

Intuitionistic Fuzzy Weakly Generalized Irresolute Mappings

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Abstract

The aim of this paper is to introduce and study the concepts of intuitionistic fuzzy weakly generalized irresolute mappings in intuitionistic fuzzy topological space. We investigate some of their properties.

Key words : Intuitionistic fuzzy topology, intuitionistic fuzzy weakly generalized closed set, intuitionistic fuzzy weakly generalized open set, intuitionistic fuzzy weakly generalized irresolute mappings, intuitionistic fuzzy $wT_{1/2}$ space and intuitionistic fuzzy wgT_q space.

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1. Introduction

After the introduction of Fuzzy set (FS) by Zadeh¹⁶ in 1965 and fuzzy topology by Chang³ in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov¹ in 1983 as a generalization of fuzzy sets. In 1997 Coker⁴ introduced the concept of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy weakly generalized irresolute mappings

in intuitionistic fuzzy topological space and studied some of their properties. We provide some characterizations of intuitionistic fuzzy weakly generalized irresolute mappings and established the relationships with other classes of early defined forms of intuitionistic mappings.

2. Preliminaries :

Definition¹ 2.1 Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the

functions $\mu_A(x):X \rightarrow [0, 1]$ and $\nu_A(x):X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in X .

Definition² 2.2 Let A and B be IFSs of the forms $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$.

Then

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition⁴ 2.3: An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFSs in X satisfying the following axioms:

- (a) $0_\sim, 1_\sim \in \tau$
- (b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- (c) $\cup G_i \in \tau$ for any arbitrary family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X .

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition⁴ 2.4: Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by $\text{int}(A) = \cup \{G / G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$ $\text{cl}(A) = \cap \{K / K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$. Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5: An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, δ) is said to be an

- (a) ⁷intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$
- (b) ⁷intuitionistic fuzzy α -closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
- (c) ⁷intuitionistic fuzzy pre-closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$
- (d) ⁷intuitionistic fuzzy regular closed set (IFRCS in short) if $\text{cl}(\text{int}(A)) = A$
- (e) ¹⁴intuitionistic fuzzy generalized closed set (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS
- (f) ¹²intuitionistic fuzzy generalized semi closed

set (IFGSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS

(g) ¹⁰intuitionistic fuzzy α generalized closed set (IF α GCS in short) if $\alpha \text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α -open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IF α OS, IFPOS, IFROS, IFGOS, IFGSOS and I α FGOS) if the complement A^c is an IFSCS, IF α CS, IFPCS, IFRCS, IFGCS, IFGSCS and IF α GCS respectively.

Definition⁸ 2.6 An IFS $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy weakly generalized closed set* (IFWGCS in short) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

The family of all IFWGCSs of an IFTS (X, τ) is denoted by IFWGCS(X).

Definition⁸ 2.7 An IFS A is said to be an *intuitionistic fuzzy weakly generalized open set* (IFWGOS in short) in (X, τ) if the complement A^c is an IFWGCS in X .

The family of all IFWGOSs of an IFTS (X, τ) is denoted by IFWGO(X).

Result⁸ 2.8 Every IFCS, IF α CS, IFGCS, IFRCS, IFPCS, IF α GCS is an IFWGCS but the converses need not be true in general.

Definition⁴ 2.9 Let f be a mapping

from a IFS X to IFS set Y . If $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle / y \in Y \}$ is an IFS in Y , then the pre-image of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle / x \in X \}$.

If $A = \{ \langle x, \lambda_A(x), \nu_A(x) \rangle / x \in X \}$ is an IFS in X , then the image of A under f denoted by $f(A)$ is the IFS in Y defined by $f(A) = \{ \langle y, f(\lambda_A(y)), f_-(\nu_A(y)) \rangle / y \in Y \}$ where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

Definition⁵ 2.10 Let f is a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an *intuitionistic fuzzy continuous* (IF continuous in short) if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition 2.11 Let f is a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

(a) ⁷*intuitionistic fuzzy semi continuous* (IFS continuous in short) if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$

(b) ⁷*intuitionistic fuzzy α continuous* (IF α continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$

(c) ⁷*intuitionistic fuzzy pre continuous* (IFP continuous in short) if $f^{-1}(B) \in \text{IFPO}(X)$ for every $B \in \sigma$

(d) ⁶*intuitionistic fuzzy completely continuous* if $f^{-1}(B) \in \text{IFRO}(X)$ for every $B \in \sigma$.

Result⁷ 2.12 Every IF continuous mapping is an IF α -continuous mapping and every IF α -continuous mapping is an IFS continuous mapping as well as an IFP continuous mapping, but the separate converses need not

be true in general.

Definition⁶ 2.13 A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an *intuitionistic fuzzy γ continuous* (IF γ continuous in short) if $f^{-1}(B)$ is an IF γ OS in (X, τ) for every $B \in \sigma$.

Definition¹⁴ 2.14 Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an *intuitionistic fuzzy generalized continuous* (IFG continuous in short) if $f^{-1}(B) \in \text{IFGCS}(X)$ for every IFCS B in Y .

Result¹⁴ 2.15 Every IF continuous mapping is an IFG continuous mapping but the converse need not be true in general.

Definition¹² 2.16 A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an *intuitionistic fuzzy generalized semi continuous* (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ) .

Definition¹¹ 2.17 A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be an *intuitionistic fuzzy alpha generalized continuous* (IF α G continuous in short) if $f^{-1}(B)$ is an IF α GCS in (X, τ) for every IFCS B of (Y, σ) .

Definition⁸ 2.18 An IFTS (X, τ) is said to be an intuitionistic fuzzy $_wT_{1/2}$ (IF $_wT_{1/2}$ in short) space if every IFWGCS in X is an IFCS in X .

Definition⁸ 2.19 An IFTS (X, τ) is said to be an intuitionistic fuzzy $_wgT_q$ (IF $_wgT_q$ in short) space if every IFWGCS in X is an IFPCS in X .

Definition¹⁴ 2.20 An IFTS (X, τ) is said to be an intuitionistic fuzzy $T_{1/2}$ (IFT $_{1/2}$ in short) space if every IFGCS in X is an IFCS in X .

Definition¹² 2.21 An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{\alpha\alpha}T_{1/2}$ (IF $_{\alpha\alpha}T_{1/2}$ in short) space if every IF α GCS in X is an IFCS in X .

Definition¹² 2.22 Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an *intuitionistic fuzzy irresolute* (IF irresolute in short) if $f^{-1}(B) \in \text{IFCS}(X)$ for every IFCS B in Y .

Definition¹² 2.23 Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an *intuitionistic fuzzy generalized irresolute* (IFG irresolute in short) if $f^{-1}(B) \in \text{IFGCS}(X)$ for every IFGCS B in Y .

Definition⁹ 2.24 A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy weakly generalized continuous* (IFWG continuous in short) if $f^{-1}(B)$ is an IFWGCS in (X, τ) for every IFCS B of (Y, σ) .

Definition¹² 2.25 An IFTS (X, τ) is called an intuitionistic fuzzy $_gT_{1/2}$ (IF $_gT_{1/2}$ in short) space if every IFGSCS in X is an IFGCS in X .

3. Intuitionistic Fuzzy Weakly Generalized Irresolute Mappings :

In this section we introduce intuitionistic fuzzy weakly generalized irresolute mappings

and studied some of its properties. Also we prove that the composition of IFWG irresolute mapping is IFWG irresolute.

Definition 3.1: Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy weakly generalized interior and an intuitionistic fuzzy weakly generalized closure are defined by
 $\text{wgint}(A) = \cup \{ G / G \text{ is an IFWGOS in } X \text{ and } G \subseteq A \}$
 $\text{wgcl}(A) = \cap \{ K / K \text{ is an IFWGCS in } X \text{ and } A \subseteq K \}.$

Theorem 3.2: If A is an IFS in X , then $A \subseteq \text{wgcl}(A) \subseteq \text{cl}(A)$.

Proof: The result follows from the definition 3.1.

Theorem 3.3: If A is an IFWGCS in X then $\text{wgcl}(A) = A$.

Proof: Since A is an IFWGCS, $\text{wgcl}(A)$ is the smallest IFWGCS which contains A , which is nothing but A . Hence $\text{wgcl}(A) = A$.

Theorem 3.4: If A is an IFWGOS in X then $\text{wgint}(A) = A$.

Proof: Since A is an IFWGOS, $\text{wgint}(A)$ is the largest IFWGOS which contains A , which is nothing but A . Hence $\text{wgint}(A) = A$.

Definition 3.5: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy weakly generalized irresolute* (IFWG irresolute in short) if $f^{-1}(A)$ is an IFWGCS in (X, τ) for every IFWGCS A of (Y, σ) .

Theorem 3.6: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping, then f is IFWG continuous mapping but not conversely.

Proof: Let f be an IFWG irresolute mapping. Let A be any IFCS in Y . Since every IFCS is an IFWGCS, A is an IFWGCS in Y . By hypothesis $f^{-1}(A)$ is an IFWGCS in X . Hence f is an IFWG continuous mapping.

Example 3.7: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \langle x, (0.2, 0.4), (0.7, 0.4) \rangle$, $T_2 = \langle y, (0.4, 0.3), (0.6, 0.7) \rangle$. Then $\tau = \{0_-, T_1, 1_-\}$ and $\sigma = \{0_-, T_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFWG continuous mapping. Let IFS $B = \langle y, (0.2, 0.3), (0.7, 0.7) \rangle$ is an IFWGCS in Y . But $f^{-1}(B)$ is not an IFWGCS in X . Therefore f is not an IFWG irresolute mapping.

Theorem 3.8: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping where (Y, σ) is an $\text{IF}_w T_{1/2}$ space. Then the following statements are equivalent
 (a) f is IFWG irresolute
 (b) f is IFWG continuous

Proof:

(a) \Rightarrow (b): Is obviously true from the theorem 3.6.

(b) \Rightarrow (a): Let f be an IFWG continuous mapping. Let A be an IFWGCS in (Y, σ) . Since (Y, σ) is an $\text{IF}_w T_{1/2}$ space, A is an IFCS in (Y, σ) and by hypothesis $f^{-1}(A)$ is an IFWGCS in (X, τ) . Therefore f is IFWG irresolute.

Theorem 3.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into IFTS Y . Then the following conditions are equivalent

- (a) f is IFWG irresolute mapping
 (b) $f^{-1}(B)$ is an IFWGOS in X for every IFWGOS B in Y .

Proof:

(a) \Rightarrow (b): Let B be an IFWGOS in Y . This implies B^c is an IFWGCS in Y . Then $f^{-1}(B^c)$ is an IFWGCS in X , by hypothesis. Since $f^{-1}(B^c) = (f^{-1}(B))^c$, implies $f^{-1}(B)$ is an IFWGOS in X .

(b) \Rightarrow (a): Let B be an IFWGCS in Y . By our assumption $f^{-1}(B^c)$ is an IFWGOS in X for every IFWGOS B^c in Y . But $f^{-1}(B^c) = (f^{-1}(B))^c$, which in turn implies $f^{-1}(B)$ is an IFWGCS in X . Hence f is IFWG irresolute mapping.

Theorem 3.10: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping then f is an IF irresolute mapping if X is an $IF_w T_{1/2}$ space.

Proof: Let A be an IFCS in Y . Then A is an IFWGCS in Y . Therefore $f^{-1}(A)$ is an IFWGCS in X , by hypothesis. Since X is an $IF_w T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X . Hence f is an IF irresolute mapping.

Theorem 3.11: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping then f is an IFP irresolute mapping if X is an $IF_{wg} T_q$ space.

Proof: Let A be an IFCS in Y . Then A is an IFWGCS in Y . Therefore $f^{-1}(A)$ is an IFWGCS in X , by hypothesis. Since X is an $IF_{wg} T_q$ space, $f^{-1}(A)$ is an IFPCS in X . Hence f is an IFP irresolute mapping

Theorem 3.12: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \delta)$ be an IFWG irresolute

mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG irresolute mapping.

Proof: Let A be an IFWGCS in Z . Then $g^{-1}(A)$ is an IFWGCS in Y , by hypothesis. Since f is an IFWG irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence $g \circ f$ is an IFWG irresolute mapping.

Theorem 3.13: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IFWG continuous mapping then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFWGCS in Y , by hypothesis. Since f is an IFWG irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence $g \circ f$ is an IFWG continuous mapping.

Theorem 3.14: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IF continuous mapping then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFCS in Y , by hypothesis. Since every IFCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y . But f is an IFWG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence $g \circ f$ is an IFWG continuous mapping.

Theorem 3.15: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an $IF\alpha$ continuous mapping then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IF α CS in Y , by hypothesis. Since every IF α CS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y . But f is an IFWG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence gof is an IFWG continuous mapping.

Theorem 3.16: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IF α G continuous mapping then $\text{gof}: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IF α GCS in Y , by hypothesis. Since every IF α GCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y . But f is an IFWG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence gof is an IFWG continuous mapping.

Theorem 3.17: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IFG continuous mapping then $\text{gof}: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG continuous mapping¹³.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFGCS in Y , by hypothesis. Since every IFGCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y . But f is an IFWG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence gof is an IFWG continuous mapping¹⁵.

Theorem 3.18: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IFP continuous mapping then

$\text{gof}: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFPCS in Y , by hypothesis. Since every IFPCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y . But f is an IFWG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence gof is an IFWG continuous mapping.

Theorem 3.19: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an intuitionistic fuzzy completely continuous then $\text{gof}: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFRCS in Y , by hypothesis. Since every IFRCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y . But f is an IFWG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence gof is an IFWG continuous mapping.

Theorem 3.20: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFWG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IFGS continuous where Y is an $\text{IF}_g T_{1/2}$ space then $\text{gof}: (X, \tau) \rightarrow (Z, \delta)$ is an IFWG continuous mapping.

Proof: Let A be an IFCS in Z . Then $g^{-1}(A)$ is an IFGSCS in Y , by hypothesis. Since Y is an $\text{IF}_g T_{1/2}$ space, $g^{-1}(A)$ is an IFGCS in Y . Since every IFGCS is an IFWGCS, $g^{-1}(A)$ is an IFWGCS in Y . But f is an IFWG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFWGCS in X . Hence gof is an IFWG continuous mapping.

Theorem 3.21: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is IFWG irresolute, then $f(\text{wgcl}(A)) \subseteq \text{cl}(f(A))$ for every IFS A of X .

Proof: Let A be an IFS of X . Then $\text{cl}(f(A))$ is an IFCS of Y . Since every IFCS is an IFWGCS, $\text{cl}(f(A))$ is an IFWGCS in Y . Since f is IFWG irresolute, $f^{-1}(\text{cl}(f(A)))$ is an IFWGCS in X . Clearly $A \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $\text{wgcl}(A) \subseteq \text{wgcl}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$. Hence $f(\text{wgcl}(A)) \subseteq \text{cl}(f(A))$ for every IFS A of X .

Theorem 3.22: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is IFWG irresolute, then $\text{wgcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS B of Y .

Proof: Let B be an IFS of Y . Then $\text{cl}(B)$ is an IFCS of Y . since every IFCS is an IFWGCS, $\text{cl}(B)$ is an IFWGCS in Y . By hypothesis, $f^{-1}(\text{cl}(B))$ is an IFWGCS in X . Clearly $B \subseteq \text{cl}(B)$ implies $f^{-1}(B) \subseteq f^{-1}(\text{cl}(B))$. Therefore $\text{wgcl}(f^{-1}(B)) \subseteq \text{wgcl}(f^{-1}(\text{cl}(B))) = f^{-1}(\text{cl}(B))$. Hence $\text{wgcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS B of Y .

Theorem 3.23: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into IFTS Y . Then the following conditions are equivalent

- (a) f is an IFWG irresolute mapping
- (b) $f^{-1}(B)$ is an IFWGOS in X for every IFWGOS in Y
- (c) $f^{-1}(\text{wgint}(B)) \subseteq \text{wgint}(f^{-1}(B))$ for every IFS B of Y
- (d) $\text{wgcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS B of Y .

Proof:

(a) \Rightarrow (b): Is obviously true from the Theorem 3.9.

(b) \Rightarrow (c): Let B be an IFS in Y and $\text{wgint}(B) \subseteq B$. Then $f^{-1}(\text{wgint}(B)) \subseteq f^{-1}(B)$. Since $\text{wgint}(B)$ is an IFWGOS in Y , $f^{-1}(\text{wgint}(B))$ is an IFWGOS in X , by hypothesis. Therefore $\text{wgint}(f^{-1}(B)) \supseteq \text{wgint}(f^{-1}(\text{wgint}(B))) = f^{-1}(\text{wgint}(B))$. Hence $f^{-1}(\text{wgint}(B)) \subseteq \text{wgint}(f^{-1}(B))$ for every IFS B of Y .

(c) \Rightarrow (d): Is obvious by taking complement in (c).

(d) \Rightarrow (a): Let B be an IFWGCS in Y and $\text{wgcl}(B) = B$. Hence $f^{-1}(B) = f^{-1}(\text{wgcl}(B)) \supseteq \text{wgcl}(f^{-1}(B))$. Therefore $\text{wgcl}(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFWGCS in X . Thus f is an IFWG irresolute mapping.

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