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Effect of measurement error on sampling plan under known coefficient of variation

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Abstract

In the present paper we investigated that in what way the operating characteristic (OC) function of the usual variables sampling inspection plans with known coefficient of variation (CV) based on the assumption of normality, error free case and independent observation, are effected when the characteristic of an item possesses a measurement error.

Key words : Variable Sampling, Inspection plan, Measurement Error, Coefficient of Variation.

1. Introduction

Acceptance sampling is the testing or the inspection of selected items from a given lot followed by acceptance or rejection of that lot on the basis of the results of the test and its indication of the lot's quality. It is assumed that a lot's quality is determined by the proportion of defective items in the lot. Variable plans, however, requires that the characteristic of interest be a continuous variable. The characteristics are measured and its actual value is recorded. In variable sampling plans, an underlying process distribution form is assumed. Then the proportion defective in the lot can be estimated by estimating the parameters of that design. The variable model thus requires more restrictive assumptions on the manufacturing process. If these assumptions can be justified, there would be a substantial saving in sample size corresponding to a given sampling risk.

Coefficient of variation (CV) is widely used as a measure of variability in engineering experiments although standard error of CV is well known[2,4]. When there is no prior knowledge available regarding the value of the population mean, it is well known that the mean of all the available data (assumed independently drawn) leads to the best estimate. However, in many practical problems, the experimenter has some guessed estimates regarding the values of the parameters either due to past experience or due to his acquaintance with the behavior of the system under consideration. Bayesian approach to the acceptance sampling plans is based on the assumption of utilizing a prior knowledge. Searls[3] has provided an estimator $\bar{\bar{x}}$ of μ which utilizes a prior knowledge concerning the value of the population CV. An estimator $\bar{\bar{x}}$ shown to have a smaller mean square error (MSE) than the sample mean $\bar{x}$.

David *et al.*[1], were the first to study the effect of measurement error on single sampling inspection plan by variables. It appears that no attempt has been made in the direction of single sampling plan with known CV under measurement error. In this section, acceptance sampling plans for variables with known CV in presence of measurement error are considered. The development of this section is based on well recognized fact that measurement are inadvertently subject to error. This leads to the immediate conclusion that what we obtain as sample value (or observed measurement) on a particular unit is not the same as its process value (or true measurement). We assume the most convenient from of linear relationship between observed measurement X and true measurement X that is X=x + e, where the variable e denotes random error of measurement. Though the relationship $\bar{X} = \bar{x} + \bar{e}$ between sample mean of the variates X, x and e is trivial, practice, we agree that σ_X^2 is not the same as σ_P^2 where σ_X^2 and σ_P^2 are the variances of X and x. At this juncture one remark appears to be very pertinent, to those author who are of the view that in an actual process the variability of the process is a combine measure of product variability and measurement or that the variability of measurement error is included as a part of the total variability of the process, this approach may appear to be unreasonable and unsound. We do not subscribe to this view since this leads to the conclusion that process variance can be pre-specified and will vary from one measuring device to another. If the measurement error is large enough, replication of measurements could sufficiently reduce the error. We make the assumption that the random variable e follows a normal law with mean zero and variance σ_e^2 and is independent of x. This in conjunction with the normality assumption for x values implies that the observed measurements X will also be normally distributed with mean μ and variance $\sigma_X^2 = \sigma_P^2 + \sigma_e^2$.

3. *Model description :*

The procedure of acceptance sampling consists in the first drawing a random sample of size n from the lot submitted for inspected and measuring the item quality x of the sampled items. Let $x_1, x_2, \dots, x_n$ denote the values of the characteristic x. An estimator $\bar{\bar{x}}$ has been constructed by Searls[3],where

$$\bar{\bar{x}} = w\ \Sigma_{j=1}^{n} x_j\ , \qquad (2.1)$$

where w is the scalar and is so that the MSE, $E\left(\bar{\bar{x}} - \mu\right)^2$ is minimum. Searls[3] has shown that $w = 1/(n + v^2)$ and hence

$$MSE(\bar{x}) = \sigma^2/(n+v^2). \qquad (2.2)$$

Let the relation between the true measurement x and observed measurement X be expressed as $X = x + e$, where e is random error and is independent of x. Thus $e \sim N(0, \sigma_e^2)$. Therefore,

$E(X) = E(x) + E(e) = E(x) = \mu$ and
$V(X) = V(x) + V(e) = \sigma_p^2 + \sigma_e^2 = \sigma_X^2 (say)$.

The relation between size of measurement error r and the correlation coefficient ρ is obtained as

$$\rho = \frac{r}{\sqrt{1+r^2}}, \qquad (2.3)$$

where $= \sigma_p / \sigma_e$.

After some simplification the MSE of observed mean $\bar{x}$ is given by

$$MSE(\bar{x}) = \mu^2 \left[w^2 n^2 \left(1 + \frac{v^2}{n\rho^2}\right) - 2wn + 1 \right], \qquad (2.4)$$

where $v = \sigma_p / \mu$ is the coefficient of variation; The $MSE(\bar{x})$ of equation (2.4) is minimum when $w = \left(1/\left((v^2/\rho^2) + n\right)\right)$. Substituting the value of w in equation (2.4) we get the minimum $MSE(\bar{x})$ as

$$MSE(\bar{x}) = \frac{\sigma_p^2}{(n\rho^2 + v^2)} . \qquad (2.5)$$

In connection with a single sampling variables plan, when CV is known, the following symbols will be used.

L= Lower specification limit
U= Upper specification limit
k= acceptance parameter
$\bar{x}$ = weighted sample mean for n

$$F(x) = \int_{-\infty}^{x} \left(\frac{1}{\sqrt{2\pi}}\right) Exp\left(-\frac{1}{2}z^2\right) dz,$$

where $z \sim N(0,1)$. We now calculate the OC function of single sampling plan. The acceptance criterion for weighted mean with known CV plan is, accept the lot if $\bar{x} + k\sigma_x \leq U$ and reject the lot otherwise for the upper specification limit. When used with L, the acceptance criterion is, accept the lot if $\bar{x} - k\sigma_x \geq L$ and reject the lot otherwise. The value of n and k are determined for a given set of values of the procedure's risk α, consumer's risk β, $AQL(p_1)$ and $LTPD(p_2)$ by the formulae

$$n = \left[\left(K_\alpha + K_\beta\right)/K_{p_1} - K_{p_2}\right]^2 \qquad (2.6)$$

$$k = \left[\left(K_\alpha K_{p_2} + K_\beta K_{p_1}\right)/\left(K_\alpha + K_\beta\right)\right] \qquad (2.7)$$

If p is the proportion defective in the lot

$$\frac{U-\mu}{\sigma_x} = K_p ,$$

where

$$\int_{K_p}^{\infty} \left(\frac{1}{\sqrt{2\pi}}\right) exp\left(-\frac{1}{2}z^2\right) dz = p. \qquad (2.5)$$

The expression for probability of acceptance *i.e.* OC function of the plan is

$$L(p) = Prob.\left[\,\bar{x} + k\sigma \leq U = \mu + K_p\sigma_x\right] \qquad (2.6)$$

Following Schilling (1982) the OC function with known CV under measurement error works out to be as

$$L(p) = \Phi\left[\sqrt{(\rho^2 n + v^2)}\left(K_p - k\right)\right], \qquad (2.7)$$

Formulae for n and k, satisfying the requirements that the probability of ejecting a lot of acceptable quality p_1 is α and the probability of accepting a lot of rejectable quality p_2 is β are respectively given by equation (2.6) and (2.7).

3. Discussion of the Results

To determine the general effects of measurement error we consider producer's and consumer's oriented plans specified by $p_1 = 0.05, \alpha = 0.05, p_2 = 0.30 \ and \ \beta = 0.10$. The values of n and k are 7 and 1.015 calculated from equation (2.3) and (2.4) respectively. The values of OC function for the above plan have been calculated with known CV for different sizes of measurement error by using equation (2.7). The relationship between size of the measurement error and correlation coefficient between true and observed values are given by the equation (2.3). These values of OC function with known CV and r = ∞, 2, 4 and 6 are presented in Table 1 and Table 2.

Table 1. Values of OC function for known CV under Measurement Error r=∞, 2.

p	r=∞					r=2				
	v=1	v=2	v=3	v=4	v=5	v=1	v=2	v=3	v=4	v=5
0.02	0.9983	0.9997	1.0000	1.0000	1.0000	0.9960	0.9993	1.0000	1.0000	1.0000
0.04	0.9813	0.9927	0.9984	0.9998	1.0000	0.9700	0.9885	0.9975	0.9997	1.0000
0.06	0.9366	0.9633	0.9846	0.9952	0.9989	0.9162	0.9522	0.9802	0.9940	0.9986
0.08	0.8650	0.9021	0.9407	0.9693	0.9863	0.8407	0.8858	0.9315	0.9653	0.9844
0.10	0.7746	0.8117	0.8568	0.8994	0.9342	0.7522	0.7948	0.8453	0.8926	0.9296
0.13	0.6236	0.6441	0.6720	0.7034	0.7357	0.6121	0.6346	0.6645	0.6979	0.7309
0.15	0.5242	0.5283	0.5342	0.5409	0.5483	0.5219	0.5264	0.5326	0.5397	0.5471
0.17	0.4317	0.4200	0.4039	0.3852	0.3654	0.4382	0.4255	0.4083	0.3885	0.3684
0.20	0.3119	0.2826	0.2440	0.2028	0.1634	0.3288	0.2962	0.2543	0.2098	0.1690
0.22	0.2461	0.2103	0.1657	0.1221	0.0848	0.2674	0.2267	0.1773	0.1292	0.0898
0.25	0.1677	0.1294	0.0866	0.0512	0.0270	0.1920	0.1465	0.0971	0.0565	0.0299
0.28	0.1108	0.0759	0.0419	0.0191	0.0072	0.1346	0.0910	0.0497	0.0221	0.0085
0.30	0.0826	0.0519	0.0249	0.0093	0.0028	0.1049	0.0649	0.0307	0.0112	0.0034
0.33	0.0519	0.0282	0.0107	0.0029	0.0006	0.0707	0.0379	0.0142	0.0037	0.0007
0.37	0.0267	0.0117	0.0031	0.0005	0.0001	0.0403	0.0174	0.0046	0.0007	0.0001
0.40	0.0156	0.0058	0.0012	0.0001	0.0000	0.0257	0.0093	0.0018	0.0002	0.0000

Table 2. Values of OC function for known CV under Measurement Error r=4,6.

p	r=4					r=6				
	v=1	v=2	v=3	v=4	v=5	v=1	v=2	v=3	v=4	v=5
0.02	0.9979	0.9996	1.0000	1.0000	1.0000	0.9982	0.9997	1.0000	1.0000	1.0000
0.04	0.9786	0.9917	0.9982	0.9998	1.0000	0.9801	0.9922	0.9983	0.9998	1.0000
0.06	0.9315	0.9605	0.9835	0.9948	0.9988	0.9343	0.9620	0.9841	0.9950	0.9988
0.08	0.8587	0.8978	0.9382	0.9681	0.9858	0.8622	0.9002	0.9396	0.9688	0.9861
0.10	0.7686	0.8071	0.8537	0.8974	0.9329	0.7719	0.8096	0.8554	0.8985	0.9336
0.13	0.6205	0.6415	0.6700	0.7017	0.7344	0.6222	0.6429	0.6711	0.7026	0.7351
0.15	0.5235	0.5278	0.5337	0.5406	0.5479	0.5239	0.5281	0.5340	0.5408	0.5481
0.17	0.4335	0.4215	0.4051	0.3862	0.3662	0.4325	0.4207	0.4044	0.3857	0.3658
0.20	0.3165	0.2863	0.2468	0.2050	0.1649	0.3140	0.2843	0.2453	0.2038	0.1641
0.22	0.2518	0.2147	0.1689	0.1243	0.0862	0.2487	0.2123	0.1672	0.1231	0.0854
0.25	0.1741	0.1339	0.0894	0.0528	0.0278	0.1706	0.1314	0.0879	0.0519	0.0274
0.28	0.1169	0.0798	0.0440	0.0200	0.0076	0.1136	0.0777	0.0429	0.0195	0.0074
0.30	0.0883	0.0552	0.0264	0.0099	0.0029	0.0852	0.0534	0.0255	0.0096	0.0028
0.33	0.0566	0.0307	0.0116	0.0031	0.0006	0.0540	0.0293	0.0111	0.0030	0.0006
0.37	0.0299	0.0131	0.0035	0.0006	0.0001	0.0281	0.0123	0.0033	0.0006	0.0001
0.40	0.0180	0.0066	0.0013	0.0001	0.0000	0.0166	0.0061	0.0012	0.0001	0.0000

From the Table 1 and Table 2 it is evident that the values of OC with known CV increases with increasing size of the measurement error, for p= 0.1, v=1 and r=, 2, 4,6 and ∞ (error free case) the OC functions are 0.7522, 0.7686, 0.7719 and 0.7746 respectively. But for known CV plans, the producer's risk increases and consumer's risk decreases with increasing size of the measurement error. From tables it is seen that there are large variation between r=2 and ∞ for fixed values of CV. In general, it can be said that the effect of measurement error on the OC of sampling plan is negligible when both r and v increases. It may be inferred that use of normal theory sampling plan with known CV is not valid for measurement error case. Even when there is slight departure from true observation it is advisable to take into account the measurement error of the parent population while choosing the sampling plan with known CV.

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