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Stratified Rivlin-Ericksen fluid effect on MHD free convection flow with heat and mass transfer past a vertical porous plate

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Abstract

A study of the effect of stratified Rivlin-Ericksen fluid on MHD free convection flow past a vertical porous plate with heat and mass transfer and neglecting induced magnetic field in comparison to applied magnetic field is investigated. The velocity, temperature and concentration distributions are derived and discussed numerically with the helps of graphs and tables. It is observed that velocity increases with the increase in G_r(Grashof number) and K (Permeability parameter), but it decreases with the increase in M (Magnetic parameter). The velocity increases with the increase in λ_o (Visco-elastic parameter) up to y = 2, then decreases.

Key words : Heat and mass transfer, Free convection, MHD flow, Porous medium, Vertical plate, Stratified Rivlin-Ericksen fluid.

Introduction

The convection problem in a porous medium has important applications in geo-thermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear rector, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Free convective flow past a vertical plate has been studied extensively by Ostrach[9]. Siegel[12] investigated the transient free convection from a vertical flat plate. Cheng and Lau[4] and Cheng and Teckchandani[5] obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. In all the above mentioned studies, the effect of porosity, permeability and the thermal resistance of the medium is ignored or treated as constant. However, porosity measurements by Benenati and Broselow[3] show that porosity is not constant but varies from the surface of the plate to its interior to which as a result permeability also varies. In case of unsteady free convective flow, Soundalgekar[14] studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by Chen *et al.*[6]. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi[10]. Bejan and Khair[2] have investigated the vertical free convective boundary layer flow embedded in a porous medium resulting from the combined heat and mass transfer. Lin and Wu[7] analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical and chemical species in dilute and aqueous solutions. Rushi Kumar and Nagarajan[11] studied the mass transfer effects of MHD free convection flow of incompressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by Manohar and Nagarajan[8]. Sivaiah *et al.*[13] studied heat and mass transfer effects on MHD free convective flow past a vertical porous plate. Recently, Agrawal *et al.*[1] have discussed the effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate.

In the present section we have considered the problem of Agrawal *et al*.[1] with stratified Rivlin-Ericksen fluid.

Mathematical Analysis :

We study the two-dimensional free convection and mass transfer flow of stratified Rivlin-Ericksen fluid past an infinite vertical porous plate under the following assumptions:

- The plate temperature is constant
- Visco-elastic and Darcy's resistance terms are taken into account with constant permeability of the medium.
- Boussinesq's approximation is valid.
- The suction velocity normal to the plate is constant and can be written as,

$$v^1 = -U_0$$

A system of rectangular co-ordinates O (x^1, y^1, z^1) is taken, such that $y^1 = 0$ on the plate and z^1 axis is along its leading edge. All the fluid properties considered constant except that the influence of the density variation with temperature is considered. The influence of the density variation in other terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is considered negligible. The variations of density, viscosity, elasticity and thermal conductivity are supposed to be of the form

$$\rho = \rho_o e^{-b^1 y^1}, \qquad \mu = \mu_o e^{-b^1 y^1},$$

$$\sigma = \sigma_o e^{-b^1 y^1}, \quad k_T = k_o e^{-b^1 y^1},$$

where ρ_o, μ_o, σ_o and k_o are the coefficients of density, viscosity, elasticity and thermal conductivity respectively at $y^1 = 0$, $b^1 > 0$ represents the stratification factor.

Under these conditions, the problem is governed by the following system of Equations:

Equation of continuity:

$$\frac{\partial v^1}{\partial y^1} = 0 \tag{1}$$

Equation of Momentum:

$$\rho\left[\frac{\partial u^1}{\partial t^1} + v^1\frac{\partial u^1}{\partial y^1}\right] = \rho g\beta(T^1 - T^1_\infty) + \rho g\beta^1(C^1 - C^1_\infty) + \left(1 + \lambda^1_o\frac{\partial}{\partial t^1}\right)\frac{\partial}{\partial y^1}\left(\mu\frac{\partial u^1}{\partial y^1}\right) - \left(\sigma B_0^2 + \frac{\nu}{K^1}\right)u^1 \tag{2}$$

Equation of Energy:

$$\frac{\partial T^1}{\partial t^1} + v^1\frac{\partial T^1}{\partial y^1} = \frac{1}{\rho C_p}\frac{\partial}{\partial y^1}\left(k_T\frac{\partial T^1}{\partial y^1}\right) \tag{3}$$

Equation of concentration:

$$\frac{\partial C^1}{\partial t^1} + v^1\frac{\partial C^1}{\partial y^1} = D\left(\frac{\partial^2 C^1}{\partial {y^1}^2}\right) \tag{4}$$

where u^1, v^1 are the velocity components. T^1, C^1 are the temperature and concentration components, ν is the kinematic viscosity, ρ is the density, σ is the electric conductivity, B_0 is the magnetic induction, k_T is the thermal conductivity and D is the concentration diffusivity, C_p is the specific heat at constant pressure, λ^1_o is the coefficient of visco-elastic fluid.

The boundary conditions for the velocity, temperature and concentration fields are:

$$u^1 = 0, T^1 = T^1_w, C^1 = C^1_w \text{ at } y^1 = 0$$
$$u^1 = 0, T^1 = T_\infty, C^1 = C_\infty \text{ at } y^1 \to \infty \tag{5}$$

Let us introduce the non-dimensional variables

$$u = \frac{u^1}{U_0}, \quad t = \frac{t^1 U_0^2}{\nu_o}, \quad y = \frac{y^1 U_0}{\nu_o},$$

$$\theta = \frac{T^1 - T^1_\infty}{T^1_w - T^1_\infty}, \quad C = \frac{C^1 - C^1_\infty}{C^1_w - C^1_\infty}$$

$$K = \frac{K^1 U_0^2}{{\nu_o}^2}, \quad P_r = \frac{\mu_o C_p}{k_o}, \quad S_c = \frac{\nu_o}{D},$$

$$M = \frac{\sigma_o B_0^2 \nu_o}{\rho_o U_0^2},$$

$$N_0 = \frac{\beta^1(C^1_w - C^1_\infty)}{\beta(T^1_w - T^1_\infty)}, G_r = \frac{\nu_o g\beta(T^1_w - T^1_\infty)}{U_0^3},$$

$$b = \frac{b^1\nu_o}{U_o} \quad \lambda_o = \frac{\lambda^1_o U_0^2}{\nu}$$

where P_r is the Prandtl number, G_r is the Grashof number, N_0 is the buoyancy ratio, S_c is the Schmidt number, M is the magnetic parameter, K is the permeability parameter, b is the stratification parameter, λ_o is the visco-elastic parameter. Other physical variables have their usual meaning.

Introducing the non-dimensional quantities describes above, the governing equations reduce to

$$\frac{\partial u}{\partial t}-(1-b)\frac{\partial u}{\partial y}=G_r(\theta+N_0C)$$

$$+\left(1+\lambda_o\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2}-\left(M+\frac{1}{K}\right)u \quad (6)$$

$$P_r\frac{\partial\theta}{\partial t}-(P_r-b)\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial y^2} \quad (7)$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \quad (8)$$

and the corresponding boundary conditions are

$$u=0,\theta=1,C=1 \text{ at } y=0 \quad (9)$$

$$u=0,\theta=0,C=0 \text{ at } y\to\infty$$

Method of Solution :

We assume the solution of eq. (6), (7), (8) as

$$u(y,t)=u_0(y)e^{-nt},$$

$$\theta(y,t)=\theta_0(y)e^{-nt},$$

$$C(y,t)=C_0(y)e^{-nt} \quad (10)$$

Using eq.(10) in eq. (6), (7), (8) and we get

$$(1-\lambda_o n)u_0''+(1-b+bn\lambda_o)u_0'-\left[\left(M+\frac{1}{K}-n\right)\right]u_0$$
$$=-G_r\theta_0-G_rN_0C_0 \quad (11)$$

$$\theta_0''+(P_r-b)\theta_0'+nP_r\theta_0=0 \quad (12)$$

$$C_0''+S_cC_0'+S_cnC_0=0 \quad (13)$$

Now the corresponding boundary conditions are

$$u_0=0,\theta_0=1,C_0=1 \text{ at } y=0$$

$$u_0=0,\theta_0=0,C_0=0 \text{ at } y\to\infty \quad (14)$$

Equations (11) to (13) are ordinary linear differential equations, now u_0,θ_0 and C_0 with boundary conditions (14) are

$$u_0=(A_1+A_2)e^{-m_3y}-A_1e^{-m_1y}-A_2e^{-m_2y} \quad (15)$$

$$\theta_0=e^{-m_1y} \quad (16)$$

$$C_0=e^{-m_2y} \quad (17)$$

where

$$m_1=\frac{(P_r-b)+\sqrt{(P_r-b)^2-4nP_r}}{2}$$

$$m_2=\frac{S_c+\sqrt{S_c^2-4S_cn}}{2}$$

$$m_3=\frac{(1-b+bn\lambda_o)+\sqrt{(1-b+bn\lambda_o)^2+4\left(M+\frac{1}{K}-n\right)(1-\lambda_o n)}}{2(1-\lambda_o n)}$$

$$A_1 = \frac{G_r}{\left[\left(1-\lambda_o n\right)m_1^2 - (1-b+bn\lambda_o)m_1 - \left(M + \frac{1}{K} - n\right)\right]}$$

$$A_2 = \frac{G_r N_0}{\left[\left(1-\lambda_o n\right)m_2^2 - (1-b+bn\lambda_o)m_2 - \left(M + \frac{1}{K} - n\right)\right]}$$

Hence, The equations for u, and C will be as follows

$$u\left(y,t\right) = \left[(A_1 + A_2)e^{-m_3 y} - A_1 e^{-m_1 y} - A_3 e^{-m_2 y}\right]e^{-nt} \quad (18)$$

$$\theta\left(y,t\right) = e^{-m_1 y}e^{-nt} \quad (19)$$

$$C\left(y,t\right) = e^{-m_2 y}e^{-nt} \quad (20)$$

Skin Friction:

The skin friction coefficient at y = 0 is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left[-m_3(A_1 + A_2) + m_1 A_1 + m_2 A_2\right]e^{-nt} \quad (21)$$

Result and Discussion

Fluid velocity distribution of fluid flow is tabulated in Table 1 and plotted in Fig. 1 having six graphs at P_r= 0.71, S_c = 0.4, n = 0.1, t = 0.1, N_0 = 1.5, b = 0.1 for following different value of G_r, M, K and λ_o.

	G_r	M	K	λ_o
For Graph-1	2	0.02	100	0.5
For Graph-2	4	0.02	100	0.5
For Graph-3	2	0.04	100	0.5
For Graph-4	2	0.02	1000	0.5
For Graph-5	2	0.02	100	2.0
For Graph-6	2	0.02	100	3.0

It is observed from Fig. 1 that all velocity graphs are increasing sharply up to y = 1.2 after that velocity in each graph begins to decrease and tends to zero with the increasing in y. It is also observed from Fig. 1 that velocity increases with the increase in G_r and K, but it decreases with the increase in M. When on increasing the value of λ_o velocity increases up to y = 2, then after it velocity decreases.

The temperature and concentration do not change with the change in above parameters taken for velocity.

Table 1. Value of velocity u for Fig. 1 at $P_r = 0.71$, $S_c = 0.4$, $n = 0.1$, $t = 0.1$, $N = 1.5$, $b = 0.1$ and different values of G_r, M, K and λ_o.

y	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	0	0	0	0	0	0
1	19.01812	38.04229	15.75076	21.34781	20.41984	21.30725
2	23.08858	46.18427	18.87094	25.99729	23.14461	22.94389
3	21.74672	43.49995	17.58977	24.55098	20.78498	19.97418
4	18.79249	37.59043	15.07953	21.26215	17.41203	16.44798
5	15.66661	31.33763	12.49684	17.75685	14.24134	13.33654

Table 2. Value of skin friction τ for Fig. 2 at $P_r = 0.71$, $S_c = 0.4$, $n = 0.1$, $N_0 = 1.5$, $b = 0.1$ and different values of G_r, M, K and λ_o.

t	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	32.76920	65.54910	27.58090	36.65335	38.60740	43.49050
0.2	32.12033	64.25114	27.03476	35.92757	37.84292	42.62933
0.4	31.48430	62.97888	26.49944	35.21615	37.09358	41.78521
0.6	30.86087	61.73182	25.97471	34.51883	36.35908	40.95781
0.8	30.24978	60.50945	25.46038	33.83531	35.63912	40.14679
1	29.65080	59.31128	24.95623	33.16532	34.93342	39.35183

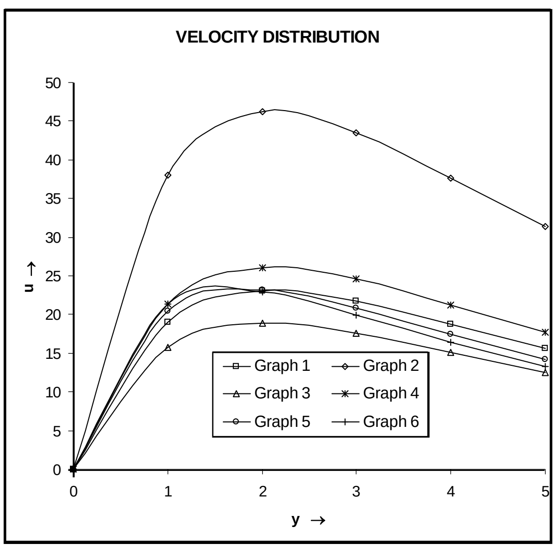

Fig. 1.

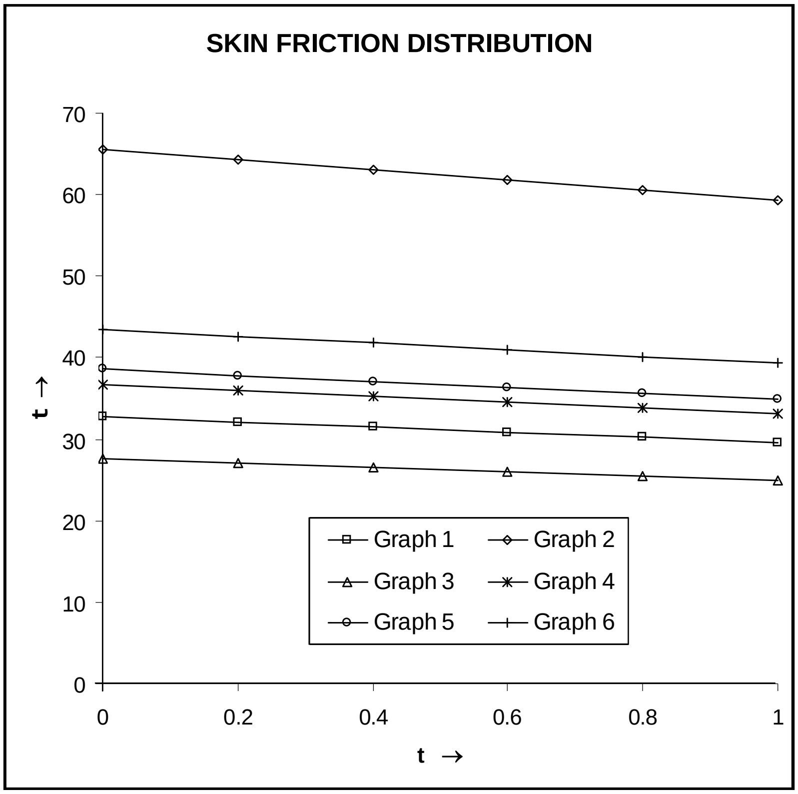

Fig. 2.

The skin friction distribution is tabulated in Table 2 and plotted in Fig. 2 having six graphs. It is observed from Fig. 2 that skin friction increases with the increase in G_r, K and λ_o, but it decreases with the increase in M.

Particular case :

When λ_o is equal to zero, this problem reduces to the problem of Agrawal *et al*. (2012).

Conclusion

1. The velocity increases with the increase in λ_o (Visco-elastic parameter) up to y = 2, then decreases.
2. The skin friction increases with the increase in λ_o.

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