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Mathematics for modelling

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Abstract

This article is an investigation of Mathematics from the point of view of Application of Mathematics for Modelling.

Key words: Model, Modelling, Mathematical Modelling.

Introduction

Science and technology of all kinds are concerned with describing the behavior of, and relationships among observations. What distinguishes the various fields are the quantities being described and related.

In Medicine we are interested in measurements such as blood pressure metabolic rate, heart rate, body temperature as well as treatment measurements such as drug dosage and measurements describing surgical procedures.

In Mechanical Engineering, the measurements we deal with positions and velocities of mechanical system components such as various types of engines.

In Chemistry we focus on the rates of formation and breakdown of different chemical bonds.

In all of these systems we need to describe the quantities of interest and find appropriate descriptions of how they are related.

The function of the Mathematics developed in this work is to provide the most useful framework that we can for accomplishing these aims.

The historical and epistemological reflection on the application of Mathematics to the Modelling. Mathematical modelling is an essential tool for understanding the world.

In other words Mathematical Modelling[1-2] is the technique of translating real world problems in to mathematical problems.

Modelling today generates attention and interest because of increasing use of Mathematical Models in all sciences and their high level of sophistication.

The goal of our paper is to give opportunities to Physicists, Biologists, Mathematicians and Historians of science to take information and reflect on the way Mathematics and Mathematical Models are used in the natural sciences today and in past.

Preliminaries: Basic terminology and properties of Mathematical Modelling are given in this paper. For other definitions readers can refer to Borrowed From V.A. Bokil[1] Dr. Gerd De Vries[2], Judha, Rosenblatt And Stoughton Bell[3], Dilwyn, Edwards & Mike Hamson[4]

The real world refers to :

Engineering, Physics, Physiology, Ecology, Wildlife, management, Chemistry, Economics, Sports.

Questions of real world :

- A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria.

Find the number of bacteria after 4 hours?

- Suppose that an object is falling in the atmosphere near the sea level. Assume that the drag is proportional to the velocity with the drag coefficient of 2 kg/sec and that the mass of the object is 10 kg.

Model :

- A description or analogy used to help visualize something (e.g. An atom) that can be directly observed.
- A system of postulates, data and inferences presented as a mathematical description of an entity or state of affairs.
- A miniature representation of something.
- A pattern of something to be made.
- An example imitation or emulation.
- Set of equation is called model for the system.
- Models are abstractions of reality!
- Models are representation of a particular thing, idea or condition.

Mathematical Model :

- **Is a description of system using mathematical concepts and language?**
- **A representation in mathematical terms of the behaviors of real devices and objects.**
- **Mathematical model are characterized by assumptions about:**
    1. Variables (the thing which change)
    2. Parameters (the thing which do not change)
    3. Functional forms (the relationship between the two)

- *Mathematical models are simplified representation of some real-world entity.*

    1. Can be equation or computer code.
    2. Are intended to mimic essential features which leaving out inessentials.

Types of Mathematical Models :

- *Deterministic vs. Stochastic models :*
    - Deterministic models have no components[3-4] that are inherently uncertain, *i.e.*, No parameters in the model are characterized by probability distributions, as opposed to stochastic models.
    - For fixed starting values, a deterministic

model will always produce the same result. A stochahastic model will produce many different results depending on the actual values that the random variables take in each realization.

◆ *Static vs. Dynamic models :*

◆ Static models are at an equilibrium or steady state, as opposed to dynamic models which change with respect to time.

◆ *Continuous vs. Discrete models :*

◆ Differential vs. Difference equations

◆ *Individual vs. Structured models :*

◆ Structured models based on age, size, stage, etc.

◆ *Mechanistic vs. Statistical models :*

◆ Statistical or empirical models are usually regression based. They provide a quantitative summary of the observed relationship among a set of measured variables.

◆ A mechanistic or scientific model begins with a description of how nature might work, and proceeds from this description to a set of predictions relating the independent and dependent variables.

◆ ***Qualitative vs. Quantitative models :***

◆ Qualitative models lead to a detailed, numerical predicition about responses, whereas qualitative models lead to general descriptions about the responses.

Modelling :

• Is an activity, a cognitive a activity. In which we think about and make models to describe how devices or objects of interest behave

Mathematical Modelling :

The process of developing a mathematical model is termed mathematical modelling.

In other words, the process of writing an equation describing the situation is called the mathematical modelling.

Mathematical Modelling is the use of Mathematics to :

- Describe real-world phenomena
- Investigative important questions about the observed world
- Explain real-world phenomena
- Test ideas
- Make predictions about the real world

One can think of mathematical modelling as an activity or process that allows a mathematician to be a chemist, an ecologist, an economist, a physiologist..............

Instead of undertaking experiments in the real world, a modeller undertakes experiments on mathematical representations of the real world.

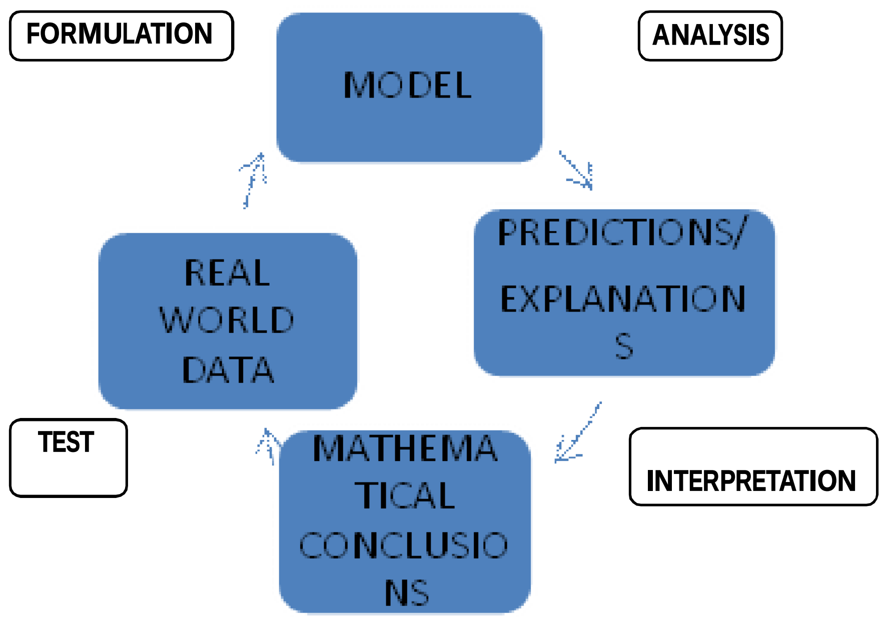
The process of Mathematical Modelling :

.... Is a series of steps taken to convert an idea first into a conceptual model and then into a quantitative model.

☆ A conceptual model represents our ideas about how the system works. It is expressed visually in a model diagram, typically involving boxes (state variables) and arrows (material flows or causal effects).

☆ Equations are developed for the rates of each process and are combined to form a quantitative model consisting of dynamic (*i.e.*, Varying with time) equations for each state variable.

☆ The dynamic equations can then be studied mathematically or translated into computer code to obtain numerical solutions for state variable trajectories.



THERE IS NO BEST MODEL, ONLY BETTER MODELS

Queries to write successful Model :

1. Identify the real problem. Identify the problem variables.

- ❑ What do we need to find out? What are the problems asking for?

2. Construct appropriate relation between the variables- a differential equation

- ❑ What is dependent, what independent variable and what is the rate of change? Figuring out how these quantities are related will result in a differential equation that models that problem.

3. Obtain the mathematical solution

- ❑ Recognize the type of the equation. Decide if you can solve it analytically ('by hand') or if you need to use the technology. In both cases, decide on the method that you will use (e.g. Is the equation separable, linear or some other type; could Euler's method, ode45 or other numerical solution be found).

4. Interpret the mathematical solution.

- ❑ After solving the equation, check if the mathematical answer make sense? Do the predictions agree with real data? Do the values have correct sign? Correct units? Correct size? Check effectiveness: could a simpler model be used? Have I found a right balance between greater precision (*i.e.* Greater complexity) and simplicity?

Examples of real-world questions that can be investigated with mathematical models

@ **A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8000 bacteria. Find the number of bacteria after 4 hours?**

Discussion

Identifying variables:

let y stands for the bacteria culture and t stands for time passed.

The first part of the problem "A bacteria culture starts with 500 bacteria…. "tells us that y (0) = 500.

The second part" …… and grows at a rate proportional to its size.
" is the key for getting the mathematical model.
Recall that the rate is the derivative and that "….. Is proportional to..."

Corresponds to "equal to constant multiple of …." So,

the equation relating the variables is $\frac{dy}{dt} = ky$.

The Solution of this differential equation is $y = y_0 e^{kt}$ Since y_o=500,

it remains to determine the proportionality constant *k*.

From the condition " After 3 hours there are 8000 bacteria. 'we obtain that 8000=$500e^{3k}$

Which gives us that? $k = \frac{1}{3}\ln 16 = .924.$

Thus, the number of bacteria after t hours can be described by $y=500e^{324t}$

- **Solution.**
Using the function we have obtained, we find the number of bacteria after 4 hours to be

$y\,(4) = 20159$ bacteria.

Why do mathematical modelling?

☆ *Scientific understanding:*

★ A model embodies a hypothesis about the study system, and lets you compare that hypothesis with data.

★ A model is often most useful when it fails to fit the data, because that says that some of your ideas about the study system are wrong.

★ Mathematical models and computer simulations are useful experimental tools for building and testing theories, assessing quantitative conjectures, answering specific questions, determining sensitivities to changes in parameter values and estimating key parameters from data.

☆ *Clarification :*

★ The model formulation process clarifies assumptions, variables, and parameters

★ the process of formulating an ecological model is extremely helpful for organizing one's thinking, bringing hidden assum-ptions to light and identifying data needs…… do you really have all the necessary pieces?!

☆ *Using our scientific understanding to manage the world :*

★ Forecasting disease or pest outbreaks

★ Designing man-made systems, for example, biological pwest control,

bioengineering
- ★ managing existing systems such as agriculture or fisheries
- ★ OPTIMIZING MEDICAL TREATMENTS

☆ *Simulated experimentation :*

Realistic experimenting may be impossible
- ★ Experiments with infectious disease spread in human populations are often impossible, unethical or expensive.
- ★ We cannot mange endangered species by trial and error.
- ★ We dare not set dosage for clinical trials of new drugs on humans or set safe limits for exposure to toxic substances without proper knowledge of the consequences.

☆ *The curse of dimensionality :*
- ★ Sometimes a purely experimental approach is not feasible because the data requirements for estimating a model grow rapidly in the number of variables.
- ★ Modeling using computer programs is cheap

Challenges in mathematical modelling :

"……….Not to produce the most comprehensive descriptive model **But**

To produce the simplest possible model that incorporates the major features of the phenomenon of interest"

HOWARD EMMONS

LIMITATIONS OF MODELS

"Hence, our truth is the intersection of independent lies." Levin's, r. 1966. The strategy of model building in population biology. Am. Sci. 54:421-431.

LEVIN'S

- ★ The bottom line is that models are abstractions of reality. LEVINS (1966) points out that modeling is essentially a tradeoff between: generality, realism and precision.
- ★ The usefulness of any particular model depends on the modeler's goals. To describe general ecological principles, it is usually necessary to sacrifice realism and precision. To describe a particular population, it is usually necessary to sacrifice generality.

Conclusion

- ★ Modeling clarifies what the underlying assumptions are
- ★ Model analysis and simulation predictions suggest crucial data that should be gathered
- ★ Model analysis and simulation suggest control strategies that could be implemented.
- ★ Modeling is not prefect and usually is a simplification of reality. Remember a model is only as a good as its assumptions are!

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