Ultra Scientist Vol. 24(3)A, 543-548 (2012).

Simultaneous dual series equations involving the product of 'r' generalized batman-k functions

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(Acceptance Date 3rd December, 2012)

Abstract

The object of this paper is to obtain an exact solution for the simultaneous dual series equations involving the product of 'r' generalized Batman- k functions by multiplying factor technique.

Key words: 45F 10 Dual Series Equations, 33C45 Bateman- k function, 33D45 Basic Orthogonal polynomials and functions, 42C05 Orthogonal functions and polynomials, General Theory, 26A33 Fractional Derivatives and Integrals, 33B 15 Beta function, 34BXX Boundary value problem.

1. Introduction

Dual series equations and triple integral equations arise frequently in mixed boundary value problems of Mathematical Physics. Here solution of the following simultaneous dual series equations has been discussed:

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s}\prod_{k=1}^{r} a_{ij_k}\frac{A_{nj_k}}{\Gamma(2\beta_k+\sigma_k+ni_k+1)}\, k_{2(ni_k+\nu_k)}^{2(\alpha_k+\sigma_k)}(x_k) = f_i(x_1, x_2, \ldots\ldots, x_r), \quad 0 \le x_k < y_k \tag{1.1}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s}\prod_{k=1}^{r} b_{ij_k}\frac{A_{nj_k}}{\Gamma(2\nu_k+\sigma_k+ni_k+1)}\, k_{2(ni_k+\beta_k)}^{2(\beta_k+\sigma_k)}(x_k) = g_i(x_1, x_2, \ldots\ldots\ldots\ldots. x_r), \quad y_k < x_k < \infty \tag{1.2}$$

Where i= 1,2,....................,s and $\alpha_k+\sigma_k+1>0$, $\beta_k > \nu_k > \alpha_k- 1/2m_k$, $2\nu_k+\sigma_k+1>0$, σ_k are negative integers and m_k are non- negative integersfor k= 1,2,...............,r.

$k_\nu^\alpha(x)$ is the generalized Batman- k function defined by

$$k_\nu^\alpha(x)= \frac{2}{\pi}\int_0^{\pi/2}(2\cos\phi)^\alpha \cos(x\tan\phi - \nu\phi)\, d\phi, \quad \alpha > -1, \tag{1.3}$$

a_{ij_k} and b_{ij_k} are known constants, $f_i(x_1,x_2,\ldots\ldots,x_r)$ and $g_i(x_1,x_2,\ldots\ldots,x_r)$ are prescribed functions and A_{nj_k} are unknown coefficients to be determined for j= 1,2,............s ; k= 1,2,................,r.

2. *Some useful Results :*

The following results will be required in our investigation. First of all, we recall the following relationships[1,8] for the particular case $\alpha = \sigma = 0$, which exhibits the fact the generalized Batman- k functions are the well known confluent hyper geometric functions of Whittaker Swatson (1963):

$$e^{x} k^{2(\alpha+\sigma)}_{2(n+\alpha)}(x) = \frac{(-1)^{n-\sigma-1}}{\Gamma(2\alpha+2\sigma+2)} (2x)^{(2\alpha+2\sigma+1)} \cdot {}_1F^1 \left[\begin{matrix} \sigma - n + 1; \\ 2\alpha + 2\sigma + 2; \end{matrix} \quad 2x \right] \quad (2.1)$$

From [2.1], it is easy to deduce the orthogonality property*,

$$\int_0^\infty x^{-2\alpha-2\sigma-1} \, k^{2(\alpha+\sigma)}_{2(m+\sigma)}(x). k^{2(\alpha+\sigma)}_{2(n+\sigma)}(x) \, dx = \frac{2^{2\alpha+2\sigma}\,\Gamma(n-\sigma)}{\Gamma(2\alpha+\sigma+n+1)} \delta_{mn} \quad (2.2)$$

Where $\alpha+\sigma+1>0$ and δ_{mn} is the kronecker delta.

Also
$$\frac{d^m}{dx^m}\left\{e^{x} k^{2(\alpha+\sigma)}_{2(n+\sigma)}(x)\right\} = 2^m e^{x} k^{2(\alpha+\sigma)}_{2(n+\sigma)}(x) \quad (2.3)$$

Where m is a non- negative integer.

With the help of the relationship [2.1], one may readily obtain the following forms of the known integrals[4]:

$$\int_0^\xi e^{x}(\xi - x)^{\beta-1} k^{2(\alpha+\sigma)}_{2(n+\sigma)}(x)dx = \frac{\Gamma(\beta)}{2^\beta} e^{\xi} k^{2(\alpha+\sigma)+\beta}_{2(n+\sigma)+\beta}(\xi) \quad (2.4)$$

Where $\alpha + \sigma > -1, \beta > 0$ and

$$\int_\xi^\infty e^{-x} x^{-2\alpha-2\sigma-1}(x-\xi)^{\beta-1} k^{2(\alpha+\sigma)}_{2(n+\sigma)}(x)dx$$

$$= \frac{\Gamma(\beta)\Gamma(2\alpha-\beta+\sigma+n+1)}{\xi^{2\alpha-\beta+2\sigma+1}\Gamma(2\alpha+\sigma+n+1)} . e^{-\xi} k^{2(\alpha+\sigma)-\beta}_{2(n+\sigma)-\beta}(\xi) \quad [2.5]$$

Where $2\alpha+\sigma+n+1> \beta>0$

*Throughout this paper σ will be understood to take on negative integral values[2-5].

3. *Solution of the equations :*

Multiplying equation [1.1] by $e^{x_k}(\xi_k - x_k)^{2\nu_k - 2\alpha_k + m_k - 1}$, where m_k are non- negative integers and equation [1.2] by $e^{-x_k} x_k^{-2\beta_k - 2\sigma_k - 1}(x_k - \xi_k)^{2\beta_k - 2\nu_k - 1}$ and then integrating equations [1.1] and [1.2] with respect to x_k 'r' times over $(0,\xi_k)$ and (ξ_k,∞) respectively. On using formulas [2.4] and [2.5], we find

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s}\Pi_{k=1}^{r} a_{ij_k}\frac{A_{nj_k}}{\Gamma(2\beta_k + \sigma_k + ni_k + 1)} k_{2(ni_k+\nu_k)+m_k}^{2(\nu_k+\sigma_k)+m_k}(\xi_k)$$

$$= \Pi_{k=1}^{r}\frac{2^{2\nu_k - 2\alpha_k + m_k}}{\Gamma(2\nu_k - 2\alpha_k + m_k)} e^{-\xi_k}$$

$$. \int_0^{\xi_k} e^{x_k}(\xi_k - x_k)^{2\nu_k - 2\alpha_k + m_k - 1} f_i(x_1, x_2, \ldots, x_r) dx_k \quad (3.1)$$

Where $0 < \xi_k < y_k$, $\alpha_k + \sigma_k > -1$, $2\nu_k - 2\alpha_k + m_k > 0$, i= 1,2,..........,s and

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s}\prod_{k=1}^{r} b_{ij_k}\frac{A_{nj_k}}{\Gamma(2\beta_k + \sigma_k + ni_k + 1)} k_{2(ni+\nu)}^{2(\nu+\sigma)}(\xi_k)$$

$$=\prod_{k=1}^{r}\frac{\xi_k^{(2\nu_k + 2\sigma_k + 1)}}{\Gamma(2\beta_k - 2\nu_k)} e^{\xi_k}\int_{\xi}^{\infty} e^{-x_k} . x_k^{-2\beta_k - 2\sigma_k - 1}(x_k - \xi_k)^{2\beta_k - 2\nu_k - 1}$$

$$. g_i(x_1, x_2, \ldots \ldots \ldots, x_r) dx_k \quad (3.2)$$

Where $y_k < \xi_k < \infty$, $\beta_k > \nu_k$, $2\nu_k + \sigma_k + 1 > 0$, i= 1,2,.................,s.

Now multiplying equation [3.1] by e^{ξ_k} and differentiating the resulting equation m_k times with respect to ξ_k then, on using the derivative formula of Rainville[9], we find

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s}\prod_{k=1}^{r} b_{ij_k}\frac{A_{nj_k}}{\Gamma(2\beta_k+\sigma_k+ni_k+1)}\, k_{2(ni_k+\nu_k)}^{2(\nu_k+\sigma_k)}(\xi_k)$$

$$=\sum_{j=1}^{s}\prod_{k=1}^{r}\left[c_{ij_k}\frac{2^{2\nu_k-2\alpha_k-\xi_k}}{\Gamma(2\nu_k-2\alpha_k+m_k)}\frac{d^{m_k}}{d\xi_k^{m_k}}\int_0^{\xi_k} e^{x_k}(\xi_k-x_k)^{2\nu_k-2\alpha_k+m_k-1} . f_i(x_1,x_2,\ldots\ldots\ldots\ldots,x_r)dx_k\right] \tag{3.3}$$

Where c_{ij_k} are the elements of the matrix $[b_{ij_k}][a_{ij_k}]^{-1}$, $0<\xi_k<y_k$, $\alpha_k+\sigma_k>-1$, $2\nu_k-2\alpha_k+m_k>0$, $m_k=0,1,2,\ldots\ldots\ldots\ldots$; $i=1,2,\ldots\ldots\ldots\ldots\ldots\ldots,s$.

Hence the series equations [1.1] and [1.2] has been converted to the respective series equations [3.3] and [3.2]. The left hand sides of the series equations [3.3] and [3.2] are now identical and hence on using the orthogonality property [2.2] of Batman- k functions we find the solution of series equations [1.1] and [1.2] in the form[7-11]

$$\prod_{k=1}^{r} A_{nj_k}=\sum_{i=1}^{s}\prod_{k=1}^{r} d_{ij_k}\frac{\Gamma(2\beta_k+\sigma_k+ni_k+1)\Gamma(2\nu_k+\sigma_k+ni_k+1)}{2^{2\nu_k+2\alpha_k}\Gamma(ni_k-\sigma_k)}$$

$$\left[\left\{\sum_{j=1}^{s}\prod_{k=1}^{r}\right\}c_{ij_k}\frac{2^{2\nu_k-2\sigma_k}}{\Gamma(2\nu_k-2\alpha_k+m_k)}\int_0^{y_k}\xi_k^{-2\nu_k-2\sigma_k-1} . k_{2(ni_k+\nu_k)}^{2(\nu_k+\sigma_k)}(\xi_k)\,F_i(\xi_1,\xi_2,\ldots\ldots\ldots,\xi_r)d\xi_k + \frac{1}{\Gamma(2\beta_k-2\nu_k)}\int_{y_k}^{\infty} e^{\xi_k}\, k_{2(ni_k+\nu_k)}^{2(\nu_k+\sigma_k)}(\xi_k)\,G_i(\xi_1,\xi_2,\ldots\ldots\ldots,\xi_r)d\xi_k\right]$$

where d_{ij_k} are the elements of the matrix $[b_{ij_k}]^{-1}$ (3.4)

and

$$F_i(\xi_1, \xi_2, \ldots\ldots\ldots.., \xi_r)$$

$$= \prod_{k=1}^{r} \left[\frac{2^{2\nu_k - 2\alpha_k - \xi_k}}{\Gamma(2\nu_k - 2\alpha_k + m_k)} \frac{d^{m_k}}{d\xi_k^{m_k}} \int_0^{\xi_k} e^{x_k} (\xi_k - x_k)^{2\nu_k - 2\alpha_k + m_k - 1} . f_i(x_1, x_2, \ldots\ldots\ldots\ldots\ldots., x_r) dx_k \right]$$

where $0 \leq \xi_k < y_k$, k= 1,2,..........,r and i= 1,2,................,s. (3.5)

and $G(\xi_1, \xi_2, \ldots\ldots\ldots.., \xi_r)$

$$= \prod_{k=1}^{r} \frac{\xi_k^{(2\nu_k + 2\sigma_k + 1)}}{\Gamma(2\beta_k - 2\nu_k)} e^{\xi_k} \int_{\xi}^{\infty} e^{-x_k} . x_k^{-2\beta_k - 2\sigma_k - 1} (x_k - \xi_k)^{2\beta_k - 2\nu_k - 1} . g_i(x_1, x_2, \ldots\ldots\ldots.., x_r) dx_k$$

where $y_k < \xi_k < \infty$, k= 1,2,.................,r and i= 1,2,..................,s. (3.6)

provided $\alpha_k+\sigma_k+1 > 0$, $\beta_k > \nu_k > \alpha_k - m_k$, $2\nu_k+\sigma_k+1 > 0$, $\sigma_k+1 \leq 0$, m_k are non- negative integers[7-10] for k= 1,2,...............,r and n= 0,1,2,.............; j= 1,2,.............,s.

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