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A new algorithmic approach on multi-objective fuzzy transshipment problem

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Abstract

In this paper, Multi-objective Transportation Problem in fuzzy environment is extended to Multi-objective transshipment problem. A new fuzzy blocking algorithm and fuzzy blocking zero point algorithm are used to obtain an optimal fuzzy time and optimal fuzzy cost of the Multi-objective fuzzy transshipment problem. These algorithms are useful to multi-objective nature of fuzzy transshipment problem.

Key words: Fuzzy Transportation problem, Fuzzy time Transshipment problem, Multi-objective Fuzzy Transshipment problem

Subject Classification: 03E72.

1. Introduction

A transportation problem allows only shipments that go directly from a supply point to a demand point. In many situations, shipments are allowed between supply points or between demand points. Sometimes there may also be points (called transshipment points) through which goods can be transshipped on their journey from a supply point to a demand point. Shipping problems with any or all of these characteristics are transshipment problems. Fortunately, the optimal solution to a transshipment problem can be found by solving a transportation problem. In what follows, we define a supply point to be a point that can send goods to another point but cannot receive goods from any other point. Similarly, a demand point is a point that can receive goods from other points but cannot send goods to any other point. A transshipment point is a point that can both receive goods from other points and send goods to other points[1-4].

The time-minimizing transportation problem is a special case of a transportation problem in which a time is associated with each shipping route. Rather than minimizing cost, the objective is to minimize the maximum time to transport all supply to the destinations. In a Time Transportation Problem, the time of the transporting items from origins to destinations is minimized, satisfying certain conditions in respect of availabilities at sources and requirements at the destinations.

Multi-objective transportation is a kind of a bicriteria transportation problem. Multi-objective transportation problem which had been proposed and also, solved by Aneja and Nair[1] and until, now many researchers[6,11] also, have great interest in this problem , and some method used their special techniques in finding the solutions for two objective functions approximately approaching to the ideal solution. The concept of fuzzy set was first introduced and investigated by Zadeh[12] and fuzzy numbers and arithmetic operations with these numbers introduced by Bellman & Zadeh and Kaufmann[2,5]. Nagoor Gani[6] *et al*. solving transportation problem using fuzzy number and Nagoor Gani[8] *et al.* solved the transshipment problem in fuzzy environment.

In this paper, Section 2 deals with some preliminary definition for the fuzzy concept, Section 3 explain about the Multi-objective Fuzzy Transshipment problem and how to convert it into multi-objective fuzzy transportation problem, Section 4 gives the Algorithms to solve the above Multi- objective fuzzy transshipment problem and in Section 5 a detail Numerical problem is discussed.

2. Preliminaries:

2.1 Fuzzy set: A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = \{(x, \mu_A(x)) : x \in A, {}_A(x) \in [0,1]\}$. In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval [0, 1], called Membership function[9-10].

2.2 Fuzzy Number: A fuzzy set $\widetilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number,

(i) $\widetilde{A}$ must be a normal fuzzy set;

(ii) ${}^{\alpha}\widetilde{A}$ must be closed interval for every $\alpha \in [0,1]$

(iii) the support of $\widetilde{A}$, ${}^{0+}\widetilde{A}$, must be bounded.

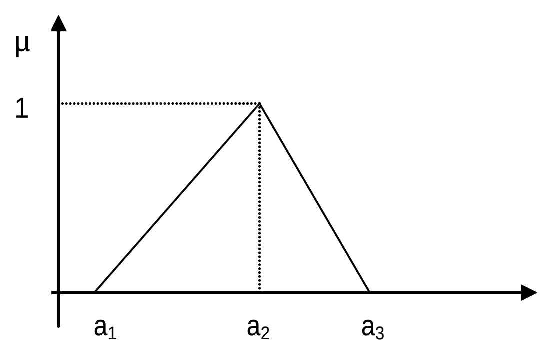
2.3 Triangular Fuzzy Number: It is a fuzzy number represented with three points as follows: $\widetilde{A} = (a_1, a_2, a_3)$. This representation is interpreted as membership functions and holds the following conditions

(i) a_1 to a_2 is increasing function

(ii) a_2 to a_3 is decreasing function

(iii) $a_1 \le a_2 \le a_3$.

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & for\ a_2 \le x \le a_3 \\ 0 & for\ x > a_3 \end{cases}$$



Triangular fuzzy number $\tilde{A}$ = (a_1, a_2, a_3)

2.4 Function Principle Operation of Triangular Fuzzy Number :

The following are the operations that can be performed on triangular fuzzy numbers:

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then,

(i) Addition: $\tilde{A} + \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$.

(ii) Subtraction: $\tilde{A} - \tilde{B} = (a_1b_3, a_2b_2, a_3b_1)$.

(iii) Multiplication: $\tilde{A} \times \tilde{B} =$ $(\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$

2.5 Ranking the fuzzy numbers :

Take $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$.

Calculate $D(\tilde{A}, M) = M - \dfrac{a_1 + 2a_2 + a_3}{4}$ &

$D(\tilde{B}, M) = M - \dfrac{b_1 + 2b_2 + b_3}{4}$

If $D(\tilde{A}, M) < D(\tilde{B}, M)$ then $\tilde{A}$ is greater than $\tilde{B}$

If $D(\tilde{A}, M) = D(\tilde{B}, M)$ then $\tilde{A}$ is equal to $\tilde{B}$

If $D(\tilde{A}, M) > D(\tilde{B}, M)$ then $\tilde{A}$ is less than $\tilde{B}$

2.6 Graded Mean Integration Method:

The graded mean integration method is used to defuzzify the triangular fuzzy number. the representation of triangular fuzzy number is $\tilde{A} = (a_1,a_2,a_3)$ and its defuzzified value is obtained by $A = \dfrac{a_1 + 4a_2 + a_3}{6}$.

3. Multi-objective Fuzzy Transshipment Problem:

The Multi-objective fuzzy transshipment problem may be written as:

$$\text{Minimize } \tilde{Z}_1 = \left[\sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} \tilde{c}_{ij}\tilde{x}_{ij}\right]$$

$$\text{Minimize } \tilde{Z}_2 = \left[Maximize\ \tilde{t}_{ij}/\tilde{x}_{ij} > 0\right]$$

$$\text{Subject to } \sum_{j=1, j\neq i}^{m+n} \tilde{x}_{ij} - \sum_{j=1, j\neq i}^{m+n} \tilde{x}_{ji} = \tilde{a}_i, \quad i = 1,2,3,\dots,m$$

$$\sum_{i=1, i\neq j}^{m+n} \tilde{x}_{ij} - \sum_{i=1, i\neq j}^{m+n} \tilde{x}_{ji} = \tilde{b}_j, \quad j = m+1, m+2, m+3, \dots, m+n$$

where $\tilde{x}_{ij} \geq 0$, i, j=1,2,3,…,m+n, j≠i.

the Multi-objective fuzzy transshipment model is transformed to Multi-objective transportation form as:

$$\text{Minimize } \tilde{Z}_1 = \left[\sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} \tilde{c}_{ij}\tilde{x}_{ij}\right]$$

$$\text{Minimize } \tilde{Z}_2 = \left[Maximize\ \tilde{t}_{ij}/\tilde{x}_{ij} > 0\right]$$

Subject to $\sum_{j=1}^{m+n} \tilde{x}_{ij} = \tilde{a}_i + \tilde{T}$, i=1,2,3,…,m

$\sum_{j=1}^{m+n} \tilde{x}_{ij} = \tilde{T}$, i=m+1,m+2,m+3,…,m+n

$\sum_{i=1}^{m+n} \tilde{x}_{ij} = \tilde{T}$, j=1,2,3,…,m

$\sum_{i=1}^{m+n} \tilde{x}_{ij} = \tilde{b}_j + \tilde{T}$, j=m+1,m+2,m+3,…,m+n

where $\tilde{x}_{ij} \geq 0$, i, j=1,2,3,…,m+n, j≠i,

the above mathematical model represents a standard balanced Multi-objective fuzzy transportation problem with (m+n) origins and (m+n) destinations. $\tilde{T}$ can be interpreted as a buffer stock at each origin and destination.

Since we assume that any amount of goods can be transshipped at each point, $\widetilde{T}$ should be large enough to take care of all transshipments. It is clear that the volume of good a transshipped at any point cannot exceed the amount produced or received and hence we take $\widetilde{T} = \sum_{i=1}^{m} \widetilde{a}_i$ or $\sum_{j=1}^{m} \widetilde{b}_j$.

4. Algorithms:

The objectives of the Multi-objective fuzzy transshipment problem have been solved one by one using fuzzy blocking algorithm and fuzzy blocking zero point algorithm. In fuzzy blocking algorithm we solve time objective and find the minimum fuzzy time requirement ($\widetilde{T}_0$) for the transportation have been obtained. Then we solve the cost objective of the problem using fuzzy zero point algorithm here we find the minimum fuzzy cost of transportation and also we obtain the minimum fuzzy time ($\widetilde{T}_M$) for the corresponding transportation. Now we have two minimum fuzzy time for the multi-objective fuzzy transportation problem, one is purely depend on fuzzy time problem and another one is depend on fuzzy cost problem. Now form the set of fuzzy times ($\widetilde{M} = [\widetilde{T}_0 , \widetilde{T}_m]$) between the obtained two minimum fuzzy times and solve the fuzzy cost problems corresponding to the obtained fuzzy times using fuzzy blocking zero point algorithm.

4.1 Fuzzy Blocking Algorithm:

Step 1: Find the maximum of the minimum of each row and column of the transformed fuzzy time transportation table and (didn't consider the diagonal fuzzy zeros). Say, $\widetilde{T}$

Step 2: Construct a reduced fuzzy time transportation table from the given table by blocking all cells having fuzzy time more than $\widetilde{T}$

Step 3: Check if each column demand is less than to sum of the supplies in the reduced fuzzy time transportation table obtained from the step 2., Also, check if each row supply is less than to sum of the column demands in the reduced fuzzy time transportation table obtained from the step 2. Don't consider the diagonal cells. If so, go to step 6. (such reduced fuzzy time transportation table is called the active fuzzy time transportation table). If not, go to step 4.

Step 4: Find a fuzzy time which is immediately next to the time $\widetilde{T}$. Say $\widetilde{U}$.

Step 5: Construct again a active fuzzy time transportation table from the fuzzy time transportation table by blocking all cells having time more than $\widetilde{U}$ and then, go to the step 3.

Step 6: Defuzzify all the fuzzy time and do allocation according to the following rules (now onwards consider the diagonal zeros):

a) Allot the maximum possible to a cell which is only one cell in the row/column. Then repeat the process till all allocations are completed.

Step 7: If (a) is not possible, select a row /a column having minimum number of unblocked cell and allot maximum possible to cell which helps to reduce the Fuzzy Buffer supply.

Step 8: Find a time which is maximum in the allotted cell and then take the fuzzy time in the same cell in the active fuzzy time transportation table (say $\tilde{T}_0$).

4.2 The Fuzzy Zero Point Algorithm :

Step 1: Construct the Multi-objective fuzzy transportation table from the Multi-objective fuzzy transshipment problem and then, convert it into a balanced one, if it is not. And defuzzify all the cost and form a new table and then goto step 2.

Step 2: Subtract the row minimum of the transformed Multi-objective fuzzy transportation table from each row entries. Don't consider the diagonal zeros till allotment table formed.

Step 3: Subtract the column minimum of the resulting Multi-objective fuzzy transportation table after using the Step 2, from each column entries.

Step 4: Check if each column fuzzy demand is less than to the sum of the fuzzy supplies whose reduced costs in that columns are zero. Also, check if each row fuzzy supply is less than to sum of the column fuzzy demands whose reduced costs in that row are zero. If so, go to Step 7. (Such reduced table is called the allotment table). If not, go to Step 5.

Step 5: Draw the minimum number of horizontal lines and vertical lines to cover all the zeros of the reduced Multi-objective fuzzy transportation table such that some entries of row(s) or/and column(s) which do not satisfy the condition of the Step 4, are not covered.

Step 6: Develop the new revised reduced Multi-objective fuzzy transportation table as follows:
(i) Find the smallest entry of the reduced cost matrix not covered by any lines.
(ii) Subtract this entry from all the uncovered entries and add the same to all entries lying at the intersection of any two lines.
and then, go to Step 4.

Step 7: Select a row /a column having minimum number of zeros and allot maximum possible to cell which helps to reduce the Fuzzy Buffer supply.

Step 8: Reform the reduced Multi-objective fuzzy transportation table after deleting the fully used fuzzy supply points and the fully received fuzzy demand points and also, modify it to include the not fully used fuzzy supply points and the not fully received fuzzy demand points.

Step 9: Repeat Step 7 to Step 8 until all fuzzy supply points are fully used and all fuzzy demand points are fully received.

Step10: Finally select the fuzzy cost corresponding to allotted cell in the transformed multi-objective fuzzy transportation table.

4.3 Fuzzy Blocking Zero Point Algorithm:

Step 1: Construct the Fuzzy Time transportation problem from the transformed Multi-objective Fuzzy Transportation Problem.

Step 2: Solve the Fuzzy time transportation problem by the Fuzzy blocking algorithm. Let the optimal solution be $\tilde{T}_0$.

Step 3: Construct the fuzzy cost transportation problem from the transformed Multi-objective fuzzy transportation problem.

Step 4: Solve the Fuzzy cost transportation problem by the Fuzzy zero point algorithm and also, find the corresponding time of transportation. Let it be $\tilde{T}_m$.

Step 5: Construct the active Fuzzy cost transportation problem for each time in $\tilde{M} = [\tilde{T}_0, \tilde{T}_m]$ and block the cells which have the fuzzy time more than $\tilde{M}$ and solve the active fuzzy cost transportation problem by using fuzzy zero point algorithm.

Step 6: For each time $\tilde{M}$, an optimal solution to the fuzzy cost transportation problem, $\tilde{X}$ is obtained from the Step 5 . Then, the vector $(\tilde{X}, \tilde{M})$ is an efficient solution to FCTP.

5. Numerical Example:

Problem: Consider the following transformed Multi-objective fuzzy transportation problem. The upper left corner in each cell gives the Fuzzy time of transportation on the corresponding route and the lower right corner in each cell gives the unit transportation cost per unit on that route.

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1	(0,0,0)	(30,40,50) (2,3,4)	(9,10,11) (13,15,17)	(11,12,13) (7,9,11)	(23,25,27) (6,8,10)	(15,24,33)
O_2	(40,50,60) (3,4,5)	(0,0,0)	(11,12,13) (5,7,9)	(13,14,15) (7,9,11)	(33,35,37) (6,8,10)	(12,21,30)
D_{2+1}	(11,12,13) (14,16,18)	(11,12,13) (2,4,6)	(0,0,0)	(31,33,35) (4,6,8)	(9,10,11) (3,5,7)	(9,15,21)
D_{2+2}	(23,25,27) (5,7,9)	(11,12,13) (4,5,6)	(9,10,11) (1,2,3)	(0,0,0)	(11,12,13) (1,2,3)	(9,15,21)
D_{2+3}	(33,35,37) (6,7,8)	(43,45,47) (3,5,7)	(23,25,27) (1,2,3)	(11,12,13) (3,5,7)	(0,0,0)	(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30)	(10,18,26)	(12,20,28)	

Now the transformed fuzzy Time Transportation Problem is given below:

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1	(0,0,0)	(30,40,50)	(9,10,11)	(11,12,13)	(23,25,27)	(15,24,33)
O_2	(40,50,60)	(0,0,0)	(11,12,13)	(13,14,15)	(33,35,37)	(12,21,30)
D_{2+1}	(11,12,13)	(11,12,13)	(0,0,0)	(31,33,35)	(9,10,11)	(9,15,21)
D_{2+2}	(23,25,27)	(11,12,13)	(9,10,11)	(0,0,0)	(11,12,13)	(9,15,21)
D_{2+3}	(33,35,37)	(43,45,47)	(23,25,27)	(11,12,13)	(0,0,0)	(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30)	(10,18,26)	(12,20,28)	

Now, applying Fuzzy Blocking algorithm. The Maximum of the minimum of each row and the minimum of each column =(11,12,13)

Now, using the step 1 to the step 5., we have the following complete allocation table:

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1	-		(9,10,11)	(11,12,13)		(15,24,33)
O_2		-	(11,12,13)			(12,21,30)
D_{2+1}	(11,12,13)	(10,12,14)	-		(9,10,11)	(9,15,21)
D_{2+2}		(11,12,13)	(9,10,11)	-	(11,12,13)	(9,15,21)
D_{2+3}				(11,12,13)	-	(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30)	(10,18,26)	(12,20,28)	

Now, using the step 6 &7 the allocations are made in the table below

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1	0 **(9,15,21)**		10 **(-28,1,30)**	12 **(20, 8,36)**		(15,24,33) (-6,9,24) (-36,8,52)
O_2		0 **(9,15,21)**	12 **(-9,6,21)**			(12,21,30) (-9,6,21)
D_{2+1}	12	12	0 **(9,15,21)**		10	(9,15,21)
D_{2+2}		12	10	0 **(-10,10,30)**	12 **(-9,15,19)**	(9,15,21) (-10,10,30)
D_{2+3}				12	0 **(9,15,21)**	(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30) (-7,16,39) (-28,1,30)	(10,18,26) (-20,8,36)	(12,20,28) (-9,5,19)	

From the above we obtain the minimum fuzzy time transportation is $\tilde{T}$ = (11,12,13) obtained from step 8.

Now, the transformed Fuzzy Cost transportation problem is given below

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1	(0,0,0)	(2,3,4)	(13,15,17)	(7,9,11)	(6,8,10)	(15,24,33)
O_2	(3,4,5)	(0,0,0)	(5,7,9)	(7,9,11)	(6,8,10)	(12,21,30)
D_{2+1}	(14,16,18)	(2,4,6)	(0,0,0)	(4,6,8)	(3,5,7)	(9,15,21)
D_{2+2}	(5,7,9)	(4,5,6)	(1,2,3)	(0,0,0)	(1,2,3)	(9,15,21)
D_{2+3}	(6,7,8)	(3,5,7)	(1,2,3)	(3,5,7)	(0,0,0)	(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30)	(10,18,26)	(12,20,28)	

Using Fuzzy zero point method the optimal solution for the transformed Fuzzy Cost transportation problem is given below

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1	0 (9,15,21)	0	8	0	**0** **(-6,9,24)**	(15,24,33) (-6,9,24)
O_2	0	0 (9,15,21)	**0** **(-26,3,32)**	**0** **(-11,3,17)**	0	(12,21,30) (-9,6,21) (-26,3,32)
D_{2+1}	15	4	**0** (9,15,21)	0	0	(9,15,21)
D_{2+2}	9	8	1	0 (9,15,21)	0	(9,15,21)
D_{2+3}	9	7	**0** **(-25,4,33)**	1	0 (-12,11,34)	(9,15,21) (-25,4,33)
	(9,15,21)	(9,15,21)	(14,22,30) (-19,18,55) (-40,3,46)	(10,18,26) (-11,3,17)	(12,20,28) (-12,11,4)	

By Fuzzy zero point method, the optimal route is $O_1 \rightarrow D_{2+3}$=(-6,9,24), $O_2 \rightarrow D_{2+1}$=(-26,3,32), $O_2 \rightarrow D_{2+2}$,=(-11,3,17), $D_{2+3} \rightarrow D_{2+1}$=(-25,4,33) with the minimum transportation fuzzy cost is (-490,128,814) and the corresponding minimum fuzzy time transportation is (23,25,27)

Now, we have $\tilde{T}_0$=(11,12,13) ; $\tilde{T}_m$= (23,25,27) and the fuzzy time $\tilde{M}$ = [(11,12,13), (13,14,15), (23,25,27)]

Now, forming the transformed fuzzy cost transportation problem for the $\tilde{M}$ value (11,12,13) and solving it using fuzzy blocking zero point algorithm.

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1			(13,15,17)	(7,9,11)		(15,24,33)
O_2			(5,7,9)			(12,21,30)
D_{2+1}	(14,15,16)	(2,4,6)			(3,5,7)	(9,15,21)
D_{2+2}		(4,5,6)	(1,2,3)		(1,2,3)	(9,15,21)
D_{2+3}				(3,5,7)		(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30)	(10,18,26)	(12,20,28)	

Using Fuzzy blocking zero point algorithm the optimal solution for the transformed Multi-objective Fuzzy Cost transportation problem for fuzzy time (11,12,13) is given below

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1	0 (9,15,21)		0	**0** **(-6,9,24)**		(15,24,33) (-6,9,24)
O_2		0 (9,15,21)	**0** **(-9,6,21)**			(12,21,30) (-9,6,21)
D_{2+1}	0	0	0 (9,15,21)		0	(9,15,21)
D_{2+2}		3	**0** **(-28,1,30)**	0 (-14,9,32)	**0** **(-9,5,19)**	(9,15,21) (-23,6,35) (-42,1,44)
D_{2+3}				0	0 (9,15,21)	(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30) (-7,16,39) (-28,1,30)	(10,18,26) (-14,9,32)	(12,20,28) (-9,5,19)	

By Fuzzy zero point method, the optimal route is $O_1 \rightarrow D_{2+2}$=(-6,9,24), $O_2 \rightarrow D_{2+1}$=(-9,6,21), $D_{2+} \rightarrow D_{2+1,}$=(-28, 1,30), $D_{2+2} \rightarrow D_{2+3}$=(-9,5,19) with the minimum transportation fuzzy cost is (-258,135,600). Now forming cost transportation Problem for the $\tilde{M}$ value (13,14,15) and solving it using Fuzzy blocking zero point algorithm.

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1			(13,15,17)	(7,9,11)		(15,24,33)
O_2			(5,7,9)	(7,9,11)		(12,21,30)
D_{2+1}	(14,15,16)	(2,4,6)			(3,5,7)	(9,15,21)
D_{2+2}		(4,5,6)	(1,2,3)		(1,2,3)	(9,15,21)
D_{2+3}				(3,5,7)		(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30)	(10,18,26)	(12,20,28)	

Using Fuzzy zero point method the optimal solution for the transformed Multi-objective Fuzzy Cost transportation problem for fuzzy time (13,14,15) is given below

	O_1	O_2	D_{2+1}	D_{2+2}	D_{2+3}	
O_1	0 (9,15,21)		**0** **(-28,1,30)**	**0** **(-20,8,36)**		(15,24,33) (-6,9,24) (-42,1,44)
O_2		0 (9,15,21)	**0** **(-9,6,21)**	8		(12,21,30) (-9,6,21)
D_{2+1}	0	0	0 (9,15,21)		0	(9,15,21)
D_{2+2}		4	0	0 (-10,10,30)	**0** **(-9,5,19)**	(9,15,21) (-10,10,30)
D_{2+3}				**0**	0 (9,15,21)	(9,15,21)
	(9,15,21)	(9,15,21)	(14,22,30) (-7,16,39) (-28,1,30)	(10,18,26) (-20,8,36)	(12,20,28) (-9,5,19)	

By Fuzzy blocking zero point method, the optimal route is $O_1 \rightarrow D_{2+1}$=(-28,1,30), $O_1 \rightarrow D_{2+2}$=(-20,8,36), $O_2 \rightarrow D_{2+1,}$= (-9,6,21), $D_{2+2} \rightarrow D_{2+3}$=(-9,5,19) with the minimum transportation fuzzy cost is (-804,139,1152). Now, the efficient solutions to the Multi-objective Fuzzy cost Transshipment problem is given below:

S.No	Efficient Routes	Values of Z	Corresponding Time
1	$O_1 \rightarrow D_{2+2}$, $O_2 \rightarrow D_{2+1}$, $D_{2+1} \rightarrow D_{2+3}$, $D_{2+2} \rightarrow D_{2+1}$;	(-258,135,600)	(11,12,13)
2	$O_1 \rightarrow D_{2+1}$, $O_2 \rightarrow D_{2+1}$, $D_{2+1} \rightarrow D_{2+3}$, $D_{2+3} \rightarrow D_{2+2}$;	(-804,139,1152)	(13,14,15)
3	$O_1 \rightarrow D_{2+3}$, $O_2 \rightarrow D_{2+1}$, $O_2 \rightarrow D_{2+2,}$, $D_{2+3} \rightarrow D_{2+1}$;	(-490,128,814)	(23,25,27)

Conclusion

In this paper, Multi-objective Transportation Problem in fuzzy environment is extended to Multi-objective transportation problem with transshipment. Multi-objective situation in transshipment problem is useful for the decision maker because it consistent of cost as well as time of transportation in single problem. The decision maker may transport their goods according to situation that the minimum time or minimum cost.

Reference

1. Aneja, Y. P. and Nair, K.P.K., Bi-criteria transportation problem, *Management Sci., 25*, 73-79 (1979).
2. Bellman, R. E., Zadeh, L. A., Decision making in a fuzzy environment, *Management Science,* 17 (1970).
3. Gupta, R., Time-Cost Transportation Problem, *Econ-math overview, 4*, 431-443 (1977).
4. Issermann, H., Linear Multi-objective transportation problem, Asia Pacific *Journal of Operational Research, 1,* 38–52 (1984).
5. Kaufmann, A., Introduction to the theory of fuzzy sets, Vol. 1 (Academic Press, New York, 1976).
6. Nagoor Gani, A., and Abdul Razak, K., Solving Transportation Problem using Fuzzy Number, *Bulletin of pure and applied science*, 23E(2), 281-289 (2004).
7. Nagoor Gani, A., and Abdul Razak, K., Two Stage Fuzzy Transportation Problem, *Journal of Physical Science, 10,* 63-69 (2006).
8. Nagoor Gani, A., Baskaran. R. and Mohamed Assarudeen.S.N., Transshipment Problem in Fuzzy Environment, Int. J. of Math. Sci. & Engg. Appls. (IJMSEA) ISSN 0973-9424, Vol. 5 No. III(May,2011), pp. 57-74
9. Pandian, P. and Natarajan, G., A New Algorithm for finding a Fuzzy Optimal Solution for Fuzzy Transportation problems, *Applied Mathematical Sciences,* Vol. *4,* No. 2, 79-90 (2010).
10. Pandian, P. and Natarajan, G., A new method for finding an optimal solution for transportation problems, *International J. of Math. Sci. & Engg. Appls.* (IJMSEA), *4*, 59-65 (2010).
11. Rita Malhotra and Puri, M.C., pricing of Multi-objectives at optimal time in a transportation problem, Combinatorial Optimization : Some aspects, Rita Malhotra *et al.* (Eds), Narosa Publishing House, New Delhi, India, (2007).
12. Zadeh, L.A., *Fuzzy sets, Information Control, 8,* 338-353 (1965).