Ultra Scientist Vol. 24(3)A, 489-500 (2012).

Another Form of Weakly Closed Sets

*M. LELLIS THIVAGAR and ** C. SANTHINI

(Acceptance Date 18th October, 2012)

Abstract

In this paper, we introduce the notions of Λ_a-sets and Λ_a-closed sets using a-open sets and a-closed sets. We also investigate several characterizations and properties of Λ_a-continuous functions, Λ_a-compact spaces and Λ_a-connected spaces.

Key words and Phrases: a-closed sets, Λ_a-sets and Λ_a-closed sets.

2010 AMS Subject Classification: 54 C05

1. Introduction

In[7], Maki introduced the concept of Λ-sets in topological spaces. Georgiou *et al*.[6] defined and investigated Λ_δ-sets and (Λ,δ)-closed sets via δ-open sets and -closed sets. Also Caldas *et al*.[2] introduced Λ_α-sets and (Λ,α)-closed sets via α-open sets and α-closed sets. In this paper, we introduce the notions of Λ_a-sets and Λ_a-closed sets via *a*-open sets and a-closed sets. The notions of Λ_a-continuity, Λ_a-compact spaces and Λ_a-connected spaces are also introduced and several characterizations of them are obtained. Finally it is proved that Λ_a-compactness and Λ_a-connectedness are preserved Λ_a-irresolute surjections.

2. *Preliminaries :*

Throughout the paper (X,τ) and (Y,σ) and (Z,η) (or simply X, Y and Z) represent topological spaces on which no separation axioms are assumed. For a subset A of (X,τ), cl(A), int(A) and A^c denote the closure of A, interior of A and the complement of A respectively.

A subset A of a topological space (X,τ) is called an a-open set[4] if $A\subset \text{int}(\text{cl}(\text{int}_\delta(A)))$.

The complement of an a-open set is called an a-closed set. The a-closure of subset A of (X,τ) is the intersection of all a-closed sets containing A and is denoted by acl(A) and the a-interior of A is the union of all a-open sets contained in A and is denoted by aint(A). The family of all a-open sets of X form a topology and is denoted by τ_a. The family of all a-open (resp. a -closed)sets of X is denoted by $aO(X,\tau)$ (resp. $aC(X,\tau)$). A subset A of a topological space (X,τ) is called an α-open set if $A \subset \text{int}(\text{cl}(\text{int}(A)))$. The complement of an α-open set is called an α-closed set. The family of all α-open sets of X is denoted by $\alpha(X,\tau)$.

Definition 2.1 A subset A of a topological space (X,τ) is called δ-closed[10] if $A=\text{cl}_\delta(A)$ where $\text{cl}_\delta(A) = \{x\in X : \text{int}(\text{cl}(U))\cap A\neq\phi,\ U\in\tau \text{ and } x\in U\}$. The complement of δ-closed set is called δ-open set. The family of all δ-open sets of X is denoted by $\delta(X,\tau)$.

Definition 2.2 A subset A of a topological space (X,τ) is called regular open[9] if $A=\text{int}(\text{cl}(A))$. The complement of regular open set is called regular closed set. The family of all regular open sets of X is denoted by $RO(X,\tau)$.

Definition 2.3 A subset A of a topological space(X,τ) is called δ-semiopen[4] if $A\subset \text{cl}(\text{int}_\delta(A))$. The complement of δ-semiopen set is called δ-semiclosed set. The family of all δ-semiopen sets of X is denoted by $\delta SO(X,\tau)$.

Definition 2.4 A subset A of a topological space(X,) is called a

(i) Λ_α-set[2] if $A=\Lambda_\alpha(A)$ where $\Lambda_\alpha(A) =\cap\{O\in\alpha(X,\tau): A\subset O\}$.

(ii) Λ_δ-set[6] if $A=\Lambda_\delta(A)$ where $\Lambda_\delta(A)=\cap\{O\in\delta(X,\tau): A\subset O\}$.

(iii) Λ_r-set[8] if $A = A_r^\Lambda$ where $\Lambda_r(A)=\cap\{O\in RO(X,\tau): A\subset O\}$.

(iv) Λ_δ^s-set[3] if $A=\Lambda_\delta^s(A)$ where $\Lambda_\delta^s(A)=\cap\{O\in\delta SO(X,\tau): A\subset O\}$.

Definition 2.5 A subset A of a topological space(X,τ) is called a

(i) (Λ,α)-closed[2] if $A=T\cap C$ where T is a Λ_α-set and C is an α-closed set.

(ii) (Λ,δ)-closed[6] if $A=T\cap C$ where T is a Λ_δ set and C is a δ-closed set.

(iii) Λ_r-closed[8] if $A=T\cap C$ where T is a Λ_r-set and C is a closed set.

(iv) $(\Lambda,s\delta)$-closed[3] if $A=T\cap C$ where T is a Λ_δ^s-set and C is a δ-semiclosed set.

Definition 2.6 A function $f:(X,\tau)\to(Y,\sigma)$ is called

(i) completely continuous[1] if $f^{-1}(V)$ is regular open in X for every open set V in Y.

(ii) a-continuous[4] if $f^{-1}(V)$ is a-open in X for every open set V in Y.

(iii) super continuous[4] if $f^{-1}(V)$ is δ-open in X for every open set V in Y.

Lemma[4,5] *2.7* Let A, B be subsets of a topological space(X,τ). Then the following hold:

(i) $A\subseteq \text{acl}(A)$.

(ii) acl(A) is an a-closed set.

(iii) A is a-closed if and only if A=acl(A).

(iv) X-acl(A)= aint(X-A).

Lemma 2.8 Let A, B be subsets of a topological space(X,τ). Then the following hold:

(i) If A $\subseteq$ B, then ac(A) $\subseteq$acl(B).
(ii)acl(acl(A))=A.

Proof:
(i) A$\subseteq$B$\subseteq$acl(B).Since acl(A) is the smallest a-closed set containing A, ac(A) $\subseteq$acl(B).
(ii) Since acl(A) is an a-closed set, the proof follows.

3. Λ_a*-sets and* V_a*-sets :*

In this section we introduce the notions of Λ_a-sets and V_a-sets in topological spaces.

Definition 3.1 Let A be a subset of a topological space (X,τ). Define the subsets Λ_a(A) and V_a(A) as follows:
$\Lambda_a(A) = \cap\{ G : G \in aO(X,\tau) : A \subset O\}$ and
$V_a(A) = \cup \{ H : H \in aC(X,\tau) : H\subset A\}$.

Lemma 3.2 Let A, B and $\{ B_i : i \in I\}$ be subsets of a topological space(X,τ). Then the following hold:
(i) A$\subseteq\Lambda_a$ (A).
(ii) If A $\subseteq$ B, then $\Lambda_a(A) \subseteq \Lambda_a(B)$.
(iii) $\Lambda_a(\Lambda_a(A)) = \Lambda_a(A)$.
(iv) If A $\in$ aO(X,), then A = (A).
(v) $\Lambda_a\cup(\{B_{i:}\ i\in I\}) = \cup\{\Lambda_a(B_i) : i \in I\}$.
(vi) $\Lambda_a(\cap(\{B_{i:}\ i\in I\}) \subseteq\cap \{\Lambda_a(B_i) : i \in I\}$.
(vii) $\Lambda_a(A^c) =(V_aA))^c$ and $V_a(A^c)=(\Lambda_a(A))^c$.

Proof:
(i) Let x $\notin\Lambda_a$(A). Then there exists G$\in$ aO(X,τ) such that A$\subseteq$ G and x $\notin$ G. Hence x $\notin$ A and so A $\subseteq \Lambda_a$ (A)
(ii) Suppose x $\notin \Lambda_a$ (B). Then there exists G $\in$ aO(X,τ) such that B$\subseteq$G and x $\notin$ G. Since A $\subseteq$ B, we have A$\subseteq$ G and x $\notin$ G and hence x $\notin\Lambda_a$ (A) which implies (ii)
(iii) From (i) and (ii) $\Lambda_a(A)\subseteq\Lambda_a(\Lambda_a(A))$. If x $\in \Lambda_a(\Lambda_a(A))$, then there exists G $\in$ aO(X,τ) such that Λ_a(A)$\subseteq$G and x$\in$G. By (i) A$\subseteq\Lambda_a$ (A)$\subseteq$G. Hence x$\in\Lambda_a$(A) and so $\Lambda_a(\Lambda_a(A))\in\Lambda_a$ (A). which implies (iii).
(iv) Follows from the definition of Λ_a (A).
(v) Let $B = \cup\{B_i : i\in I\}$.By (ii) $\Lambda_a(B_i) \subseteq \Lambda_a(B)$ for each i $\in$ I and hence $\cup\{\Lambda_a(B_i): i\in I\}\in\Lambda_a(B) = \Lambda_a (\cup\{B_i : i\in I\})$.

Conversely, suppose $x \notin \cup\{\Lambda_a(B_i) : i \in I\}$. Then $x \notin \Lambda_a(B_i)$ for each i $\in$ I. Hence there exists $G_i\in aO(X,\tau)$ such that $B_i\subseteq G_i$ and $x\notin G_i$ for each i $\in$ I. We have $\bigcup_{i\in I} B_i \subseteq \bigcup_{i\in I} G_i$ for each i $\in$ I and $\cup_{i\in I} G_i$ is an a-open set such that $x \notin \cup_{i\in I} G_i$. Thus $x\notin \Lambda_a(\cup_{i\in I}\{B_i : i\in I\})$ which shows that $\Lambda_a(\cup_{i\in I}\{B_i : i\in I\})\subseteq\cup\{\Lambda_a(B_i).: i\in I\}$.
(vi) Suppose $x \notin\cap\{\Lambda_a(B_i): i\in I\}$ then there exists $i_0\in I$, such that $x \notin\Lambda_a(B_{i_0})$. Hence there exists G $\in$ aO(X,τ) such that $B_{i_0}\subseteq$G and x $\notin$ G. Hence we have $\cap\{B_i : i \in I\}\subseteq B_{i_0}\subseteq$ G and x $\notin$ G which implies $x \notin (\cap\{B_i : i\in I\})$ and so $\Lambda_a(\cap\{B_i : i\in I\})\subseteq\cap\{\Lambda_a(B_i): i \in I\}$.
(vii) $(\Lambda_a(A))^c=X-\cap\{G \in aO(X,\tau) : A\subseteq G\}=\cup\{G^c \in aC(X,\tau) : G^c\subseteq A^c\} = V_a(A^c)$. Similarly we can prove that $(V_a(A))^c =\Lambda_a(A^c)$

Remark 3.3 In general $\Lambda_a(A\cap B) \neq\Lambda_a(A)\cap\Lambda_a(B)$ as the following example shows.

Example 3.4 Let X = {a,b,c} and τ={ϕ,{a}, {a,b},X}. Then aO(X,τ)={ϕ,X}. If A={a} and B={b}, then $\Lambda_a(A\cap B)=\phi$ whereas $\Lambda_a(A)\cap\Lambda_a(B)=X$.

Lemma 3.5 Let A, B and {B_i :i $\in$I} be subsets of a topological space (X,τ). Then the following hold:

(i) $V_a(A) \subseteq A$.

(ii) If $A \subseteq B$, then $V_aA) \subseteq V_a(B)$.

(iii) $V_a(V_a(A)) = V_a(A)$.

(iv) If $A \in aC(X,\tau)$, then $A = V_a(A)$.

(v) $V_a \cap\{B_i : i\in I\}) =\cap\{V_a(B_i) : i \in I\}$.

(vi) $V_a(\cup\{B_i : i\in I\})\supseteq\cup \{V_a(B_i) : i\in I\}$.

Proof: Follows from lemma 3.2

Definition 3.6 A subset A of a topological space (X,τ) is said to be a Λ_a-set (resp. V_a-set) if $\Lambda_a(A)$ =A.(resp. $V_a(A)=A$))

Lemma 3.7 Let A, B and {B_i: i$\in$I} be subsets of a topological space(X,τ). Then the following hold:

(i) ϕ and X are Λ_a-sets and V_a-sets.

(ii) If $A \in aO(X,\tau)$, then A is a Λ_a-set.

(iii) If A_i is a Λ_a-set for each i$\in$I, then $\bigcup_{i\in I} A_i$ is a Λ_a-set.

(iv) If A_i is a Λ_a-set for each i$\in$I, then $\bigcup_{i\in I} A_i$ is a Λ_a-set.

(v) If $A \in aC(X, \tau)$, then A is a V_a-set.

(vi) If A_i is a V_a-set for i$\in$I, then is a $\bigcup_{i\in I} A_i$ is a V_a-set.

(vii) If A_i is a V_a -set for i I, then $\bigcup_{i\in I} A_i$ is a V_a-set.

Proof: Follows from lemma 3.2 and lemma 3.5

Remark 3.8 Let $\tau^{\Lambda\alpha}$ (resp.$\tau^{V\alpha}$) denotes the family of all Λ_a-sets (resp.V_a-sets) in a topological space (X,τ), then $\tau^{\Lambda\alpha}$(resp.$\tau^{V\alpha}$) is a topology on X consisting of all a-open (resp. a-closed) sets. Clearly (X,$\tau^{\Lambda\alpha}$) and (X,$\tau^{V\alpha}$) are Alexandroff spaces (ie) arbitrary intersections of open sets are open.

4. Λ_a- *closed sets and* Λ_a- *open sets*

In this section we introduce and investigate Λ_a-closed szets in topological spaces which are stronger than (Λ,α)-closed sets and (Λ,sδ)-closed sets.

Definition 4.1 A subset A of a topological space (X,τ) is said to be Λ_a-closed if $A=T\cap C$ where T is a Λ_a-set and C is an a-closed set. The family of all Λ_a-closed sets of X is denoted by $\Lambda_aC(X,\tau)$.

Proposition 4.2 Every Λ_a-set (resp. a-closed set) is Λ_a-closed.

Remark 4.3 Converse of the above proposition need not be true as the following example shows.

Example 4.4 Let X = {a,b,c,d} and τ={ϕ, {a}, {b,c}, {a,b,c}, X}.Then {b,c} is Λ_a-closed but not a-closed .Also { a, d} is Λ_a-closed but not a Λ_a-set.

Theorem 4.5 The following statements

are equivalent for a subset A of a topological space (X,τ).

(i)A is Λ_a-closed.

(ii) A=T $\cap$acl(A) where T is a Λ_a-set.

(iii) A=Λ_a(A)$\cap$ acl(A).

Proof:

(i)$\Rightarrow$(ii)Suppose A is Λ_a-closed. Then A=T$\cap$C where T is a Λ_a-set and C is an a-closed set. Since A$\subseteq$C, by lemma 2.8, acl(A) $\subseteq$acl(C) =C and hence A = T$\cap$C$\supseteq$ T$\cap$acl(A) $\supseteq$A.Thus A=T$\cap$ acl(A).

(ii) $\Rightarrow$ (iii Suppose A=T$\cap$acl(A) where T is a Λ_a-set. Since A$\subseteq$ T, by lemma 3.2, Λ_a(A)$\subseteq\Lambda_a$ (T)= T and so A$\subseteq$ Λ_a (A)$\cap$ acl(A) = T$\cap$acl(A) = A. Thus A = Λ_a(A)$\cap$ acl(A).

(iii)$\Rightarrow$(i) Since Λ_a(A) is a Λ_a-set and acl(A) is a-closed, the proof follows.

Theorem 4.6 For a subset A of a topological space (X,τ), the following hold.

(i) Λ_a(A) $\subseteq$ Λ_δ (A).

(ii) Λ_a(A) $\subseteq$ Λ_a (A).

(iii) Λ_a(A) $\subseteq$ A_r^Λ .

(iv) Λ_δ^s (A) $\subseteq$ Λ_a (A) .

Proof:

(i)Suppose x $\notin$ Λ_δ (A). Then there exists G $\in$ δ O(X,τ) such that A$\subseteq$ G and x$\notin$G. Since every δ-open set is a-open[4], there exists G $\in$ aO(X,τ) such that A$\subseteq$G and x $\notin$ G. Hence x$\notin$ Λ_a (A) which implies

(ii) Follows from the fact that every a-open set is α-open[4].

(iii) Follows from the fact that every regular open set is a-open[4].

(iv) Follows from the fact that every a-open set is δ-semiopen[4].

Theorem 4.7:

(i) Every Λ_δ-set is a Λ_a-set.

(ii) Every Λ_a -set is a Λ_a -set.

(iii) Every Λ_r-set is a Λ_a-set.

(iv) Every Λ_a-set is a Λ_δ^s -set.

Proof:

(i) Let A be a subset A of a topological space (X,τ). Suppose A is a Λ_δ-set. Then A=Λ_δ(A). By lemma 3.2, A$\subseteq\Lambda_a$ (A).By theorem 4.6, Λ_a(A)$\subseteq\Lambda_\delta$(A)=A and so A=Λ_a(A). Thus A is a Λ_a-set.

(ii) Suppose A is a Λ_a-set. Then A=Λ_a(A). By lemma[2] 2.1, A$\subseteq\Lambda_a$(A). By theorem 4.6, Λ_a(A) $\subseteq\Lambda_a$(A)=A and so A= Λ_a(A). Thus A is a Λ_a-set.

(iii) Suppose A is a Λ_r-set. Then A = A_r^Λ . By lemma 3.2, A$\subseteq\Lambda_a$(A). By theorem 4.6, Λ_a(A) $\subseteq$ A_r^Λ =A and so A=Λ_a(A). Thus A is a Λ_a-set.

(iv) Suppose A is a Λ_a-set. Then A=Λ_a (A). By lemma[3] 2.1,A$\subseteq$ Λ_δ^s (A). By theorem 4.6, Λ_δ^s (A) $\subseteq\Lambda_a$(A)=A and so Λ_δ^s (A)= A. Thus A is a Λ_δ^s -set.

Theorem 4.8

(i)Every (Λ,δ)-closed is Λ_a-closed.

(ii) Every Λ_a- closed is (Λ,α)-closed.

(iii) Every Λ_a- closed is $(\Lambda,s\delta)$-closed.

Proof:

(i) Let A be a subset A of a topological space (X,τ). Suppose A is a (Λ,δ)-closed set. Then

$A = T \cap C$ where T is a Λ_δ-set and C is a δ-closed set. By theorem 4.7, T is a Λ_a-set. Since every δ-closed set is a-closed, C is a-closed. Hence A is Λ_a-closed.

Similarly we can prove (ii) and (iii).

Remark 4.9 The reverse implications of the above theorem need not be true as shown by the following examples.

Example 4.10 Let X = {a,b,c,d} and τ={ϕ, {a}, {b}, {a,b}, X}. Then {c} is Λ_a-closed but not (Λ,δ)-closed .

Example 4.11 Let X = {a,b,c,d} and τ={ϕ, {a}, {c}, {a,b}, {a,c},{ a,b,c},{ a,c,d}, X}. Then {a,c} is (Λ,α)-closed but not Λ_a-closed.

Example 4.12 Let X={a,b,c,d,e} and τ={ϕ, {a}, {d,e}, {a,d,e}, X}.Then {d} is $(\Lambda,s\delta)$-closed but not Λ_a-closed.

Remark 4.13 The following example shows that the concepts of Λ_a-closed sets and Λ_r-closed sets are independent of each other.

Example 4.14 Let X= {a, b, c, d} and τ={ϕ, {a}, {b, c}, {a, b, c}, X}. Then {a, b, c} is Λ_a-closed but not Λ_r-closed.

Example 4.15 Let X = {a,b,c} and τ={ϕ,{a},X}. Then {b, c} is Λ_r-closed but not Λ_a-closed

Remark 4.16 The following table shows the relationships of Λ_a-closed sets with other known sets. The symbol"1" in a cell means that a set implies the other set and the symbol "0" in a cell means that a set does not imply the other set.

set	I	II	III	IV	V
I	1	1	0	0	1
II	0	1	0	0	0
III	1	1	1	0	1
IV	0	1	0	1	1
V	0	0	0	0	1

I - Λ_a-closed, II - (Λ,α)-closed,
III- (Λ,δ)-closed, IV- Λ_r - closed
V- $(\Lambda, s\delta)$-closed

Theorem 4.17 If A_i is Λ_a-closed for each $i \in I$, then $\cap\{A_i : i \in I\}$is Λ_a-closed.

Proof: Since A_i is Λ_a-closed for each $i \in I$, $A_i = T_i \cap C_i$ where T_i is a Λ_a-set and C_i is a-closed set for each $i \in I$. We have $\cap \{A_i : i \in I\} = \cap\{ T_i \cap C_i : i \in I \} = \{\cap T_i : i \in I\} \cap \{\cap C_i : i \in I\}$. By theorem 3.7, $\{\cap T_i : i \in I\}$ is a Λ_a-set and $\{\cap C_i : i \in I\}$ is an a-closed set and hence $\cap\{A_i : i \in I\}$is Λ_a-closed.

Definition 4.18 A subset A of a topological space (X,τ) is said to be Λ_a-open if the complement of A is Λ_a-closed. The family of all Λ_a-open sets of a topological space is denoted by
$\Lambda_a O(X, \tau)$.

Theorem 4.19 The following statements are equivalent for a subset A of a topological space (X, τ).
(i) A is Λ_a-open.

(ii) A= T$\cup$C where T is a V_a-set and C is an a-open set.
(iii) A= T$\cup$aint(A) where T is a V_a-set.
(iv) A= V_a(A)$\cup$ aint(A).

Proof:
(i) $\Rightarrow$(ii) Suppose A is Λ_a-open. Then X-A =T$\cap$C where T is a Λ_a-set and C is an a-closed set. Hence A = (X -T) $\cup$ (X - C) where X -T is a V_a-set and X-C is an a-open set. which implies (ii).

(ii) $\Rightarrow$(iii) Suppose A= T$\cup$C where T is a V_a-set and C is an a-open set. Since C $\subseteq$ A and C is a-open, C = aintC $\subseteq$ aint (A) and hence A = T $\cup$ C $\subseteq$T$\cup$ aint(A) $\subseteq$A. Thus A = T$\cup$ aint(A) which implies(iii)

(iii) $\Rightarrow$(iv) Suppose A= T$\cup$aint(A) where T is a V_a-set. Since T $\subseteq$A, by lemma 3.5, V_a(T)$\subseteq$ V_a(A) and so T $\subseteq$ V_a(A). Hence A=T$\cup$aint(A) $\subseteq$$V_a$(A) $\cup$aint(A) A. Thus A= V_a(A)$\cup$ aint(A). which implies(iv).

(iv)$\Rightarrow$(i) Suppose A= V_a(A) $\cup$aint(A). Then X-A = (X- V_a(A)) $\cap$(X-aint(A)) = Λ_a(X-A) $\cap$acl(X-A). By lemma 3.2, Λ_a(X-A) is a Λ_a-set and acl(X-A)is a-closed.Hence X-A is Λ_a-closed and hence A is Λ_a-open which implies (i).

Proposition 4.20
(i)Every a-open set is Λ_a-open.
(ii)Every V_a -set is Λ_a-open.

Proof:
(i) Let A be an a-open subset of a topological space(X,τ). Then X-A is a-closed in X. By proposition 4.2, X-A is Λ_a-closed and hence A is Λ_a-open
(ii) Let A be a V_a-set in X. Then A =A$\cup\phi$. By theorem 4.19, A is Λ_a-open.

Theorem 4.21 If A_i is Λ_a-open for each i$\in$I, then $\cup$\{ A_i : i$\in$I\} is Λ_a-open

Proof: Since Ai is Λ_a-open for each i$\in$I, X-A_i is Λ_a-closed. for i$\in$I By theorem 4.17, $\cap$\{ X-A_i : i$\in$I\} is Λ_a-closed which implies X-$\cup$\{A_i : i$\in$I\} is Λ_a-closed and so $\cup$\{ A_i: i$\in$I\} is Λ_a-open.

Remark 4.22 It follows from theorem 4.8 and proposition 4.20 that the class of Λ_a-open sets lies between the topology τ_a and the class of (Λ,α)-open sets.

5. *-continuous functions :*

Definition 5.1 Let (X,τ) be a topological space and A$\subseteq$X.A point x$\in$X is called a Λ_a-cluster point of A if for every Λ_a-open set U of X containing x we have A$\cap$U$\neq\phi$. The set of all Λ_a-cluster points of A is called the Λ_a-closure of A and is denoted by Λ_a-cl(A).

Lemma 5.2 Let A and B be subsets of a topological space (X,τ). Then the following properties hold:
(i)A $\subseteq$$\Lambda_a$-cl(A).
(ii) Λ_a-cl(A)= $\cap$\{F : F is Λ_a-closed\}
(iii)If A $\subseteq$ B, then Λ_a-cl(A)$\subseteq$ Λ_a -cl(B)
(iv)A is Λ_a-closed if and only if A=Λ_a-cl(A).
(v) Λ_a-cl(A) is Λ_a-closed.

Definition 5.3 A function f: (X,τ)$\rightarrow$

(Y,σ) is called Λ_a-continuous if $f^{-1}(V)$ is a Λ_a-open subset of X for every open subset of Y.

Theorem 5.4 For a function $f: (X,\tau)\to(Y,\sigma)$, the following are equivalent:

(i) f is Λ_a-continuous
(ii) $f^{-1}(V)$ is a Λ_a-closed subset of X for every closed subset of Y.
(iii) For each $x\in X$ and for every open set V of Y containing f(x), there exists a Λ_a-open set U of X containing x such that $f(U) \subseteq V$.
(iii) $f(\Lambda_a\text{-cl}(A))\subseteq \text{cl}(f(A))$ for each subset A of X.
(iv) $\Lambda_a\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for each subset B of Y.

Proof:
(i)⇔(ii) Let V be any closed subset of Y. Then Y-V is open in Y. Since f is Λ_a-continuous, $f^{-1}(Y-V) = X-f^{-1}(V)$ is Λ_a-open in X. Consequently $f^{-1}(V)$ is a Λ_a-closed which implies (ii).

Converse is similar.

(ii) ⇔(iii) Let $x\in X$ and V be any open set of Y such that $f(x) \in V$. Then $x\in f^{-1}(V)$ and $f^{-1}(V)$ is Λ_a-open in X Let $U=f^{-1}(V)$. Then $x \in U$ and $f(U) \subseteq V$. Thus U is the required Λ_a-open set such that $x \in U$ and $f(U) \subseteq V$ which implies(iii).

Conversely V be an open set of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. By (iii) there exists a Λ_a-open set U_x in X such that $x\in U_x$ and $f(U_x) \subseteq V$. Therefore $x\in U_x \subseteq f^{-1}(V)$. and hence $f^{-1}(V) = \cup\{U_x : x\in f^{-1}(V)\}$. By theorem 4.21, $f^{-1}(V)$is Λ_a-open in X .which implies (ii).

(ii) ⇔ (iv) Let A be any subset of X. Since cl(f(A)) is closed in Y. By (ii)$f^{-1}(\text{cl}(f(A))$ is Λ_a- closed in X. Now $A \subseteq f^{-1}(f(A))\subseteq f^{-1}(\text{cl}(f(A))$. By lemma 5.2, $\Lambda_a\text{-cl}(A)\subseteq f^{-1}(\text{cl}(f(A))$ which implies $f(\Lambda_a\text{-cl}(A))\subseteq f(f^{-1}(\text{cl}(f(A)))\subseteq \text{cl}(f(A))$ which implies (iv). Conversely let V be a closed subset of Y. Then. $f^{-1}(V)\subseteq X$. By (iv) $f(\Lambda_a\text{-cl}(f^{-1}(V))\subseteq \text{cl}(f(f^{-1}(V))\subseteq \text{cl}(V)=V$. which implies $\Lambda_a\text{-cl}(f^{-1}(V)) \subseteq f^{-1}(V)$. By lemma 5.2, $f^{-1}(V) \subseteq \Lambda_a\text{-cl}(f^{-1}(V))$ and so $f^{-1}(V)= \Lambda_a\text{-cl}(f^{-1}(V))$. Hence by lemma 5.2, $f^{-1}(V)$ is Λ_a-closed in X which implies(ii).

(iv) ⇔ (v) Let B be a subset of Y.By (iv) $f(\Lambda_a\text{-cl}(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B)$. Hence $\Lambda_a\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ which implies (v).

Conversely let A be a subset of Y. Take B=f(A). By(v) $\Lambda_a\text{-cl}(A) \subseteq \Lambda_a\text{-cl}(ff^{-1}(B))\subseteq f^{-1}(\text{cl}(B)) \subseteq f^{-1}(\text{cl}(f(A))$.Hence $f(\Lambda_a\text{-cl}(A)) \subseteq \text{cl}(f(A))$ which implies (iv)

Definition 5.5 A function $f: (X,\tau) \to (Y,\sigma)$ is called
(i)Λ_a-irresolute if $f^{-1}(V)$ is a Λ_a-open subset of X for every Λ_a-open subset of Y.
(ii) quasi -Λ_a-irresolute if $f^{-1}(V)$ is a Λ_a-open subset of X for every a-open subset of Y.

Theorem 5.6 A function $f: (X,\tau)\to(Y,\sigma)$ is
(i) Λ_a-irresolute if and only if $f^{-1}(V)$ is a Λ_a-closed subset of X for every Λ_a-closed subset of Y.
(ii) quasi Λ_a-irresolute if $f^{-1}(V)$ is a Λ_a-closed subset of X for every a-closed subset of Y.

Proof:

(i) Suppose f is Λ_a-irresolute. Let V be any Λ_a-closed subset of Y. Then Y-V is Λ_a-open in Y. Since f is Λ_a-irresolute,
$f^{-1}(Y-V) = X-f^{-1}(V)$ is Λ_a-open in X and so $f^{-1}(V)$ is a Λ_a-closed in X. Converse is similar.
(ii) Suppose f is quasi-Λ_a-irresolute. Let V be any a-closed subset of Y. Then Y-V is a-open in Y. Since f is quasi-a-irresolute, $f^{-1}(Y-V) = X-f^{-1}(V)$ is Λ_a-open in X. and so $f^{-1}(V)$ is Λ_a-closed in X. Converse is similar.

Theorem 5.7 For a function f: $(X,\tau)\rightarrow(Y,\sigma)$, the following properties hold:
(i) If f is Λ_a-irresolute, then f is quasi Λ_a-irresolute.
(ii) If f is a-continuous, then f is Λ_a-continuous.
(iii) If f is a-irresolute, then f is quasi-Λ_a-irresolute.

Proof:

(i) Let V be any a-closed subset of Y. By proposition 4.2, V is Λ_a-closed subset of Y. Since f is -irresolute, $f^{-1}(V)$ is Λ_a-closed in X. Thus f is quasi Λ_a-irresolute.
(ii) Let V be any closed subset of Y. Since f is a-continuous, $f^{-1}(V)$ is a-closed in X By proposition 4.2, $f^{-1}(V)$ is Λ_a-closed in X. Thus f is Λ_a-continuous.
(iii) Let V be an a-closed subset of Y. Since f is a-irresolute, $f^{-1}(V)$ is a-closed in X By proposition 4.2, $f^{-1}(V)$ is Λ_a-closed in. Thus f is quasi-Λ_a-irresolute.

Remark 5.8 The reverse implications of the above theorem need not be true as shown by the following example.

Example 5.9 Let X = {a,b,c,d} =Y, τ={ϕ,{a},{b},{a,b},X}. and σ={ϕ,{a}, {b,c}, {a,b,c},Y}. We have aC(X,τ) = {ϕ,{c}, {d}, {c,d}, {a,c,d}, b,c,d}, X} and
Λ_aC(X,τ)=P(X). Also aC(Y,σ)={ϕ, {d}, {a,d,}, {b,c,d}, Y} and Λ_a C(Y,σ) ={ϕ,{a}, {d}, {b,c}, (a,d),{a,b,c},{b,c,d},Y}. Define a function f: $(X,\tau)\rightarrow(Y,\sigma)$ by f(a)=c, f(b)=d, f(c)=a and f(d)=b. Then f is Λ_a-continuous but not a-continuous since f^{-1}({a,d})={b,c} is not a-closed in X where {a,d} is closed in Y. Also f is quasi-Λ_a-irresolute but not a-irresolute since f^{-1}({d})={b} is not a-closed in X where {d} is a- closed in Y.

Remark 5.10 Composition of two Λ_a-continuous functions need not be Λ_a-continuous as shown by the following example.

Example 5.11 Let X= {a,b,c,d}= Y,τ= {ϕ, {a}, {b,c}, {a,b,c},X}, σ={ϕ, {a},{b}, {a,b}, Y}. and η={ϕ,{a},{a,b},Z}. Now Λ_aC(X,τ)= {ϕ,{a},{d},{b,c},(a,d),{a,b,c},{b,c,d},X}and Λ_aC(Y,σ) =P(Y}. Define a function f : $(X,\tau)\rightarrow(Y,\sigma)$ by f(a)=a, f(b)=c, f(c)=d and f(d)=b and a function g: $(Y,\sigma)\rightarrow(Z,\eta)$ by g(a)=d, g(b)=a, g(c)=b and g(d)=c. Then f and g are Λ_a-continuous functions but $g_o f$: $(X,\tau)\rightarrow(Z,\eta)$ not Λ_a-continuous since
$(g_o f)^{-1}$({c,d})={a,c} is not Λ_a-closed in X where {c,d} is closed in Z.

Theorem 5.12 Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\eta)$ be two functions. Then the following hold:
(i) If f is Λ_a-continuous and g is continuous, then $g_o f$: $(X,\tau)\rightarrow(Z,\eta)$ is Λ_a-continuous.
(ii) If f is quasi-Λ_a-irresolute and g is a-continuous,

then $g_\circ f$: $(X,\tau)\to(Z,\eta)$ is Λ_a-continuous

(iii) If f is quasi-Λ_a-irresolute and g is super - continuous, then $g_\circ f$: $(X,\tau)\to(Z,\eta)$ is Λ_a-continuous.

(iv) If f is quasi-Λ_a-irresolute and g is completely continuous, then $g_\circ f$: $(X,\tau)\to(Z,\eta)$ is Λ_a-continuous.

(v) If f is Λ_a-irresolute and g is Λ_a-continuous, then $g_\circ f$: $(X,\tau)\to(Z,\eta)$ is Λ_a-continuous.

Proof:

(i) Let V be any closed subset of Z. Since g is continuous, $g^{-1}(V)$ is closed subset of Y. Since f is Λ_a-continuous, $f^{-1}(g^{-1}(V)) = (g_\circ f)^{-1}(V)$ is a Λ_a-closed in X. Thus $g_\circ f$ is Λ_a-continuous.

(ii) Let V be any closed subset of Z. Since g is a-continuous, $g^{-1}(V)$ is a-closed subset of Y. Since f is quasi-Λ_a-irresolute, $f^{-1}(g^{-1}(V))=(g_\circ f)^{-1}(V)$ is a Λ_a-closed in X. Thus $g_\circ f$ is Λ_a-continuous.

(iii)Since every super continuous function is a-continuous[4]. the proof follows.

(iv) Since every completely continuous function is super-continuous[4], the proof follows.

(v) Let V be a closed subset of Z. Since g is Λ_a-continuous, $g^{-1}(V)$ is Λ_a-closed subset of Y. Since f is Λ_a-irresolute,
$f^{-1}(g^{-1}(V)) = (g_\circ f)^{-1}(V)$ is a Λ_a-closed in X. Thus $g_\circ f$ is Λ_a-.continuous.

Theorem 5.13 Let f: $(X,\tau)\to(Y,\sigma)$ and g: $(Y,\sigma)\to(Z,\eta)$ be two functions. Then the following hold :

(I)If f is Λ_a- irresolute and g is Λ_a- irresolute, then $g_\circ f$: $(X,\tau)\to(Z,\eta)$ is Λ_a- irresolute

(ii) If f is quasi-Λ_a-irresolute and g is a-irresolute, then $g_\circ f$: $(X,\tau)\to(Z,\eta)$ is quasi-Λ_a-irresolute.

(iii) If f is Λ_a-irresolute and g is quasi-Λ_a-irresolute, then $g_\circ f$: $(X,\tau)\to(Z,\eta)$ is quasi-Λ_a-irresolute.

Proof:

(i) Let V be a Λ_a-closed subset of Z. Since g is Λ_a-irresolute, $g^{-1}(V)$ is Λ_a-closed subset of Y. Since f is Λ_a-irresolute, $f^{-1}(g^{-1}(V))=(g_\circ f)^{-1}(V)$ is a Λ_a-closed in X. Thus $g_\circ f$ is Λ_a-.irresolute.

(ii) Let V be a a-closed subset of Z. Since g is a-irresolute, $g^{-1}(V)$ is a-closed subset of Y. Since f is quasi-Λ_a-irresolute,
$f^{-1}(g^{-1}(V))=(g_\circ f)^{-1}(V)$ is a Λ_a-closed in X. Thus $g_\circ f$ is quasi-Λ_a-.irresolute.

(iii) Let V be an a -closed subset of Z. Since g is quasi-Λ_a-irresolute, $g^{-1}(V)$ is Λ_a-closed subset of Y. Since f is Λ_a-irresolute, $f^{-1}(g^{-1}(V)) = (g_\circ f)^{-1}(V)$ is a Λ_a-closed in X. Thus gf is quasi-Λ_a-.irresolute

6. *Applications :*

In this section, we relate the concept of -continuous functions to the classes of Λ_a-compact spaces and Λ_a-connected spaces.

Definition 6.1 A topological space (X,τ) is said to be

(i) Λ_a-compact if every cover of X by Λ_a-open sets of X has a finite sub-cover.

(ii) a-compact[5] if every a-open cover of X has a finite sub-cover.

Theorem 6.2 For a topological space (X,τ), the following properties hold:

(i) If (X, τ^{Λ_a}) is compact, then X is a-compact.

(ii) If (X,τ) is Λ_a-compact, then (X, τ^{V_a}) is compact.

Proof:

(i) Let $\{F_\alpha : \alpha \in I\}$ be any a-open cover of X. By lemma 3.2, F_α is a Λ_a-set for each $\alpha \in$I. Since (X, τ^{Λ_a}) is compact, there exists a finite I_0 of I such that X= $\cup\{F_\alpha : \alpha \in I_0\}$ and hence X is a-compact.

(ii) Let $\{F_\alpha : \alpha \in I\}$ be any cover of X by V_a-sets of X. By proposition 4.20, F_α is a Λ_a-open for each $\alpha \in$ I. Since (X,τ) is Λ_a-compact, there exists a finite I_0 of I such that X= $\cup\{F_\alpha : \alpha \in I_0\}$ which shows that (X, τ^{V_a}) is compact..

Theorem 6.3 If f: (X,τ)→(Y,σ) is a Λ_a-irresolute surjection and X is Λ_a-compact, then Y is Λ_a-compact.

Proof: Let $\{F_\alpha : \alpha \in I\}$ be a cover of Y by Λ_a-open sets of Y. Since f is Λ_a-irresolute, $\{f^{-1}(F_\alpha) : \alpha \in I\}$ is a cover of X by Λ_a-open sets of X. Since X is Λ_a-compact, there exists a finite I_0 of I such that X=$\cup\{f^{-1}(F_\alpha\} : \alpha \in I_0\}$. Since f is surjective, Y= f(X) = $\cup\{F_\alpha : \alpha \in I_0\}$ and hence Y is Λ_a-compact.

Theorem 6.4 Every Λ_a-continuous image of a Λ_a-compact space is compact.

Proof: Let f : (X,τ)→(Y,σ) be a Λ_a-continuous surjection from a Λ_a-compact space X onto a topological space Y. Let $\{F_\alpha : \alpha \in I\}$ be any open cover of Y. Since f is Λ_a-continuous, $\{f^{-1}(F_\alpha) : \alpha \in I\}$ is a cover of X by Λ_a-open sets of X Since X is a -compact space there exists a finite I_0 of I such that X = $\cup\{f^{-1}(F_\alpha\} : \alpha \in I_0\}$.Since f is surjective, Y= f(X)=$\cup\{F_\alpha : \alpha \in I_0\}$ and hence Y is compact.

Definition 6.5 A topological space (X,τ) is said to be Λ_a-connected (resp.a-connected[5]) if X cannot be written as a disjoint union of two non-empty Λ_a-open (resp. a-open) sets of X.'

Theorem 6.6 For a topological space (X,τ), the following are equivalent.

(i) X is Λ_a-connected.

(ii) The only subsets of X which are both Λ_a-open and Λ_a-closed are ϕ and X.

Theorem 6.7 Every Λ_a-connected space is a-connected.

Proof: Let X be a Λ_a-connected space. Suppose X is not a-connected. Then X= A∪B where A and B are disjoint non-empty a-open sets in X. Since every a-open set is Λ_a-open, A and B are disjoint non-empty Λ_a-open sets in X, a contradiction to the fact that X is Λ_a-connected. Hence X is a-connected.

Remark 6.8 The following example shows that the converse of the above theorem need not be true.

Example 6.9 Let X={a,b,c,d} and τ= {ϕ, {a},{b,c},{a,b,c},X}.ThenΛ_aC(X,τ)={ϕ,{a},{d}, {b,c},{a,b,c},{b,c,d}, X}. Then X is a-connected but not Λ_a-connected, since X={a,d}∪{b,c} where {a,d} and {b,c} are disjoint non-empty Λ_a-open sets in X.

Theorem 6.10 If a topological space (X,τ) is Λ_a-connected., then (X, τ^{Λ_a}) is connected.

Proof: Suppose (X, τ^{Λ_a}) is connected. Then X =A∪B where A and B are disjoint non-empty Λ_a-sets in X. By proposition 4.2, A and B are Λ_a-closed sets which shows that X is not Λ_a-connected. Hence (X,) is connected.

Theorem 6.11 If f: (X,τ)→(Y,σ) is a Λ_a-continuous surjection and X is Λ_a-connected., then Y is connected.

Proof: Suppose Y is not connected. Then Y= A∪B where A and B are disjoint non-empty open sets in Y. Then we have X=f^{-1}(A)∪ f^{-1}(B) and f^{-1}(A)∩f^{-1}(B)= ϕ. Since f is Λ_a-continuous, f^{-1}(A) and f^{-1}(B) are disjoint non-empty Λ_a-open sets in X which shows that that X is not Λ_a-connected. Hence Y is connected.

Theorem 6.12 If f: (X,τ)→(Y,σ) is a Λ_a-irresolute surjection and X is Λ_a-connected, then Y is Λ_a- connected.

Proof: Suppose Y is not Λ_a-connected. Then Y= A∪B where A and B are disjoint non-empty Λ_a-open sets in Y. Then we have X= f^{-1} (A) ∪ f^{-1} (B)and f^{-1}(A)∩ f^{-1}(B)= ϕ. Since f is Λ_a-irresolute, f^{-1}(A) and f^{-1}(B) are disjoint non-empty Λ_a-open sets in X which shows that that X is not Λ_a-connected. Hence Y is Λ_a- connected.

References

1. Arya S.P., Gupta R., On strongly continuous mappings, *Kyungpook Math. J., 14,* 131-143 (1974).
2. Caldas M., Georgiou D.N., Jafari S., Study of (Λ,α)-closed sets and related notions in topological spaces, *Bulletin of the Malaysian Mathematical Sciences Society,* (2), 30(1), 23-36 (2007).
3. Caldas M., Ganster M., Georgiou D.N., Jafari S., Moshokoa S.P., δ-semiopen sets in topology, *Topology Proceedings,* Vol. *29,* No. *2,* 369-383 (2005).
4. Erdal Ekici, On a-open sets, A*-sets and decompositions of continuity and super-continuity, *Annales Univ. Sci. Budapest, 51,* 39-51 (2008).
5. Erdal Ekici, New forms of contra-continuity, Carpathian Math.,*24,* No.1, 37-45 (2008).
6. Georgiou D.N., Jafari S., Noiri Properties of (Λ,δ)-closed sets in topological spaces, *Bollettino Unione Mathematica Italiana 8),* 7-B, 745-756 (2004).
7. Hauro Maki, Generalized Λ-sets and the associated closure operator, The special issue in Commemoration of Prof. Kazusada IKEDA's Retirement, 1, Oct (1986).
8. Jeyanthi M.J., Pious Missier S., Thangavelu P., On contra Λ_r-continuous functions, *Theoretical Mathematics Applications,* Vol. 1, no. 1, 1-15 (2011).
9. Stone M.H., Application of the theory of Boolean rings in general topology, *Trans. Amer. Math. Soc., 41,* 375-381 (1937).
10. Velicko N.V., H-closed sets in topological spaces, *Amer. Mat. Sci. Transl., 78,* 103-118 (1968).