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Temperatures in the Prism and I-Function

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Abstract

The aim of this paper is to obtain the temperatures in the prism involving I-function.

1. Introduction

The I-function of one variable is defined by Saxena [3, p.366-375] and we will represent here in the following manner:

$$I^{m,n}_{p_i,q_i:r}\left[x\middle|^{[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]}\right]$$

$$=(1/2\pi\omega)\int_L \theta(s)x^s ds \qquad (1)$$

where $\omega = \sqrt{(-1)}$,

$$\theta(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right\}},$$

integral is convergent, when ($B>0$, $A \le 0$), where

$$B=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p_i}\alpha_{ji}+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q_i}\beta_{ji}, \qquad (2)$$

$$A=\sum_{j=1}^{p_i}\alpha_{ji}-\sum_{j=1}^{q_i}\beta_{ji},$$

$|\arg x| < \frac{1}{2} B\pi$, $\forall\ i \in (1, 2, \ldots, r)$.

In the present investigation we require the following results:

From Gradshteyn[2]:

$$\int_0^L(\sin\pi x/L)^{\omega-1}\sin n\pi x/L\,dx=\frac{L\sin\frac{1}{2}n\pi\,\Gamma(\omega)}{2^{\omega-1}\Gamma\{\frac{1}{2}(\omega\pm n+1)\}},$$

where n is any integer and $\omega > 0$.

2. Formulation of the Problem:

All four faces of an infinitely long rectangular prism, formed by the planes $x = 0$, $x = a$, $y = 0$ and $y = b$, are kept at temperature zero. Let the initial temperature distribution be $f(x, y)$, and derive this expression for the temperature $u(x, y, t)$ in the prism is given by [1, p.131] as follows:

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{mn} \exp\left[-\pi^2 kt\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\right]\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}, \tag{4}$$

where

$$B_{mn} = \frac{4}{ab}\int_0^b\int_0^a f(x,y)\ \sin\frac{m\pi y}{b}\sin\frac{n\pi x}{a}\,dxdy. \tag{5}$$

3. Solution in terms of I-Function:

Consider

$$f(x, y) = (\sin\frac{\pi x}{a})^{\omega-1}(\sin\frac{\pi y}{b})^{\delta-1} I^{k,l}_{p_i,q_i:r}[z(\sin\frac{\pi x}{a})^{\lambda}(\sin\frac{\pi y}{b})^{-\mu}],\ , \tag{6}$$

where $I^{k,l}_{p_i,q_i:r}[z]$ is the I-function, defined in (1).

Combining (6) and (5), making use of the definition of I-function as given in (1), changing the order of integration, after using the integral (3), we arrive at

$$B_{mn} = 2^{4-\omega-\delta}\sin\frac{m\pi}{2}\sin\frac{n\pi}{2}$$

$$\times I^{k+1,l+1}_{p_i+3,q_i+3:r}[z2^{-\lambda+\mu}|^{(1-\omega,\lambda),\ldots\ldots,(\frac{1}{2}+\frac{\delta}{2}\pm\frac{m}{2},\frac{\mu}{2})}_{(\delta,\mu),\ldots\ldots,(1/2-\omega/2\pm n/2,\lambda/2)}], \tag{7}$$

provided that $|\arg z| < (1/2)\pi B$, $\lambda \geq 0, \mu \geq 0, \mathrm{Re}(\omega) > 0, Re(\delta) > 0$, where B is given in (2).

Putting the value of B_{mn} from (7) in (4), we get following required solution of the problem:

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} 2^{4-\omega-\delta}\exp\left[-\pi^2 kt\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\right]\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$$

$$\times I^{k+1,l+1}_{p_i+3,q_i+3:r}[z2^{-\lambda+\mu}|^{(1-\omega,\lambda),\ldots\ldots,(\frac{1}{2}+\frac{\delta}{2}\pm\frac{m}{2},\frac{\mu}{2})}_{(\delta,\mu),\ldots\ldots,(1/2-\omega/2\pm n/2,\lambda/2)}]$$

$$\times \sin\frac{m\pi}{2}\sin\frac{n\pi}{2}. \tag{8}$$

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