Ultra Scientist Vol. 24(3)A, 441-448 (2012).

On Felicitous Labelings of H_n and $H_n \odot S_m$ graphs

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(Acceptance Date 3rd September, 2012)

Abstract

A simple graph G is called felicitous if there exists a 1 – 1 function $f : V(G) \to \{0,1,2, \ldots, q\}$ such that the set of induced edge labels $f^*(uv) = (f(u) + f(v)) \pmod q$ are all distinct. In this paper, H_n and $H_n \odot S_m$ graphs are shown to be felicitous.

Keywords: Graph Labeling, H_n and $H_n \odot S_m$, felicitous labeling.

1. Introduction

The graphs we consider are simple. For notation and terminology, we refer to[1]. Lee, Schmeichel and Shee[2] introduced the concept of a felicitous graph as a generalization of a harmonious graph. A graph G with q edges is called *harmonious* if there is an injection $f : V(G) \to Z_q$, the additive group of integers modulo q such that when each edge xy of G is assigned the label $(f(x) + f(y)) \pmod q$, the resulting edge labels are all distinct. A *felicitous* labeling of a graph G, with q edges is an injection $f : V(G) \to \{0,1,2, \ldots, q\}$ so that the induced edge labels $f^*(xy) = (f(x) + f(y)) \pmod q$ are distinct. Clearly, a harmonious graph is felicitous. An example of a felicitous graph which is not harmonious is the graph $K_{m,n}$, where $m, n > 1$. Throughout this paper, f denotes a 1-1 function from V(G) to a subset of the set of non-negative integers and for any edge $e = xy \in E(G)$, $f^*(e) = f(x) + f(y)$.

2. *Definitions and Basic results :*

Definition 2.1: Let H_n – graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{n+1/2}$ and $u_{n+1/2}$ by means of an edge if n is odd and the vertices $v_{n/2+1}$ and $u_{n/2}$ if n is even.

Definition 2.2 : The m – corona of a graph G, denoted by $G \odot S_m$ is a graph obtained from G by identifying the centre vertex of the star S_m at each vertex.

Remark[3,4] *2.3 :* Let G be a graph with an odd number of edges and let $f : V(G) \to \{0, 1, 2, \ldots, q\}$ be an odd edge labeling of G. Then, f is a felicitous labeling for G.

3. Main Results

Theorem 3.1 : H_n – graph is a felicitous graph.

Proof : Let $V(H_n) = \{ u_i^{(j)} : 1 \le i \le n$ and $1 \le j \le 2\}$ and $E_1 = \{(u_i^{(j)}\ u_{i+1}^{(j)}) : 1 \le i \le n-1$ and $1 \le j \le 2\}$. $E(H_n) = E_1 \cup \left\{ u^{(1)}_{\frac{n+1}{2}} u^{(2)}_{\frac{n+1}{2}} \right\}$ when n is odd and $E(H_n) = E_1 \cup \left\{ u^{(1)}_{\frac{n}{2}+1}\ u^{(2)}_{\frac{n}{2}} \right\}$ when n is even.

It is enough to show that H_n – graph admits odd edge labeling.

Define $f : V(H_n) \to \{0, 1, 2, \ldots, q = 2n-1\}$ by

For $1 \le j \le 2$, $f(u_i^{(j)}) = n(j-1)+(i-1), 1 \le i \le n$.

The labels of the edges are as follows :

For $1 \le j \le 2$, $f(u_i^{(j)}\ u_{i+1}^{(j)}) = 2n(j-1) + (2i-1)$, $1 \le i \le n-1$

$f(u^{(1)}_{\frac{n+1}{2}}\ u^{(2)}_{\frac{n+1}{2}}) = 2n - 1$ when n is odd

and $f(u^{(1)}_{\frac{n}{2}+1}\ u^{(2)}_{\frac{n}{2}}) = 2n - 1$ when n is even

Now, $f(E(G)) = \{1, 3, 5, \ldots, 2(n-1)-1, 2n-1, 2n+1, 2n+3, \ldots, 2n+2(n-1)-1\} = \{1, 3, 5, \ldots, 2n-3, 2n-1, 2n+1, 2n+3, \ldots, 4n-3\} = \{1, 3, 5, \ldots, 2q-1\}$.

Clearly, the above edge values are distinct and odd and hence G admits odd edge labeling. Therefore by 2.3, H_n – graph is a felicitous graph.

For example, a felicitous labeling of H_5 and H_4 are shown in Figure 3.1.

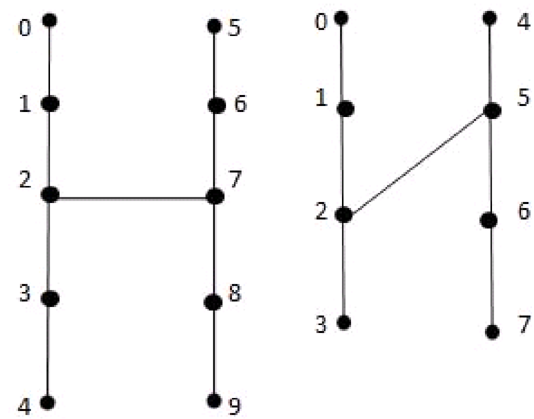

Fig. 3.1

Theorem 3.2 : $H_n \odot S_m$ is a felicitous graph for any m.

Proof : Let $V(H_n \odot S_m) = \{ u_i^{(j)} : 1 \le i \le n$ and $1 \le j \le 2\} \cup \{ v_{i_k}^{(j)} : 1 \le k \le m, 1 \le i \le n$ and $1 \le j \le 2\}$ and $E_1 = \{(u_i^{(j)} u_{i+1}^{(j)}) : 1 \le i \le n-1$ and $1 \le j \le 2\} \cup \{(u_i^{(j)}\ v_{i_k}^{(j)}) : 1 \le i \le n, 1 \le j \le 2$ and $1 \le k \le m\}$. Let $E(H_n \odot S_m) = E_1 \cup \{(u^{(1)}_{\frac{n+1}{2}}\ u^{(2)}_{\frac{n+1}{2}})\}$ when n is odd and $E(H_n \odot S_m) = E_1 \cup \{(u^{(1)}_{\frac{n}{2}+1}\ u^{(2)}_{\frac{n}{2}})\}$ when n is even.

It is enough to show that $H_n \odot S_m$ admits odd edge labeling.

Case (i) : When m = 1

Let $V(H_n \odot S_1) = \{u_i^{(j)} : 1 \le i \le n$ and $1 \le j \le 2\} \cup \{v_i^{(j)} : 1 \le i \le n$ and $1 \le j \le 2\}$ and $E_1 = \{(u_i^{(j)} u_{i+1}^{(j)}) : 1 \le i \le n$ and $1 \le j \le 2\} \cup \{(u_i^{(j)} v_i^{(j)}) : 1 \le i \le n$ and $1 \le j \le 2\}$ $E(H_n \odot S_1) = E_1 \cup \{(u_{\frac{n+1}{2}}^{(1)} u_{\frac{n+1}{2}}^{(2)})\}$ when n is odd and $E(H_n \odot S_1) = E_1 \cup \{(u_{\frac{n}{2}+1}^{(1)} u_{\frac{n}{2}}^{(2)})\}$ when n is even.

Define $f : V(H_n \odot S_1) \to \{0, 1, 2, \ldots, q = 4n-1\}$ by

$$f(u_{2i-1}^{(1)}) = 4(i-1), \quad 1 \le i \le \frac{n+1}{2}$$

$$f(u_{2i}^{(1)}) = 4i-1, \quad 1 \le i \le \frac{n-1}{2}$$

$$f(u_{2i-1}^{(2)}) = 2n + 4i - 3, \quad 1 \le i \le \frac{n+1}{2}$$

$$f(u_{2i}^{(2)}) = 2n + 4i - 2, \quad 1 \le i \le \frac{n-1}{2}$$

$$f(v_{2i-1}^{(1)}) = 4i-3, \quad 1 \le i \le \frac{n+1}{2}$$

$$f(v_{2i}^{(1)}) = 4i-2, \quad 1 \le i \le \frac{n-1}{2}$$

$$f(v_{2i-1}^{(2)}) = 2n + 4i - 4, \quad 1 \le i \le \frac{n+1}{2}$$

$$f(v_{2i}^{(2)}) = 2n + 4i - 1, \quad 1 \le i \le \frac{n-1}{2}$$

The labels of the edges are as follows :

$$f(u_i^{(1)} u_{i+1}^{(1)}) = 4i-1, \quad 1 \le i \le n-1$$

$$f(u_i^{(2)} u_{i+1}^{(2)}) = 4n+4i-1, \quad 1 \le i \le n-1$$

$$f(u_{\frac{n+1}{2}}^{(1)} u_{\frac{n+1}{2}}^{(2)}) = 4n-1 \text{ when } n \equiv 1 \pmod 2$$

$$f(u_{\frac{n}{2}+1}^{(1)} u_{\frac{n}{2}}^{(2)}) = 4n-1 \text{ when } n \equiv 0 \pmod 2$$

$$f(u_i^{(1)} v_i^{(1)}) = 4i-3, \quad 1 \le i \le n$$

$$f(u_i^{(1)} v_i^{(1)}) = 4n + 4i - 3, \quad 1 \le i \le n$$

Now, $f(E(G)) = \{3, 7, \ldots, 4(n-1)-1\} \cup \{4(n+1)-1, 4(n+2)-1, \ldots, 4(n+n-1)-1\} \cup \{4n-1\} \cup \{1, 5, 9, \ldots, 4n-3\} \cup \{4(n+1)-3, 4(n+2)-3, \ldots, 4(n+n)-3\}$

$= \{1, 3, 5, 7, \ldots, 4n-5\} \cup \{4n+3, 4n+5, \ldots, 8n-5\} \cup \{4n-1\} \cup \{1, 5, 9, \ldots, 4n-3\} \cup \{4n+1, 4n+5, \ldots, 8n-3\}$.

$= \{1, 3, 5, 7, \ldots, 4n-5, 4n-3, 4n-1, 4n+1, 4n+3, 4n+5, \ldots, 8n-5, 8n-3\} = \{1, 3, 5, \ldots, 2q-1\}$.

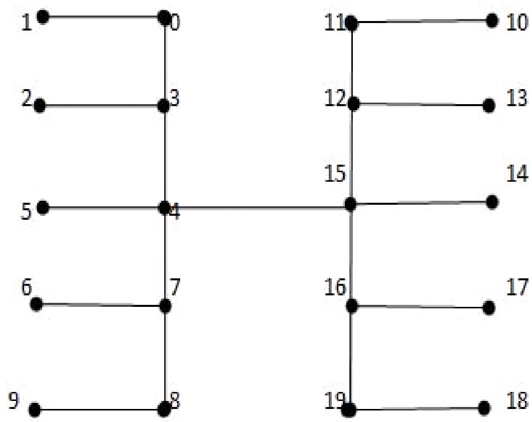

Fig. 3.2 (a) $H_5 \odot K_1$

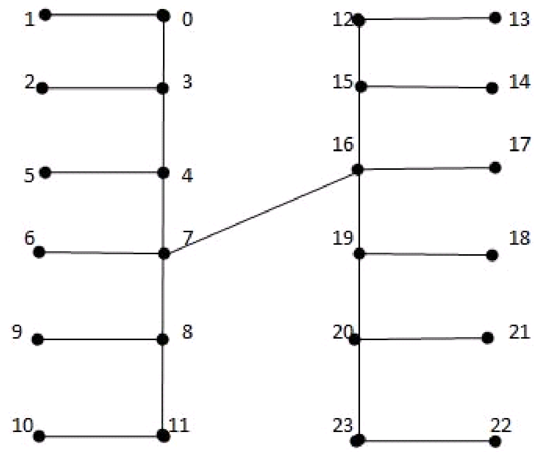

Fig. 3.2 (b) $H_6 \odot K_1$

Case (ii) : When m = 2

Let $V(H_n \odot S_2) = \{u_i^{(j)}, v_i^{(j)}, w_i^{(j)} : 1 \le i \le n \text{ and } 1 \le j \le 2\}$ and $E_1 = \{(u_i^{(j)} u_{i+1}^{(j)}) : 1 \le i \le n \text{ and } 1 \le j \le 2\} \cup \{(u_i^{(j)} v_i^{(j)}) : 1 \le i \le n \text{ and } 1 \le j \le 2\} \cup \{(u_i^{(j)} w_i^{(j)}) : 1 \le i \le n$ and $1 \le j \le 2$. $E(H_n \odot S_2) = E_1 \cup \{(u_{\frac{n+1}{2}}^{(1)} u_{\frac{n+1}{2}}^{(2)})\}$ when n is odd and $E(H_n \odot S_2) = E_1 \cup \{(u_{\frac{n}{2}+1}^{(1)} u_{\frac{n}{2}}^{(2)})\}$ when n is even.

Define $f : V(H_n \odot S_2) \to \{0, 1, 2, \ldots, q = 6n - 1\}$ by

$f(u_i^{(1)}) = 3i - 2, \quad 1 \le i \le n$

$f(u_i^{(2)}) = 3(n + i - 1) + 1, \quad 1 \le i \le n$

$f(v_i^{(1)}) = 3i - 3, \quad 1 \le i \le n$

$f(v_i^{(2)}) = 3(n + i - 1), \quad 1 \le i \le n$

$f(w_i^{(1)}) = 3i - 1, \quad 1 \le i \le n$

$f(w_i^{(2)}) = 3(n + i - 1) + 2, \quad 1 \le i \le n$

The labels of the edges are as follows :

$f(u_i^{(1)} u_{i+1}^{(1)}) = 6i - 1, \quad 1 \le i \le n-1$

$f(u_i^{(2)} u_{i+1}^{(2)}) = 6(n + i) - 1, \quad 1 \le i \le n-1$

$f(u_i^{(1)} v_i^{(1)}) = 6i - 5, \quad 1 \le i \le n$

$f(u_i^{(2)} v_i^{(2)}) = 6(n + i) - 5, \quad 1 \le i \le n$

$f(u_i^{(1)} w_i^{(1)}) = 6i - 3, \quad 1 \le i \le n$

$f(u_i^{(2)} w_i^{(2)}) = 6(n + i) - 3, \quad 1 \le i \le n$

$f(u^{(1)}_{\frac{n+1}{2}} u^{(2)}_{\frac{n+1}{2}}) = 6n-1$ when $n \equiv 1 \pmod 2$

$f(u^{(1)}_{\frac{n}{2}+1} u^{(2)}_{\frac{n}{2}}) = 6n-1$ when $n \equiv 0 \pmod 2$

Now, $f(E(G)) = \{5, 11, \dots, 6(n-1)-1\} \cup \{6(n+1)-1, 6(n+2)-1, \dots, 6(n+n-1)-1\} \cup \{1, 7, \dots, 6n-5\} \cup \{6(n+1)-5, 6(n+2)-5, \dots, 6(2n)-5\} \cup \{3, 9, \dots, 6n-3\} \cup \{6(n+1)-3, 6(n+2)-3, \dots, 6(2n)-3\} \cup \{6n-1\}$

$= \{5, 11, \dots, 6n-7\} \cup \{6n+5, 6n+11, \dots, 12n-7\} \cup \{1, 7, \dots, 6n-5\} \cup \{6n+1, 6n+7, \dots, 12n-5\} \cup \{3, 9, \dots, 6n-9, 6n-3\} \cup \{6n+3, 6n+9, \dots, 12n-3\} \cup \{6n-1\}.$

$= \{1, 3, 5, 7, \dots, 6n-7, 6n-5, 6n-3, 6n-1, 6n+1, 6n+3, 6n+5, 6n+7, 6n+9, 6n+11, \dots, 12n-7, 12n-5, 12n-3\} = \{1, 3, 5, \dots, 2q-1\}.$

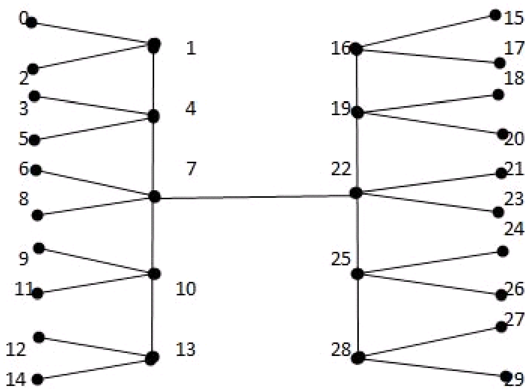

Fig. 3.3 (a) $H_5 \odot S_2$

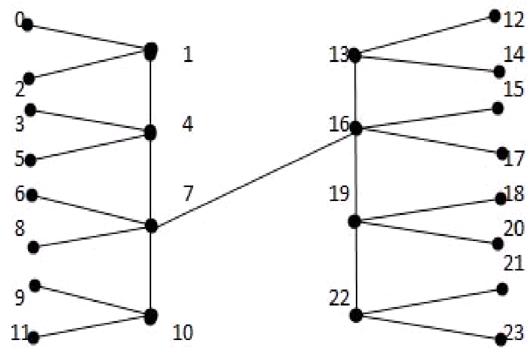

Fig. 3.3 (b) $H_4 \odot S_2$

Case (iii) : When $m \geq 3$ and $n \equiv 1 \pmod 2$

Let $V(H_n \odot S_m) = \{u_i^{(j)} : 1 \leq i \leq n$ and $1 \leq j \leq 2\} \cup \{v_{i_k}^{(j)} : 1 \leq k \leq m, 1 \leq i \leq n$ and $1 \leq j \leq 2\}$ and $E(H_n \odot S_m) = \{(u_i^{(j)} u_{i+1}^{(j)}) : 1 \leq i \leq n-1$ and $1 \leq j \leq 2\} \cup \{(u_i^{(j)} v_{i_k}^{(j)}) : 1 \leq i \leq n, 1 \leq j \leq 2$ and $1 \leq k \leq m\} \cup \{(u^{(1)}_{\frac{n+1}{2}} u^{(2)}_{\frac{n+1}{2}})\}$.

Define $f : V(H_n \odot S_m) \rightarrow \{0, 1, 2, \dots, q = 2n(m+1)-1\}$ by

$f(u^{(1)}_{2i-1}) = 2(m+1)(i-1)+1, \quad 1 \leq i \leq \frac{n+1}{2}$

$f(u^{(1)}_{2i}) = (m+1)(2i-1)+(m-1), \quad 1 \leq i \leq \frac{n-1}{2}$

$f(u^{(2)}_{2i-1}) = (m+1)(n+2i-2)+(m-1), \quad 1 \leq i \leq \frac{n+1}{2}$

$f(u^{(2)}_{2i}) = (m+1)(n+2i-1)+1, \quad 1 \leq i \leq \frac{n-1}{2}$

For $1 \le i \le \frac{n+1}{2}$, $f(u^{(1)}_{(2i-1)_k}) = 2(m + 1)(i - 1) + 2(k - 1)$, $1 \le k \le m$

For $1 \le i \le \frac{n-1}{2}$, $f(u^{(1)}_{(2i)_k}) = 2(m + 1)(i - 1) + (2k + 1)$, $1 \le k \le m$

For $1 \le i \le \frac{n+1}{2}$, $f(u^{(2)}_{(2i-1)_k}) = (m + 1)(n + 2i - 3) + (2k + 1)$, $1 \le k \le m$

For $1 \le i \le \frac{n-1}{2}$, $f(u^{(2)}_{(2i)_k}) = (m+1)(n + 2i - 1) + 2(k - 1)$, $1 \le k \le m$

The labels of the edges are as follows :

$$f(u^{(1)}_{i}u^{(1)}_{i+1}) = 2(m + 1)(i - 1) + (2m + 1), \quad 1 \le i \le n - 1$$

$$f(u^{(2)}_{i}u^{(2)}_{i+1}) = 2(m + 1)(n+i-1)+(2m + 1), \quad 1\le i \le n - 1$$

For $1 \le i \le n$,

$$f(u^{(1)}_{i}u^{(1)}_{i_k}) = 2(m + 1)(i - 1) + (2k - 1), \quad 1 \le k \le m$$

$$f(u^{(2)}_{i}u^{(2)}_{i_k}) = (m + 1)(n + 2i + 1) + (2k - 1), \quad 1 \le k \le m$$

$$f(u^{(1)}_{\frac{n+1}{2}}\, u^{(2)}_{\frac{n+1}{2}}) = 2n(m + 1) - 1$$

Now, $f(E(G)) = \{2m + 1, 2(m + 1) + 2m + 1, 4(m + 1) + 2m + 1, \ldots, 2(m + 1)(n - 2) + 2m + 1\} \cup \{2n(m + 1) + 2m + 1, 2(n + 1)(m + 1) + 2m + 1, \ldots, 2(m + 1)(2n - 2) + 2m + 1\} \cup \{1, 3, 5, \ldots, 2m - 1, 2(m + 1) + 1, 2(m + 1) + 3, \ldots, 2(m + 1) + 2m - 1, \ldots, 2(m + 1)(n - 1) + 1, 2(m + 1)(n - 1) + 3, \ldots, 2(m + 1)(n - 1) + 2m - 1\} \cup \{2n(m + 1) + 1, 2n(m + 1) + 3, \ldots, 2(m + 1)(n + 1) + 2m - 1, \ldots, 2(m + 1)(2n - 1) + 1, 2(m + 1)(2n - 1) + 3, \ldots, 2(m + 1)(2n - 1) + 2m - 1\} \cup \{2n(m + 1) - 1\}$.

$= \{2m + 1, 4m + 3, 6m + 5, \ldots, 2mn + 2n - 3\} \cup \{2mn + 2n + 2m + 1, 2mn + 2n + 4m + 3, \ldots, 4mn + 4n - 2m - 3\} \cup \{1, 3, 5, \ldots, 2m - 1, 2m + 3, 2m + 5, \ldots, 4m + 1, \ldots, 2mn - 2m + 2n - 1, 2mn - 2m + 2n + 1, \ldots, 2mn + 2n - 3\} \cup \{2mn + 2n + 1, 2mn + 2n + 3, \ldots, 2mn + 2m + 2n - 1, 2mn + 2m + 2n + 3, 2mn + 2m + 2n + 5, \ldots, 2mn + 2n + 4m - 1, 4mn - 2m + 4n - 1, 4mn - 2m + 4n + 1, \ldots, 4mn + 4n - 3\} \cup \{2mn + 2n - 1\}$.

$= \{1, 3, 5, \ldots, 2m - 1, 2m + 1, 2m + 3, 2m + 5, \ldots, 4m + 1, 4m + 3, \ldots, 6m + 5, \ldots, 2mn - 2m + 2n - 1, 2mn - 2m + 2n + 1, \ldots, 2mn + 2n - 3, 2mn + 2n - 1, 2mn + 2n + 1, 2mn + 2n + 3, \ldots, 2mn + 2m + 2n - 1, 2mn + 2n + 2m + 1, \ldots, 2mn + 2m + 2n + 3, 2mn + 2m + 2n + 5, \ldots, 2mn + 4m + 2n + 1, 2mn + 4m + 2n + 3, \ldots, 4mn - 2m + 4n - 3, 4mn - 2m + 4n - 1, 4mn - 2m + 4m + 1, \ldots, 4mn + 4n - 3\}$.

$= \{1, 3, 5, \ldots, 2m - 1, 2m + 1, 2m + 3, 2m + 5, \ldots, 4m + 1, 4m + 3, \ldots, 6m + 5, \ldots, 2n(m + 1) - 2m - 1, 2n(m + 1) - 2m + 1, \ldots, 2n(m + 1) - 3, 2n(m + 1) - 1, 2n(m + 1) + 1, 2n(m + 1) + 3, \ldots, 2n(m + 1) + 2m - 1, 2n(m + 1) + 2m + 1, 2n(m + 1) + 2m + 3, 2n(m + 1) + 2m + 5, \ldots, 2n(m + 1) + 4m + 1, 2n(m + 1) + 4m + 3, \ldots, 4n(m + 1) - 2m - 3, 4n(m + 1) - 2m - 1, 4n(m + 1) - 2m + 1, \ldots, 4n(m + 1) - 3\} = \{1, 3, 5, \ldots, 2q - 1\}$.

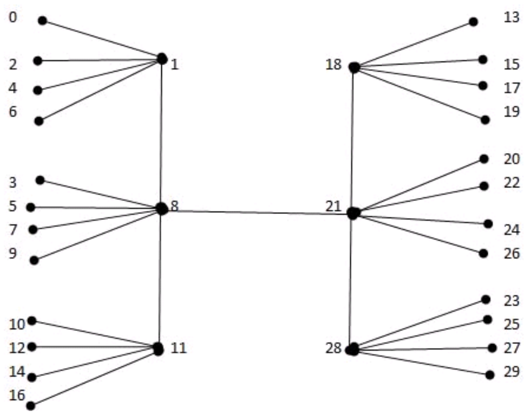

Fig. 3.4 $H_3 \odot S_4$

Case (iv) :When $m \geq 3$ and $n \equiv 0 \pmod 2$)

Let $V(H_n \odot S_m) = \{u_i^{(j)} : 1 \leq i \leq n$ and $1 \leq j \leq 2\} \cup \{v_{i_k}^{(j)} : 1 \leq k \leq m, 1 \leq i \leq n$ and $1 \leq j \leq 2\}$ and $E(H_n \odot S_m) = \{(u_i^{(j)} u_{i+1}^{(j)})$: $1 \leq i \leq n-1$ and $1 \leq j \leq 2\} \cup \{(u_i^{(j)} v_{i_k}^{(j)})$: $1 \leq i \leq n$, $1 \leq j \leq 2$ and $1 \leq k \leq m\} \cup \{(u^{(1)}_{\frac{n}{2}+1} u^{(2)}_{\frac{n}{2}})\}$.

Define $f : V(H_n \odot S_m) \to \{0, 1, 2, \ldots, q = 2n(m+1) - 1\}$ by

$f(u^{(1)}_{2i-1}) = 2(m+1)(i-1) + 1, \; 1 \leq i \leq \frac{n+1}{2}$

$f(u^{(1)}_{2i}) = (m+1)(2i-1) + (m-1), \quad 1 \leq i \leq \frac{n-1}{2}$

$f(u^{(2)}_{2i-1}) = (m+1)(n+2i-2) + 1, \; 1 \leq i \leq \frac{n+1}{2}$

$f(u^{(2)}_{2i}) = (m+1)(n+2i-1) + (m-1),$

$1 \leq i \leq \frac{n-1}{2}$

For $1 \leq i \leq \frac{n+1}{2}$, $f(u^{(1)}_{(2i-1)_k}) = 2(m+1)(i-1) + 2(k-1), \quad 1 \leq k \leq m$

For $1 \leq i \leq \frac{n-1}{2}$, $f(u^{(1)}_{(2i)_k}) = 2(m+1)(i-1) + (2k+1), \quad 1 \leq k \leq m$

For $1 \leq i \leq \frac{n+1}{2}$, $f(u^{(2)}_{(2i-1)_k}) = (m+1)(n+2i-2) + 2(k-1), \quad 1 \leq k \leq m$

For $1 \leq i \leq \frac{n-1}{2}$, $f(u^{(2)}_{(2i)_k}) = (m+1)(n+2i-2) + (2k+1), \quad 1 \leq k \leq m$

The labels of the edges are as follows :

$f(u_i^{(1)} u_{i+1}^{(1)}) = 2(m+1)(i-1) + (2m+1),$

$1 \leq i \leq n-1$

$f(u_i^{(2)} u_{i+1}^{(2)}) = 2(m+1)(n+i-1) + (2m+1),$

$1 \leq i \leq n-1$

For $1 \leq i \leq n$,

$f(u_i^{(1)} u_{i_k}^{(1)}) = 2(m+1)(i-1) + (2k-1),$

$1 \leq k \leq m$

$f(u_i^{(2)} u_{i_k}^{(2)}) = (m+1)(n+i+1) + (2k-1),$

$1 \leq k \leq m$

$f(u^{(1)}_{\frac{n+1}{2}} u^{(2)}_{\frac{n+1}{2}}) = 2n(m+1) - 1$

Now, $f(E(G)) = \{2m+1, 2(m+1) + 2m+1, 4(m+1) + 2m+1, \ldots, 2(m+1)(n-2) + 2m + 1\} \cup \{2n(m+1) + 2m+1, 2(n+1)(m+1) + 2m+1, \ldots, 2(m+1)(2n-2) + 2m+1\} \cup \{1, 3, 5, \ldots, 2m-1, 2(m+1)+1, 2(m+1) + 3, \ldots, 2(m+1) + 2m-1, \ldots, 2(m+1)(n-1) + 1, 2(m+1)(n-1) + 3, \ldots, 2(m+1)(n-1) + 2m-1\} \cup \{2n(m+1)+1, 2n(m+1) + 3, \ldots, 2(m+1)(n+1) + 2m-1, \ldots, 2(m+1)(2n-1) + 1, 2(m+1)(2n-1) + 3, \ldots,$

$2(m + 1) (2n – 1)+2m – 1\} \cup \{2n(m + 1) – 1\}$.

$= \{2m + 1, 4m +3, 6m + 5, \ldots, 2mn + 2n – 3\} \cup \{2mn + 2n + 2m + 1, 2mn + 2n+4m+3, \ldots, 4mn + 4n – 2m – 3\} \cup \{1, 3, 5, \ldots, 2m – 1, 2m + 3, 2m + 5, \ldots, 4m + 1, \ldots, 2mn – 2m + 2n – 1, 2mn – 2m + 2n + 1, \ldots, 2mn + 2n – 3\} \cup \{2mn + 2n + 1, 2mn + 2n + 3, \ldots, 2mn + 2m + 2n – 1, 2mn + 2m + 2n + 3, 2mn + 2m + 2n + 5, \ldots, 2mn + 2n + 4m – 1, 4mn – 2m + 4n – 1, 4mn – 2m + 4n + 1, \ldots, 4mn + 4n – 3\} \cup \{2mn + 2n – 1\}$.

$= \{1, 3, 5, \ldots, 2m – 1, 2m + 1, 2m + 3, 2m + 5, \ldots, 4m + 1, 4m + 3, \ldots, 6m + 5, \ldots, 2mn – 2m + 2n – 1, 2mn – 2m + 2n + 1, \ldots, 2mn + 2n – 3, 2mn + 2n – 1, 2mn + 2n + 1, 2mn + 2n + 3, \ldots, 2mn + 2m + 2n – 1, 2mn + 2n + 2m + 1, \ldots, 2mn + 2m + 2n + 3, 2mn + 2m + 2n + 5, \ldots, 2mn + 4m + 2n + 1, 2mn + 4m + 2n + 3, \ldots, 4mn – 2m + 4n – 3, 4mn – 2m + 4n – 1, 4mn – 2m + 4m + 1, \ldots, 4mn + 4n – 3\}$.

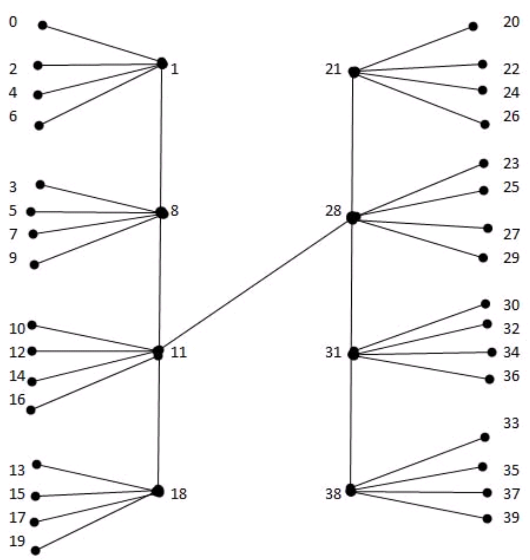

Fig. 3.5 $H_4 \odot S_4$

$= \{1, 3, 5, \ldots, 2m – 1, 2m + 1, 2m + 3, 2m + 5, \ldots, 4m + 1, 4m + 3, \ldots, 6m + 5, \ldots, 2n(m + 1) – 2m – 1, 2n(m + 1) – 2m + 1, \ldots, 2n (m + 1) – 3, 2n(m + 1) – 1, 2n(m + 1) + 1, 2n(m + 1) + 3, \ldots, 2n(m + 1) + 2m – 1, 2n(m + 1) + 2m + 1, 2n(m + 1) + 2m + 3, 2n(m + 1) + 2m + 5, \ldots, 2n(m + 1) + 4m + 1, 2n(m + 1) + 4m + 3, \ldots, 4n(m + 1) – 2m – 3, 4n(m + 1) – 2m – 1, 4n(m + 1) – 2m + 1, \ldots, 4n(m + 1) – 3\} = \{1, 3, 5, \ldots, 2q – 1\}$.

In all the above cases, the edge values are distinct and odd and hence G admits odd edge labeling. Therefore by 2.3, $H_n \odot S_m$ is a felicitous graph for any m.

4. References

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