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On binary continuity and binary separation axioms

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Abstract

Recently the authors introduced the concept of binary topology between two sets and investigate its basic properties where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. In this paper we introduce and study binary-T_0, binary-T_1 and binary-T_2 axioms that are analogous to the separation axioms of topology. Binary continuity is also characterized in this paper.

Key words: Binary topology, binary open, binary closed, binary closure, binary-T_0, binary-T_1, binary-T_2 and binary continuity.

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1. Introduction

The concept of binary topology from X to Y is introduced by the authors[2]. The concepts of binary closed, binary closure, binary interior and binary continuity are also introduced[2]. Further the concepts of base and sub base of a binary topological space are introduced and investigated in[3]. The purpose of this paper is to introduce separation axioms in binary topological spaces and characterize their basic properties. Section 2 deals with basic concepts. Binary continuity is discussed in section 3. Section 4 is dealt with binary-T_0, binary-T_1 and binary-T_2 spaces. Throughout the paper, $\wp(X)$ denotes the power set of X.

2. *Preliminaries :*

Let X and Y be any two nonempty sets. The authors defined in[2] that a binary topology from X to Y is a binary structure $\mathcal{M} \subseteq \wp(X)\times \wp(Y)$ that satisfies the following axioms.

(i) $(\varnothing, \varnothing)$ and $(X, Y) \in \mathcal{M}$,

(ii) $(A_1\cap A_2, B_1\cap B_2)\in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$,

(iii) If $\{(A_\alpha,B_\alpha):\alpha\in\Delta\}$ is a family of members of $\mathcal{M}$, then

$$\left(\bigcup_{\alpha\in\Delta} A_\alpha , \bigcup_{\alpha\in\Delta} B_\alpha\right) \in \mathcal{M}.$$

If $\mathcal{M}$ is a binary topology from X to Y then the triplet (X, Y, $\mathcal{M}$) is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space (X, Y, $\mathcal{M}$). The elements of $X \times Y$ are called the binary points of the binary topological space (X, Y, $\mathcal{M}$). If Y=X then $\mathcal{M}$ is called a binary topology on X in which case we write (X, $\mathcal{M}$) as a binary space. The examples of binary topological spaces are given in[2].

Suppose (X, ρ) and (Y, σ) are two topological spaces. Let $\rho\times\sigma=\{(A, B): A\in\rho, B\in\sigma\}$, $\tau(\rho\times\sigma) =\{A\subseteq X : (A, B)\in\rho\times\sigma$, for some $B\subseteq Y\}$ and $\tau'(\rho\times\sigma) =\{B\subseteq Y: (A, B)\in\times\sigma)$ for some $A\subseteq X\}$. The following lemma is very useful in sequel.

Lemma 2.1. Suppose (X, ρ) and (Y, σ) are two topological spaces. Then $\rho\times\sigma$ is a binary topology from X to Y such that[2] $\tau(\rho\times\sigma) = \rho$, $\tau'(\rho\times\sigma) = \sigma$.

3. *Binary continuity :*

The concept of binary continuity between a topological space and a binary topological space is introduced and studied in[2]. Binary continuity is also investigated in[3]. Binary continuity is further characterized here.

Definition 3.1. Let $f:Z\to X\times Y$ be a function. Let $A\subseteq X$ and $B\subseteq Y$. We define $f^{-1}(A, B) = \{z\in Z: f(z)(A, B)\}$.[2]

Definition 3.2. Let (X,Y, $\mathcal{M}$) be a binary topological space and let (Z,τ) be a topological space. Let $f: Z\to X\times Y$ be a function. Then f is called binary continuous[2] if $f^{-1}(A, B)$ is open in Z for every binary open set (A, B) in $X\times Y$.

The following proposition will be useful in characterizing binary continuity.

Proposition 3.3. Let $f:Z\to X\times Y$ be a function. Let $A\subseteq X$ and BY. Suppose $p_1:X\times Y\to X$, $p_2:X\times Y\to Y$ are projections on X and Y respectively. Then $f^{-1}(A, B) = (p_1\circ f)^{-1}(A)\cap (p_2\circ f)^{-1}(B)$.

Proof. $f^{-1}(A, B)=\{z\in Z: f(z)\in(A, B)\}$
$=\{z \in Z: (p_1\circ f)(z)\in A \text{ and } (p_2\circ f)(z)\in B\}$
$=\{z\in Z:(p_1\circ f)(z)\in A\}\cap\{z\in Z:(p_2\circ f)(z)\in B\}=(p_1\circ f)^{-1}(A)\cap(p_2\circ f)^{-1}(B)$. □

Definition 3.4. A relation R from X to Y is called a partial function from X to Y if for all $x\in X$, $y, y' \in Y$, $(x,y) \in R$ and $(x, y') \in R \Rightarrow y = y'$.

It is easy to see that f is a partial function from X to Y if and only if there is a subset A of X such that $f: A\to Y$ is a function from A to Y. Every function $f: A\to Y$ induces a partial function from X to Y. The notation $f: X \xrightarrow{*} Y$ denotes a partial function from X to Y.

Suppose (X,τ) and (Y,σ) are topological spaces. Then $\{A\times B:A\in\tau, B\in\sigma\}$ is a base for the product topology ρ on $X \times Y$. We use the notation τ^+ = the set of all non empty open sets in τ. Then $\tau^+\times\sigma^+$ is a proper subset of $\tau\times\sigma$.

A partial function $f: \tau\times\sigma \xrightarrow{*} \rho$ is defined by $f(A, B) = A\times B$ for all elements $(A, B) \in\tau^+\sigma^+$. Now $A\times B = \varnothing$, if atleast one of A, B is $\varnothing$. We can identify the sets $(A, \varnothing)$, $(\varnothing, B)$ and $(\varnothing,\varnothing)$ in $\tau\times\sigma \setminus \tau^+\times\sigma^+$ by a single class of elements in $\tau\times\sigma$. Since $\tau^+\times\sigma^+$ can be thought of a proper sub collection of ρ, we take $\tau\times\sigma$ itself as a proper subcollection of ρ

Proposition 3.5. Every continuous function with respect to the product topology is binary continuous. But the converse is not true.

Proof. Let $f:Z\to X\times Y$ be a continuous function with respect to the product topology. Let (A, B) be a binary open set in $X\times Y$. Therefore (A, B) can be identified with some open set $G = A\times B$ in the product space $X\times Y$. Since f is continuous, $f^{-1}(G)$ is open in Z that implies $f^{-1}(A, B)$ is open in Z. This implies f is binary continuous. The converse of the Proposition 3.5 is not true as shown below.

Let $X=\{a, b\}$, $Y=\{1, 2\}$ and $Z=\{x, y\}$. $\eta=\{\varnothing, Z, \{x\}\}$ is a topology on Z, $\tau=\{\varnothing, X, \{a\}\}$ is a topology on X and $\sigma=\{\varnothing, Y, \{2\}\}$ is a topology on Y. $X\times Y =\{(a, 1), (a, 2), (b, 1), (b, 2)\}$. Define $f: Z\to X\times Y$ by $f(x) =(a, 1)$ and $f(y)= (b, 2)$. The product topology ρ on $X\times Y$ is given by $\rho=\{\varnothing, X\times Y, \{(a, 2), (b, 2)\}, \{(a, 2)\}, \{(a, 1),(a, 2)\}\}$. By using Lemma 1.1, $\tau\times\sigma=\{(\varnothing,\varnothing), (\varnothing,\{1\}), (\varnothing,Y), (X,\varnothing), (X,Y), (X,\{2\}), (\{a\},\varnothing), (\{a\},\{2\}), (\{a\},Y)\}$ is a binary topology from X to Y. Clearly f is binary continuous but f is not continuous with respect to its product topology as $f^{-1}\{(a, 2), (b, 2)\} =\{y\}$ which is not open in Z. □

Proposition 3.6. Let (X_1, τ_1), (X_2, τ_2) and (Z, τ) be topological spaces. Suppose $f:Z\to X_1\times X_2$ is a function and $p_1:X_1\times X_2X_1$, $p_2:X_1\times X_2\to X_2$ are projections on X_1 and X_2 respectively. If $p_1\circ f:(Z, \tau)\to(X_1, \tau_1)$ and $p_2\circ f:(Z, \tau)\to(X_2, \tau_2)$ are continuous functions, then $f :Z\to X_1 \times X_2$ is a binary continuous function from (Z, τ) to $(X_1, X_2, \tau_1 \times\tau_2)$

Proof. Suppose $p_1\circ f:(Z, \tau)\to(X_1, \tau_1)$ and $p_2\circ f:(Z, \tau)\to(X_2, \tau_2)$ are continuous. Let $(A,B)\in\tau_1\times\tau_2$. Therefore, $A\in\tau_1$, $B\in\tau_2$. Since $p_1\circ f : (Z, \tau)\to(X_1, \tau_1)$ and $p_2\circ f : (Z, \tau)\to(X_2, \tau_2)$ are continuous, we have $(p_1 \circ f)^{-1}(A)$ is open in Z and $(p_2 \circ f)^{-1}(B)$ is open in Z. Then by using Proposition 3.3, $f^{-1}(A, B)= (p_1\circ f)^{-1}(A)\cap (p_2\circ f)^{-1}(B)$ which is open in Z. This proves that f is binary continuous. □

Binary-T_0 and binary-T_1 :

In this section the concepts of binary-T_0 and binary-T_1 separation axioms are introduced and their basic properties are discussed.

Definition 4.1. A binary topological space $(X,Y, \mathcal{M})$ is called a binary -T_0 if for any two binary points $(x_1, y_1), (x_2, y_2)\in X\times Y$ with $x_1 \neq x_2$, $y_1\neq y_2$, there exists $(A, B)\in \mathcal{M}$ such that exactly one of the following holds.
(i) $(x_1, y_1)\in(A, B)$, $(x_2, y_2)\in(X\setminus A, Y\setminus B)$ and
(ii) $(x_1, y_1)\in(X\setminus A,Y\setminus B)$, $(x_2, y_2)\in(A,B)$.

Definition 4.2. A binary topological space $(X,Y, \mathcal{M})$ is called a binary-T_1 if for every $(x_1, y_1), (x_2, y_2)\in X\times Y$ with $x_1\neq x_2$, $y_1\neq y_2$, there exist $(A, B), (C, D)\in \mathcal{M}$, with $(x_1, y_1)\in(A, B)$ and $(x_2, y_2)\in(C, D)$ such that $(x_2, y_2)\in(X\setminus A,Y\setminus B)$ and $(x_1, y_1)\in(X\setminus C, Y\setminus D)$.

Definition 4.3. The binary points (x_1, y_1), $(x_2, y_2)\in X\times Y$ are distinct if $x_1 \neq x_2$, $y_1\neq y_2$.

Example 4.4. Consider $X =\{a, b, c\}$, $Y = \{1, 2\}$. Clearly $\mathcal{M} =\{(\varnothing, \varnothing), (\{a\},\{1\}), (\{b\},\{2\}), (\{a, b\},\{1, 2\}), (X,Y)\}$ is a binary topology from X to Y. Since there exists no binary open set (A, B) in $\mathcal{M}$ such that (a, 2) $\in$ (A, B) and (b, 1) $\in$(X \A, Y\B), $\mathcal{M}$ is not binary-T_0.

Example 4.5. The discrete binary topology from X to Y is binary-T_0.

Example 4.6. The indiscrete binary topology from X to Y is not binary-T_0.

Example 4.7. Consider X ={ a, b},Y = {1,2}. $\mathcal{M}$ ={($\emptyset$,$\emptyset$), ({a},{1}), ({b},{1}), (X,{1}), ($\emptyset$,{1}), (X,Y)} is a binary topology from X to Y. As the only distinct pair of binary points of $X\times Y$ are (a,1), (b,2) and (a ,2), (b,1), the space is binary-T_0.

Example 4.8. X={a,b}, Y={1, 2}. $\mathcal{M}$ ={($\emptyset$, $\emptyset$), ({a},{1}), (X,Y)} is a binary topology from X to Y. Then (a, 1), (b, 2)$\in X\times Y$. Clearly (X,Y) is the only binary open set with (a,1)$\in$(X,Y) and (b, 2) $\in$(X,Y).This implies that $\mathcal{M}$ is not binary-T_1.

Example 4.9. The indiscrete binary topology from X to Y is not binary-T_1.

Example 4.10. The discrete binary topology from X to Y is binary-T_1.

Proposition 4.11. If the binary topological space $(X,Y, \rho\times\sigma)$ is binary-T_0, then (X, ρ) and (Y, σ) are T_0.

Proof. We assume that $(X,Y, \rho \times \sigma)$ is binary-T_0. Let $x_1, x_2\in X$ and $y_1,y_2\in Y$ with $x_1 \neq x_2$, $y_1\neq y_2$. Since $(X,Y, \rho \times \sigma)$ is binary-T_0, there exists $(A, B)\in\rho \times \sigma$ such that either $(x_1, y_1)\in(A, B)$; $(x_2, y_2)\in(X\backslash A,Y\backslash B)$ or $(x_1, y_1) \in (X\backslash A,Y\backslash B)$; $(x_2, y_2)\in(A,B)$. This implies that either $x_1\in A$; $x_2 \in X\backslash A$; $y_1 \in B$; $y_2\in Y\backslash B$ or $x_1\in X\backslash A$; $x_2\in A$;$y_1\in Y\backslash B$; $y_2\in B$. This implies either $x_1\in A$; $x_2 \in X\backslash A$ or $x_1\in X\backslash A$; $x_2\in A$ and either $y_1\in B$; $y_2\in Y\backslash B$ or $y_1\in Y\backslash B$; $y_2\in B$. Since $(A, B)\in\rho\times \sigma$, we have $A\in\rho$and $B\in\sigma$. This proves that (X, ρ) and(Y, σ) are T_0. □

Remark 4.12. The converse of the above proposition is not true as shown below.

Let X={a, b,c}, Y={1, 2,3}. Clearly τ={$\emptyset$,X, {a},{b},{a, b}}is a topology on X and σ={$\emptyset$, Y,{2},{2,3}}is a topology on Y. Also, (X, τ) and (Y, σ) are T_0- spaces.

Now $\tau\times\sigma$ ={($\emptyset$,$\emptyset$), ($\emptyset$,Y),($\emptyset$,{2}), ($\emptyset$,{2,3}), ({a},$\emptyset$), ({a},{2}), ({a},{2,3}), ({a},Y) ({b}, $\emptyset$), ({b},{2}), ({b},{2,3}), ({b},Y), ({a,b}, $\emptyset$), ({a, b}, {2}), ({a,b}, {2,3}), ({a ,b},Y), (X, $\emptyset$), (X,{2}), (X,{2,3}), (X, Y)}.

The distinct points (a,1)and (b,2)$\in X\times Y$, but there is no binary open set (A,B) in $\tau\times \sigma$ such that (a,1)$\in$ (A,B) and (b,2)$\in$ (X\A,Y\B)

Definition 4.13. A binary topological space $(X,Y, \tau\times\sigma)$ is called binary-T_0 with respect to the first coordinate if for every pair of binary points (x_1,α), (y_1,α) there exists $(A,B)\in \tau\times \sigma$ with $x_1\in A$, $y_1\notin A$, $\alpha\in B$.

Definition 4.14. A binary topological space $(X,Y, \tau\times\sigma)$ is called binary-T_0 with respect to the second coordinate if for every pair of binary points (β, x_2),(β, y_2) there exists $(A,B)\in \tau\times \sigma$ with $\beta\in A$,$x_2 \in B$ and $y_2 \notin B$.

Proposition 4.15. If $(X,Y, \tau\times\sigma)$ is binary-T_0 with respect to the first and the second coordinates, then $(X,Y, \tau\times\sigma)$ is binary-T_0 .

Proof. Let $(X,Y, \tau\times\sigma)$ be binary-T_0 with respect to the first and the second coordinates. Let $(x_1, y_1), (x_2, y_2) \in X\times Y$ with $x_1\neq x_2, y_1\neq y_2$. Take $\alpha\in Y$ and $\beta\in X$. Then $(x_1, \alpha), (x_2,\alpha) \in X\times Y$. Since $(X,Y, \tau\times\sigma)$ is binary-T_0 with respect to the first coordinate, by using Definition 4.13, there exists $(A,B)\in \tau\times \sigma$ with $x_1\in A$, $x_2\notin A$, $\alpha\in B$. Since $(\beta,y_1), (\beta,y_2,) \in X\times Y$, by using the above arguments and using Definition 4.14, there exists $(C,D)\in \tau\times \sigma$ with $y_1 \in D$, $y_2 \notin D$, $\beta\in C$ Therefore, $(x_1, y_1)\in(A, D)$ and $(x_2, y_2)\in(X\backslash A,Y\backslash D)$. Hence $(X,Y, \rho\times\sigma)$ is binary-T_0. □

Proposition 4.16. (X, τ) and (Y, σ) are T_1 spaces if and only if the binary topological space $(X ,Y, \tau\times \sigma)$ is binary-T_1.

Proof. Assume that (X, ρ) and (Y, σ) are T_1 spaces. Let $(x_1,y_1),(x_2, y_2)\in X\times Y$ with $x_1 \neq x_2$, $y_1\neq y_2$. Since (X,ρ) is T_1, there exist $A, B\in\rho$, $x_1\in A$ and $x_2 \in B$ such that $x_1 \notin B$ and $x_2 \notin A$. Also, since (Y,σ) is T_1, there exist $C, D\in\sigma$, $y_1\in C$, and $y_2 \in D$ such that $y_1\notin D$ and $y_2 \notin C$. Thus, $(x_1, y_1)\in(A,C)$ and $(x_2, y_2) \in(B, D)$ with $(x_1, y_1)\in(X\backslash B,Y\backslash D)$ and $(x_2, y_2) \in(X\backslash A, Y\backslash C)$. This implies that $(X ,Y, \rho \times \sigma)$ is binary-T_0. Conversely assume that $(X,Y, \rho \times \sigma)$ is binary-T_1. Let $x_1, x_2 \in X$ and $y_1, y_2 \in Y$ with $x_1 \neq x_2$, $y_1\neq y_2$. Therefore, $(x_1, y_1), (x_2, y_2) \in X\times Y$. Since $(X,Y, \rho\times\sigma)$ is binary-T_1, there exist (A,B) and $(C, D)\in\rho \times \sigma$, $(x_1, y_1) \in (A,B)$ and $(x_2, y_2)\in(C, D)$ such that $(x_1, y_1) \in(X\backslash C,Y\backslash D)$ and $(x_2, y_2) \in(X\backslash A,Y\backslash B)$ .Therefore, $(x_1\in A$ and $x_2 \in C)$ and $(x_1\in X\backslash C$ and $x_2 \in X\backslash A)$ and $(y_1\in B$ and $y_2\in D)$ and $(y_1\in Y\backslash D$ and $y_2 \in Y\backslash B)$. Since (A, B) and $(C, D)\in\rho \times \sigma$, we have $A, C \in\rho$ and $B, D \in\sigma$ .This proves that (X, ρ) and (Y, σ) are T_1 spaces. □

Proposition 4.17. The binary topological space $(X, Y, \mathcal{M})$ is binary-T_1 if and only if every binary point $\wp(X)\times\wp(Y)$ is binary closed.

Proof. Assume that $(X, Y, \mathcal{M})$ is binary-T_1. Let $(x, y)\in X\times Y$. Let $(\{x\},\{y\})\in \wp(X)\times\wp(Y)$. We shall show that $(\{x\},\{y\})$ is binary closed. It is enough to show that $(X\backslash\{x\}, Y\backslash\{y\})$ is binary open. Let $(a, b)\in (X\backslash\{x\}, Y\backslash\{y\})$. This implies that $a \in X\backslash\{x\}$ and $b\in Y\backslash\{y\}$. Hence $a \neq x$ and $b \neq y$. That is, (a, b) and (x, y) are distinct binary points of $X\times Y$.

Since $(X, Y, \mathcal{M})$ is binary-T_1, there exist (A, B) and $(C, D)\in \mathcal{M}$, $(a, b) \in(A, B)$ and $(x, y) \in(C, D)$ such that $(a, b)(X\backslash C,Y\backslash D)$ and$(x, y)\in(X\backslash A,Y\backslash B)$.

Therefore, $(A,B)\subseteq(X\backslash\{x\}, Y\backslash\{y\})$. Hence $(X\backslash\{x\},Y\backslash\{y\})$ is a binary neighborhood of (a, b). This implies $(\{x\},\{y\})$ is binary closed.

Conversely assume that $(\{x\},\{y\})$ is binary closed for every $(x, y) \in X\times Y$.
Let $(x_1,y_1),(x_2, y_2)\in X\times Y$ with $x_1 \neq x_2$, $y_1\neq y_2$. Therefore, $(x_2, y_2)\in(X\backslash\{x_1\}, Y\backslash\{y_1\})$ and $\in (X\backslash\{x_1\}, Y\backslash\{y_1\})$ is binary open.

Also $(x_1, y_1)\in(X\backslash\{x_2\}, Y\backslash\{y_2\})$ and$\in(X\backslash\{x_2\}, Y\backslash\{y_2\})$ is binary open.
This shows that $(X, Y, \mathcal{M})$ is binary-T_1. □

Definition 4.18. Two binary open sets (A, B) and (C, D) are said to be disjoint if $(A\cap C,B\cap D)=(\varnothing, \varnothing)$. That is $A\cap C=\varnothing$ and $B\cap D=\varnothing$.

Definition 4.19. A binary topological space $(X,Y, \mathcal{M})$ is called a binary-T_2 if for every $(x_1, y_1), (x_2, y_2) \in X \times Y$ with $x_1 \neq x_2, y_1 \neq y_1$, there exist disjoint binary open sets (A,B) and (C, D) such that $(x_1, y_1) \in (A,B)$ and $(x_2, y_2) \in (C,D)$.

From the above definition, we have $(x_1, y_1) \in (X\backslash C, Y\backslash D)$ and $(x_2, y_2) \in (X\backslash A, Y\backslash B)$.

Proposition 4.20. (X, τ) and (Y, σ) are T_2 spaces if and only if the binary topological space $(X,Y, \tau \times \sigma)$ is binary-T_2.

Proof. Assume that (X, τ) and (Y, σ) are T_2 spaces. Let $(x_1,y_1), (x_2, y_2) \in X \times Y$ with $x_1 \neq x_2, y_1 \neq y_2$. Since (X, τ) is T_2, there exist disjoint open sets $A, B \in \tau$, $x_1 \in A$ and $x_2 \in B$ such that $x_1 \notin B$ and $x_2 \notin A$. Also, since (Y,σ) is T_2, there exist disjoint open sets $C, D \in \sigma$, $y_1 \in C$, and $y_2 \in D$ such that $y_1 \notin D$ and $y_2 \notin C$. Thus, $(x_1, y_1) \in (A,C)$ and $(x_2, y_2) \in (B, D)$ with $(x_1, y_1) \in (X\backslash B, Y\backslash D)$ and $(x_2, y_2) \in (X\backslash A, Y\backslash C)$. Since A and B are disjoint ,we have $A \cap B = \emptyset$. Also Since C and D are disjoint we have $C \cap D = \emptyset$. Thus $(A\cap B, C \cap D) = (\emptyset, \emptyset)$. Hence (A,C) and (B,D)are disjoint binary open sets. This implies that $(X,Y, \tau \times \sigma)$ is binary-T_2.

Conversely we assume that $(X,Y, \tau \times \sigma)$ is binary-T_2.

Let $x_1, x_2 \in X$ and $y_1, y_2 \in Y$ with $x_1 \neq x_2, y_1 \neq y_2$. Therefore, $(x_1, y_1), (x_2, y_2) \in X \times Y$. Since $(X ,Y, \tau \times \sigma)$ is binary-T_2, there exist disjoint binary open sets (A,B) and $(C, D) \in \tau \times \sigma$ with $(x_1, y_1) \in (A,B)$ and $(x_2,y_2) \in (C, D)$. That is, $(x_1, y_1) \in (X\backslash C, Y\backslash D)$ and $(x_2, y_2) \in (X\backslash A, Y\backslash B)$. Therefore, $(x_1 \in A$ and $x_2 \in C)$ and $(x_1 \in X\backslash C$ and $x_2 \in X\backslash A)$ and $(y_1 \in B$ and $y_2 \in D)$ and $(y_1 \in Y\backslash D$ and $y_2 \in Y\backslash B)$. Since (A, B) and $(C, D) \in \tau \times \sigma$, we have $A, C \in \tau$ and $B, D \in$. This proves that (X, τ) and (Y, σ) are T_2. $\square$

Let $(X, Y, \mathcal{M})$ be a binary topological space. Let $(A , B) \subseteq (X, Y)$. Define $\mathcal{M}_{(A, B)} = \{(A \cap U, B \cap V): (U, V) \in \mathcal{M}\}$. Then $\mathcal{M}_{(A, B)}$ is a binary topology from A to B. The binary topological space $(A, B, \mathcal{M}_{(A,B)})$ is called a binary sub-space of $(X, Y, \mathcal{M})$.

Proposition 4.21. Every subspace of a binary-T_i space is binary-T_i for i=0,1,2.

Conclusion

The separation axioms namely T_0, T_1 and T_2 are extended to binary topological spaces. It is note worthy to see that binary-T_2 $\Rightarrow$ binary-T_1 $\Rightarrow$bibary-T_0.

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