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The fuzzified concepts of several ideals of semi groups

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Abstract

The paper is addressed to the introduction of fuzzified ceoncepts of some ideals of semi group developed by Atanassov (1980). Some properties of such ideals have also been investigated.

Key words : Intutionistic fuzzy (Atanassov), Ideal, Intutionistic fuzzy bi-ideal.

1. Introduction

The concept of intutionistic fuzzy set was introduced by Atanassov[1,2] as a generalization of the notion of fuzzy set introduced by Zadeh[8]. The concept of [1,2] ideals in semi group was introduced by Lajos[3]. In this paper we consider the intutionistic fuzzification of the concept of several ideals of semi group $\tilde{S}$ of Atanassov[1] are investigate some of their properties.

2. *Pre-Requisties :*

Let $\tilde{S}$ by a semi group. By a sub-semi group of $\tilde{S}$, we mean a non empty sub set $\tilde{A}$ of $\tilde{S}$ such that $\tilde{A}^2 \subseteq \tilde{A}$ and by a left ideal (right) of $\tilde{S}$, we mean a non empty sub set $\tilde{A}$ of $\tilde{S}$ such that $\tilde{S}\tilde{A} \subseteq \tilde{A} \left(\tilde{A}\tilde{s} \subseteq \tilde{A} \right)$. By simple ideal, we mean a non empty sub set of $\tilde{S}$ which is both left and right ideal of $\tilde{S}$.

A sub semi group $\tilde{A}$ of a semi-group $\tilde{S}$ is called a bi-ideal of $\tilde{S}$, if $\tilde{A}\tilde{S}\tilde{A} \subseteq \tilde{A}$. A sub semi group $\tilde{A}$ of $\tilde{S}$ is called a ideal of $\tilde{S}$, if $\tilde{A}\tilde{S}\tilde{A}^2 \subseteq \tilde{A}$. A semi group $\tilde{S}$ is called regular if $a \in a^2 \tilde{S} a^2$ for all $x \in \tilde{S}$ [Atanassov[2]]. A sub semi group is said to be regular if $\forall a \in \tilde{S}, \Im b \in \tilde{S}$ such that a =aba and ab= ba

[Atanassov[2]]. $\tilde{S}$ is completely regular iff $\tilde{S}$ is a union of groups iff $\tilde{S}$ is regular. $\tilde{S}$ is said to be left (right) duo if every left (right) ideal of $\tilde{S}$ is a simple ideal.

An intutionistic fuzzy $\tilde{A}$ in a non empty set X is an object of the form

$$\tilde{A} = \left\{\left(a, \lambda_{\tilde{A}}(a), \mu_{\tilde{A}}(a) : a \in x\right)\right\}$$

Where the fucntion $\lambda_{\tilde{A}} : x \to [0,1]$, $\mu_{\tilde{A}} : x \to [0,1]$ denote the degree of membership such that

$$0 \le \lambda_{\tilde{A}}(a) + \mu_{\tilde{A}}(a) \le 1 \quad \forall a \in x$$

An intutionistic fuzzy set can also be specified by the ordered pair $\left(\lambda_{\tilde{A}}, \mu_{\tilde{A}}\right)$ in I^x X I^x. For simplicity we can use the symbol $\tilde{A} = \left(\lambda_{\tilde{A}}, \mu_{\tilde{A}}\right)$.

3. Fuzzy Ideals :

Definition (3.1) :

Set $\tilde{S}$ by a semi group. An intutionistic fuzzy set $\tilde{A} = \left(\lambda_{\tilde{A}}, \mu_{\tilde{A}}\right)$ in S is called an intutionistic fuzzy sub-semi group of $\tilde{S}$ if

(i) $\lambda_{\tilde{A}}(ab) \ge \min\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(b)\}$ and

(ii) $\mu_{\tilde{A}}(ab) \ge \max\{\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(b)\}$ for all $a, b \in \tilde{S}$

Definition (3.2) :

An intutionistic fuzzy set $\tilde{A} = (\lambda_{\tilde{A}}, \mu_{\tilde{A}})$ in $\tilde{S}$ is called an intutionistic fuzzy left ideal of $\tilde{S}$ if $\lambda_{\tilde{A}}(ab)\lambda_{\tilde{A}}(b)$ and $\mu_{\tilde{A}}(ab) \le \mu_{\tilde{A}}(b)$ for all $a, b \in \tilde{S}$. An intutionistic right ideal is defined in analogous way.

An intutionistic fuzzy set $\tilde{A}$ is called an intutionistic fuzzy ideal of $\tilde{S}$ it if it is both intutionistic fuzzy left as well as right ideal.

Any intutionistic fuzzy left (right) ideal of $\tilde{S}$ is an intutionistic fuzzy sub semi group $\tilde{S}$.

Definition (3.3):

An intutionistic fuzzy semi group $\tilde{A}$ of $\tilde{S}$ is called an intutionistic fuzzy bi-ideal of $\tilde{S}$ if

(i) $\lambda_{\tilde{A}}(acb) \ge \min\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(b)\}$

(ii) $\mu_{\tilde{A}}(acb) \le \max\{\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(b)\}$

Example (3.4) :

Let an intutionistic fuzzy set $\tilde{A} = \left(\lambda_{\tilde{A}}, \mu_{\tilde{A}}\right)$ in S by

$$\lambda_{\tilde{A}}(x) = 0.6$$

$$\lambda_{\tilde{A}}(y) = 0.5$$

$$\lambda_{\tilde{A}}(z) = 0.4$$

$$\lambda_{\tilde{A}}(p)=\lambda_{\tilde{A}}(q)=0.3$$
$$\mu_{\tilde{A}}(x)=0.3$$
$$\mu_{\tilde{A}}(y)=0.3$$
$$\mu_{\tilde{A}}(z)=0.4$$
$$\mu_{\tilde{A}}(p)=0.5$$
$$\mu_{\tilde{A}}(q)=0.6$$

We can Vow by that $\tilde{A}$ us an intutionistic fuzzy bi-ideal of $\tilde{S}$ by routine calculation, where $\tilde{S}=\{x,y,z,p,q\}$ is a semi group are given by Table

	x	y	z	p	q
x	x	x	x	x	x
y	x	x	x	x	x
z	x	x	z	z	z
p	x	x	z	p	q
q	x	x	z	z	q

Theorem (3.5) :

Every intutionistic fuzzy left ideal (right) of $\tilde{S}$ is an intutionistic fuzzy bi-ideal of $\tilde{S}$

Proof :

let $\tilde{A}=\left(\lambda_{\tilde{A}},\ \mu_{\tilde{A}}\right)$ be an intutionistic fuzzy left ideal of $\tilde{S}$ and $c,a,b\in\tilde{S}$. Then $\lambda_{\tilde{A}}(acb)=\lambda_{\tilde{A}}((ac)b)\geq\lambda_{\tilde{A}}(b)\geq\min\{\lambda_{\tilde{A}}(a),\lambda_{\tilde{A}}(b)\}$

and $\mu_{\tilde{A}}(acb)=\mu_{\tilde{A}}((ac)b)\leq\mu_{\tilde{A}}(b)\leq\max\{\mu_{\tilde{A}}(a),\mu_{\tilde{A}}(b)\}$

$\Rightarrow\tilde{A}$ is an intutionistic fuzzy bi-ideal of $\tilde{S}$. The right case is proved in similar way.

Theorem (3.6) :

Let $\tilde{S}$ be a regular semi-group. If every ideal of $\tilde{S}$ is a right (left) ideal of $\tilde{S}$, then every intutionistic fuzzy bi-ideal of $\tilde{S}$ is an intutionistic fuzzy right (left) ideal of $\tilde{S}$.

Proof :

Suppose that every bi-ideal of $\tilde{S}$ is a right ideal of $\tilde{S}$. Let $\tilde{A}=\left(\lambda_{\tilde{A}},\mu_{\tilde{A}}\right)$ be the intutionistic fuzzy bi-ideal of $\tilde{S}$ and $a,b\in S$, then $a\tilde{S}a$ is a bi-ideal of $\tilde{S}$ and so $a\tilde{S}a$ is a right ideal of $\tilde{S}$. Since $\tilde{S}$ is regular we have $ab\in\left(a\tilde{S}a\right)\tilde{S}\subseteq a\tilde{S}a$ which implies that ab = aca for some $c\in\tilde{S}$. Since $\tilde{A}$ is an intutionistic fuzzy bi-ideal of $\tilde{S}$ it follows that

$$\lambda_{\tilde{A}}(ab)=\lambda_{\tilde{A}}(aca)\geq\min((\lambda_{\tilde{A}}(a),$$

$\lambda_{\tilde{A}}(a)) = \lambda_{\tilde{A}}(a)$

and $\mu_{\tilde{A}}(ab) = \mu_{\tilde{A}}(aca) \leq \max(\mu_{\tilde{A}}(a),$

$\mu_{\tilde{A}}(a) = \mu_{\tilde{A}}(a)$

$\Rightarrow \tilde{A}$ is an intutionsitic fuzzy right ideal of $\tilde{S}$.

Definition (3.7):

An intutionistic fuzzy sub semi-group $\tilde{A} = \left(\lambda_{\tilde{A}}, \mu_{\tilde{A}}\right)$ of $\tilde{S}$ is called fuzzy ideal of $\tilde{S}$ [Atanassov[1-2]] if

(i) $\lambda_{\tilde{A}}\left(ac(bd)\right) \geq \min\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(b), \lambda_{\tilde{A}}(d)\}$

(ii) $\mu_{\tilde{A}}\left(ac(bd)\right) \leq \max\{\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(b), \mu_{\tilde{A}}(d)\}$

for all $a, b, c, d \in \tilde{S}$.

Theorem (3.8) :

Every intutionistic fuzzy[4,5] bi-ideal is an intutionistic fuzzy Atanassov ideal.

Proof :

Let $\tilde{A} = \left(\lambda_{\tilde{A}}, \mu_{\tilde{A}}\right)$ be an intutionistic fuzzy bi-ideal of $\tilde{S}$. Then

$$\lambda_{\tilde{A}}\left(ac(bd)\right) = \lambda_{\tilde{A}}\left((acb)\,d\right)$$
$$\geq \min\{\lambda_{\tilde{A}}(acb),\ \lambda_{\tilde{A}}(d)\}$$
$$\geq \min[\max\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(b)\}, \lambda_{\tilde{A}}(d)]$$
$$= \min\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(b), \lambda_{\tilde{A}}(d)\}$$
$$\leq \max\{\lambda_{\tilde{A}}(acb),\ \lambda_{\tilde{A}}(d)\}$$
$$\leq \max[\max\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(b), \lambda_{\tilde{A}}(d)\}]$$
$$= \min\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(b), \lambda_{\tilde{A}}(d)\}$$

Hence $\tilde{A}$ is an intutionistic fuzzy Atanassov ideal of $\tilde{S}$.

Theorem (3.9) :

If $\tilde{S}$ is a regular semi group, then every intutionistic fuzzy Atanassov ideal of $\tilde{S}$ is an intutionistic fuzzy bi-ideal of $\tilde{S}$.

Proof :

Suppose that a semi group $\tilde{S}$ is regular and $\tilde{A} = (\lambda_{\tilde{A}}, \mu_{\tilde{A}})$ be an intutionistic fuzzy Atanassov ideal of $\tilde{S}$. Let $c, a, b \in \tilde{S}$

Since $\tilde{S}$ in regular, we have $ac \in (a\tilde{S}a)$

$\tilde{S} \subseteq a\tilde{S}a$

$\Rightarrow ac = asa \quad \text{for some } s \in S.$

Thus

$$\lambda_{\tilde{A}}(acb) = \lambda_{\tilde{A}}\left(((asa)), b\right) = \lambda_{\tilde{A}}\left(as(ab)\right)$$
$$\geq \min\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(b)\}$$
$$= \min\{\lambda_{\tilde{A}}(a),\ \lambda_{\tilde{A}}(b)\}$$

and $\mu_{\tilde{A}}(acb) = \mu_{\tilde{A}}\left(((asa)), b\right)$

$$= \mu_{\tilde{A}}\left(as(ab)\right)$$

$$\leq \max\{\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(a), \mu_{\tilde{A}}(b)\}$$
$$= \max\{\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(b)\}.$$

$\therefore \tilde{A}$ is an intutionistic bi-ideal of $\tilde{S}$

Theorem (3.10) :

Let $\tilde{A} = \left(\lambda_{\tilde{A}}, \mu_{\tilde{A}}\right)$ be an intutionistic fuzzy bi-ideal of $\tilde{S}$, if $\tilde{S}$ is [Atanassov[2]] regular, then $\tilde{A}(a) = \tilde{A}(a^2) \quad \forall a \in \tilde{S}$.

Proof :

Let $a \in \tilde{S}$, then $\exists x \in \tilde{S}$ such that $a = a^2 x a^2$.

Hence

$\lambda_{\tilde{A}}(a) = \lambda_{\tilde{A}}(a^2 x a^2) \geq \min\{\lambda_{\tilde{A}}(a^2), \lambda_{\tilde{A}}(a^2)\}$

$= \lambda_{\tilde{A}}(a^2) \geq \min\{\lambda_{\tilde{A}}(a), \lambda_{\tilde{A}}(a)\} = \lambda_{\tilde{A}}(a)$

and $\mu_{\tilde{A}}(a) = \mu_{\tilde{A}}(a^2 x a^2) \leq \max\{\mu_{\tilde{A}}(a^2),$

$= \mu_{\tilde{A}}(a^2) \leq \max\{\mu_{\tilde{A}}(a), \mu_{\tilde{A}}(a)\} = \mu_{\tilde{A}}(a)$

$\Rightarrow \lambda_{\tilde{A}}(a) = \lambda_{\tilde{A}}(a^2)$ and $\mu_{\tilde{A}}(a) = \mu_{\tilde{A}}(a^2)$ so that

$$\tilde{A}(a) = \tilde{A}(a^2)$$

Definition (3.11) :

A semi group $\tilde{S}$ is said to be intutionistic fuzzy left duo (right) if every intutionistic fuzzy left ideal (right) of $\tilde{S}$ is an intutionistic fuzzy[4-7] ideal of $\tilde{S}$

Theorem (3.12) :

Let $\tilde{S}$ be a regular semi group. If $\tilde{S}$ is left duo (right) then $\tilde{S}$ is intutionistic fuzzy left (right) duo.

Proof :

Suppose that $\tilde{S}$ is a left duo and $\tilde{A} = \left(\lambda_{\tilde{A}}, \mu_{\tilde{B}}\right)$ be any intutionistic fuzzy left ideal of $\tilde{S}$. Let $a, b \in \tilde{S}$. Then since the left ideal $\tilde{S}a$ is a two sided ideal of $\tilde{S}$ and since $\tilde{S}$ is regular, we have

$$a, b \in (a\tilde{S}b) \subseteq \tilde{S} \Rightarrow \tilde{S} \subseteq \tilde{S}a$$

$\Rightarrow \exists a\, c \in \tilde{S}$ such that ab = ca. As $\tilde{A}$ is an intutionistic fuzzy left ideal of $\tilde{S}$, we get

$\lambda_{\tilde{A}}(ab) = \lambda_{\tilde{A}}(ca) \geq \lambda_{\tilde{A}}(a)$ and

$$\mu_{\tilde{A}}(ab) = \mu_{\tilde{A}}(ca) \leq \mu_{\tilde{A}}(a)$$

$\Rightarrow \tilde{A}$ is an intutionistic fuzzy right ideal of $\tilde{S}$ and so $\tilde{S}$ is intutionistic fuzzy left duo. The right case can be proved in analogous way[4-7].

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