $\mathbf{V_1}$ denotes the casualty of $\mathbf{R_1}$ on $\mathbf{V_1}$ called relations.

Every edge in the fuzzy relational map is weighed by a number in the set **{0,1}**. Let $\mathbf{W_{11}}$ be the weight of the edge $\mathbf{R_1\ V_1}$, where $\mathbf{W_{11}}$ **€ {0,1}.** $\mathbf{W_{11}>0}$ if increase in R_1 implies increase in $\mathbf{V_1}$ or decrease in $\mathbf{R_1}$ implies decrease in $\mathbf{V_1}$ **i.e**. casualty of $\mathbf{R_1}$ on $\mathbf{V_1}$ is 1.

If $\mathbf{W_{11}= 0}$ then **R1** has no effect on $\mathbf{V_1}$. We avoid the case when one increase and other decreases.

Definition (2, 1): When the nodes of the fuzzy relational mappings are fuzzy sets, they are called fuzzy nodes.

Definition (2.2) : Fuzzy relational maps with edge weights **{0,1}** are called **Simple Fuzzy Relational maps.**

3. *Relational matrix :*

Let $\mathbf{R_1, R_2, \ldots\ldots R_n}$ be the nodes of the domain space **R** of a fuzzy relational map and $\mathbf{V_1, V_2, \ldots\ldots V_m}$ be the nodes of the range space **V**. Let the matrix **M** be defined as $\mathbf{M=[W_{ij}]}$ where $\mathbf{W_{ij}}$ is the weight of the directed edge $\mathbf{R_iV_j}$ **or** $\mathbf{V_iR_j}$, then **M** is called the relational matrix of fuzzy relational maps.

Let $\mathbf{R_1, R_2, \ldots\ldots R_n}$ and $\mathbf{V_1, V_2, \ldots\ldots V_m}$ denote the nodes of fuzzy relational maps.

Let $\mathbf{A=(a_1, a_2, \ldots. a_n)}$**, ai € {0,1}.** Then **A** is called the instantaneous state vector of the domain space which denotes the **on-off** position of the nodes at any instant.

Similarly let $\mathbf{B=\{ b_1, b_2, \ldots b_m\}}$**,** $\mathbf{b_i}$ **€{ 0,1},** then **B** is called the instantaneous state Vector of the range space which denotes the **on-off** position of the nodes at any instant.

$\therefore$ $\mathbf{a_i = 0}$ **if ai is off**

= **1 if it is on (i = 1,2,………n)**

$\mathbf{b_i = 0}$ **if bi is off**

= **1 if it is on (i = 1,2,……m)**

Definition (3.1) : Let $\mathbf{R_iV_j}$ **(or** $\mathbf{V_iR_j}$**)** be the edges of the maps. The edges form a directed cycle. **A** fuzzy relational map is said to be cycle if it possesses a directed cycle, otherwise acycle. A fuzzy relational map with cycles is said to have a feed back[4,5].

Definition (3.2) : When there is any feed back in fuzzy relational maps, it is called a dynamic system.

Definition (3.3) : In $\mathbf{R_iV_j}$**,** if $\mathbf{R_i}$ is switched on and casualty flows through edges of cycle and it again causes $\mathbf{R_i}$ **or** $\mathbf{V_j}$, we say that the dynamical system goes round and round. This is true for any node $\mathbf{R_i}$ or $\mathbf{V_j}$ **(i=1, 2,………n, j= 1,2, ……..m)**. The equilibrium state of this dynamical system is called the hidden pattern.

Definition (3.4) : If the equilibrium[4,5] state of a dynamical system is a unique state vector, it is called a fixed point. Consider a fuzzy relational map with $\mathbf{V_1, V_2, \ldots. V_m}$ and $\mathbf{R_1, R_2, \ldots\ldots R_n}$ as nodes. Let us start the dynamical system by switching $\mathbf{V_i}$ **or** $\mathbf{R_j}$. Let us assume that the maps settles down with $\mathbf{V_1}$ and $\mathbf{V_m}$ (or $\mathbf{R_1}$ and $\mathbf{R_n}$) *i.e.* the state vector remains at **(1,0,0…0)** in **V**, then this state of vector is called fixed point[4-6].

Definition (3.5) : If the fuzzy relational maps settle down with a state vector repeating the form

$$\mathbf{A_1 \longrightarrow A_2 \longrightarrow A_3 \longrightarrow \ldots \longrightarrow A_1 \longrightarrow A_1}$$

or $$\mathbf{B_1 \longrightarrow B_2 \longrightarrow B_3 \longrightarrow \ldots \longrightarrow B_1 \longrightarrow B_1},$$

then the equilibrium is called limit cycle.

4. *Determination of hidden pattern :*

Let $\mathbf{V_1, V_2, \ldots V_m}$ and $\mathbf{R_1, R_2, \ldots R_n}$ be the fuzzy relational maps with feed back. Let **M** be the relational matrix[6].

Let us find a hidden pattern. When R_1 is switched on given an input vector **(1,0,..0)** in **R** the data should pass through the matrix **M**. Let $\mathbf{A_j M = \{ s_1, s_2, \ldots s_m \}}$ after updating the resultant vector, we get $\mathbf{A_1 M} €\mathbf{V}$. Now let $\mathbf{B = A_1 M}$, we pass on **B** into M^T and obtain $\mathbf{BM^T}$. We update the vector $\mathbf{BM^T}$ **€ R.**

This process is repeated till we get a limit cycle.

Definition (4.1) : A finite number of fuzzy relational maps can be combined together to produce the joint effect of all the fuzzy relational maps.

Let $\mathbf{M_1, M_2, M_3, .. M_p}$ be the relational matrices of the fuzzy relational mappings with nodes $\mathbf{V_1, V_2, \ldots V_m}$ **and** $\mathbf{R_1, R_2, \ldots R_n}$, then the combined fuzzy relational maps is represented by the relational matrix

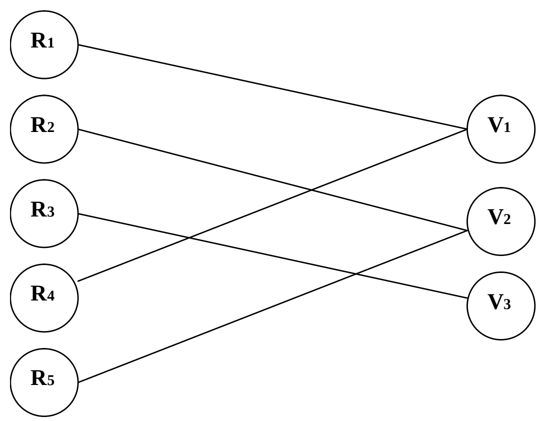
$$\mathbf{M = M_1 + M_2 + \ldots + M_p}.$$

Example (4.2) : Let us consider the relationship between a doctor and a patient. Let the domain Space belong to doctors be $\mathbf{R_1, R_2, R_3, R_4, R_5}$ and the range space belong to their Patients be $\mathbf{V_1, V_2, V_3}$

We describe the nodes as follows :-

Domain Space	**Range Space**
R_1—> **Treatment is nice**	V_1—>**Healthy Patient**
R_2—> **Treatment is poor**	V_2—> **Thin Patient**
R_3—> **Treatment is average**	V_3—>**Average Patient.**
R_4—> **Treatment is kind**	
R_5—> **Treatment is rude**	

The relational graph of the doctor patient model is as follows :-



The relational matrix **M** of the map is

$$\mathbf{M} = \quad \mathbf{0} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

If **A= (10000**) is passed on in the relational matrix **M** the vector **AM=(100)** means that the patient is a healthy patient.

Now let **AM=B, BM** $^{\mathbf{T}}$ **= (10010)** which means that the treatment is good and the doctor is kind.

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