Ultra Scientist Vol. 24(3)A, 591-600 (2012).

Simultaneous triple integral equations involving fox's H-functions

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(Acceptance Date 17th December, 2012)

Abstract

The formal solution of certain simultaneous triple integral equations involving Fox's H-functions is obtained by the method of fractional integration. By the application of fractional integration operators, the given simultaneous triple integral equations are transformed into three others with a common kernel and the problem then reduced to that of solving one integral equation.

Key words : 45-XX Integral Equations, 45F10 Simultaneous Triple Integral Equations, 33C60 Fox' H function, 45 H05 (Special kernels) unsymmetrical Fourier kernel, 33C XX Hypergeometric Function, 44A15 Special Transforms (Mellin), 26A33 Fractional derivatives and Integrals, 44A20 Transforms of special functions, 45E99 Suitable contour Barnes type.

1. Introduction

Fox c[6] defined the H-function as follows:

$$H_{p,q}^{m,n}[z] = H_{p,q}^{m,n}\left[z \mid \begin{matrix}(a_j,\alpha_j)\\(b_j,\beta_j)\end{matrix}\right] = \frac{1}{2\Pi i}\int_L \phi(s)\ z^s\, ds$$

where

$$\phi(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\ \prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\ \prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)} \qquad [1.1]$$

z is not equal to zero, an empty product is to be interpreted as unity and the following simplified assumptions are made[2-5]:

(i) m, n, p and q are integers satisfying $0 \le m \le q$, $n \le p$.

(ii) $\alpha_j{}'s : (j = 1,2,\ldots\ldots\ldots\ldots,p)$ and $\beta_j{}'s : (j = 1,2,\ldots\ldots\ldots\ldots,q)$ are positive numbers.

(iii) a_j=(j=1,2, ..., p), b_j:(j=1,2, ..., q) are complex numbers such that no pole of $\Gamma(b_h-\beta_h s)$, h=1,2,……,m coincides with any pole of $\Gamma(1-a_j+\alpha_j s)$, j=1,2, ..., n. i.e. $\alpha_j(b_h+\upsilon)\neq \beta_h(a_j+\eta-1)$ for ν,=0,1,2,…

………. and h=1,2,………,m and j=1,2,……….,n.

(iv) All the poles of the integrand in [1.1] are simple.

(v) The contour L is Mellin Barnes type which runs from σ-i∞ to σ+i∞ in S(=σ+i, σ and t being real) plane such that the points $s=\frac{(b_h+\upsilon)}{\beta_h}$, h=1,2,…………,m; ν=0,1,2, ……… which are the poles of $\Gamma(b_n-\beta_h s)$ lie to the right and the points $s=\frac{(a_j-\eta-1)}{\alpha_j}$, j=1,2,………….,n ; η=0,1,2,……… which are the poles of $\Gamma(1-a_j+\alpha_j s)$ lie to the left of L. Such a contour L is possible on account of (iii).

(vi) The conditions for the convergence of the integral [5.1.1] can be found in the research paper of Braaksma[1].

In an earlier section, Fox, c.[6] introduced the H-function in the following manner

$$H^{q,p}_{2p,2q}\left[x\,\Big|\,\begin{matrix}(1-a_i,\alpha_j)\ ,(a_i-\alpha_i\ ,\alpha_i)\\ (b_j,\beta_j)\quad ,(1-b_j-\beta_j,\beta_j)\end{matrix}\right]$$

$$=H^{q,p}_{2p,2q}(x)=\frac{1}{2\Pi i}\int_L \chi_{p,q}(s)\,x^{-s}\,ds$$

where

$$\chi_{p,q}=\prod_{i=1}^{q}\frac{\Gamma(b_i+s\beta_i)}{\Gamma(b_i+\beta_i-s\beta_i)}\prod_{j=1}^{p}\frac{\Gamma(a_j-s\alpha_j)}{\Gamma(a_j-\alpha_j+s\alpha_j)}\qquad [1.2]$$

behaves as Symmetrical Fourier kernel

2. Results used in the proof of the sequel:

Mellin Transform:

$$M\{f(x)\}=F(s)=\int_0^\infty f(x)x^{s-1}\,dx\quad [2.1]$$

Inverse Mellin Transform:

$$M^{-1}\{F(s)\}=f(x)$$

$$=\frac{1}{2\Pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}F(s)\ x^{-s}ds\qquad [2.2]$$

For s=σ+it , x>0

Parseval's Theorem for Mellin Transform:

If M {f (u)} =F(s) and M {a (u)} =A(s) Then

$$M\{a(ux)\}=x^{-s}A(s)\quad \text{and}$$

$$\int_0^\infty f(ux)\ a(u)du=\frac{1}{2\Pi i}\int_L x^{-s}\,F(s)A(1-s)ds\qquad [2.3]$$

Fox's Beta Formulae:

Fox defined Beta formulae by following fractional integrals[5-10]:

$$\int_0^x (x^{1/c}-\upsilon^{1/c})^{d-e-1}\,\upsilon^{\frac{e}{c}-s-1}d\upsilon$$

$$=\frac{c\Gamma(d-e)\Gamma(e-cs)}{\Gamma(d-cs)}x^{\frac{d}{c}-\frac{1}{c}-s}\qquad[2.4]$$

Provided d>e and $\frac{e}{c}>\sigma$ where s = σ+it and 0< x < 1

$$\int_x^\infty (\upsilon^{1/c}-x^{1/c})^{d-e-1}\,\upsilon^{\frac{1}{c}-\frac{d}{e}-s-1}d\upsilon$$

$$=\frac{c\Gamma(d-e)\Gamma(e+cs)}{\Gamma(d+cs)}x^{-\frac{e}{c}-s}\qquad [2.5]$$

Provide d>e and $\frac{e}{c}>\sigma$ where s= σ+it and x>1

Fractional Erdelyi-Kober Operator:

Fox used the following generalized Erdelyi-Kober operators:

$$T[\gamma,\epsilon:\ m]\{f(x)\} = \frac{m}{\Gamma\gamma} x^{-\gamma m-\epsilon+m-1}\int_0^x (x^m-\upsilon^m)^{\gamma-1}\,\upsilon^e f(\upsilon)d\upsilon \qquad [2.6]$$

Where 0<x<1

$$R[\gamma,\epsilon:\ m]\ \{f(x)\} = \frac{m}{\Gamma\gamma} x^{\epsilon}\int_x^\infty (\upsilon^m-x^m)^{\gamma-1}\,\upsilon^{-\epsilon-\gamma m+m-1} f(\upsilon)d\upsilon \qquad [2.7]$$

Where x > 1

The operator T exists. If $f(x)\in L_p(0,\infty)$, p>1 , $\gamma>0$ and $\in>(1-p)/p$ and If , f(x) can be differentiated sufficient number of times then the operator T exists for both negative and positive value of γ. The operator R exists. If $f(x)\in L_p(0,\infty)$, $p\geq1$ and If, f(x) can be differentiated sufficient number of times then the operator R exists. If $m > \in > -1/p$ while γ can take any negative or positive value[1-5].

A Theorem For Mellin Transform:

If M{f(u)}=F(s) and M{g(u)}=G(s) then

$$\int_0^\infty g(u)\,f(u)du = \frac{1}{2\Pi i}\lim_{\substack{T\to\infty \\ \&\ \sigma_0=\mathrm{Re}(s)}}\int_{\sigma_0-iT}^{\sigma_0+iT} G(s)\,F(1-s)ds$$

$$= \frac{1}{2\Pi i}\int_L G(s)\,F(1-s)ds \qquad [2.8]$$

Thus If g (ux) is considered to be a function of u with x as a parameter, where x > 0

Then $M\{g(ux)\}= x^{-s}G(s)$ [2.9]

From [2.8] and [2.9], we have

$$\int_0^\infty g(ux)\,f(u)du = \frac{1}{2\Pi i}\lim_{\substack{T\to\infty \\ \&\ \sigma_0=\mathrm{Re}(s)}}\int_{\sigma_0-iT}^{\sigma_0+iT} x^{-s}G(s)\,F(1-s)ds$$

$$= \frac{1}{2\Pi i}\int_L x^{-s}G(s)\,F(1-s)ds \qquad [2.10]$$

Additional conditions for the validity of [2.10] are that

F(s) $\epsilon L_p(\sigma_0 - i\infty, \sigma_0 + i\infty)$

and $x^{1-\sigma_0} g(x)\ \epsilon L_p(0,\infty)\ p \geq 1$ where L_p denotes the class of functions g(x) such that

$$\int_0^\infty |g(x)|^p \frac{dx}{x} < \infty \qquad [2.11]$$

3. Solution of simultaneous triple integral equations involving fox's H-functions as symmetrical fourier kernel :

Here we consider following integral equations:

$$\int_0^\infty H_{2p,2q}^{q,p}\left[ux \left| \begin{matrix} (1-\alpha_i^k,\alpha_i)\ ,(\alpha_i^k-\alpha_i\ ,\alpha_i) \\ (b_j^k,\beta_j)\ \ ,(1-b_j^k-\beta_j\ ,\beta_j) \end{matrix}\right.\right] \sum_{h=1}^n a_{hk}\, f_h(u)du = L_k(x), \qquad [3.1]$$

$$0 < x < \xi$$

$$\int_0^\infty H_{2p,2q}^{q,p}\left[ux \left| \begin{matrix} (1-c_i^k,\alpha_i)\ ,(a_i^k-\alpha_i\ ,\alpha_i) \\ (b_j^k,\beta_j)\ \ ,(1-d_j^k-\beta_j\ ,\beta_j) \end{matrix}\right.\right] \sum_{h=1}^n b_{hk}\, f_h(u)du = M_k(x), \qquad [3.2]$$

$$\xi < x < \eta$$

$$\int_0^\infty H_{2p,2q}^{q,p}\left[ux \left| \begin{matrix} (1-c_i^k,\alpha_i)\ ,(c_i^k-\alpha_i\ ,\alpha_i) \\ (d_j^k,\beta_j)\ \ ,(1-d_j^k-\beta_j\ ,\beta_j) \end{matrix}\right.\right] \sum_{h=1}^n c_{hk}\, f_h(u)du = N_k(x), \qquad [3.3]$$

$$\eta < x < \infty$$

Where a_{hk}, b_{hk} and c_{hk} are well known constants and $L_k(x)$, $M_k(x)$ and $N_k(x)$ are prescribed functions for h=1,2,………n , k=1,2,……….n. Here f_h (u) is unknown function in integral equations [3.1],[3.2] and [3.3] for h=1,2,…………..,n and $0 < \xi < 1, \xi < 1 < \eta$ and $1 < \eta < \infty$.

Now applying Fox's result to integral equations [3.1], [3.2] and [3.3] then we have from Persaval theorem of Mellin transform6-[12]

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^q \Gamma(b_i^k + s\beta_i)}{\prod_{i=1}^q \Gamma(b_i^k + \beta_i - s\beta_i)} \frac{\prod_{j=1}^p \Gamma(a_j^k - s\alpha_j)}{\prod_{j=1}^p \Gamma(a_j^k - \alpha_j + s\alpha_j)} x^{-s} \sum_{h=1}^n a_{hk}\, F_h\,(1-s)ds = L_k(x)$$

Where $0 < x < \xi$ and k=1,2,………….,n. [3.4]

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(b_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(d_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p}\Gamma\left(c_j^k-s\alpha_j\right)}{\prod_{j=1}^{p}\Gamma\left(a_j^k-\alpha_j+s\alpha_j\right)}x^{-s}\sum_{h=1}^{n}b_{hk}\,F_h\,(1-s)ds = M_k(x)$$

where $\xi < x < \eta$ and k=1, 2, …………., n. [3.5]

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(d_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(d_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p}\Gamma\left(c_j^k-s\alpha_j\right)}{\prod_{j=1}^{p}\Gamma\left(c_j^k-\alpha_j+s\alpha_j\right)}x^{-s}\sum_{h=1}^{n}c_{hk}\,F_h(1-s)ds = N_k(x)$$

where $\eta < x < \infty$ and k=1,2,………….,n. [3.6]

In integral equation [3.4] replacing x by υ and multiplying both sides of the equation [3.4] by

$$(x^{1/\alpha_p}-\upsilon^{1/\alpha_p})^{\alpha_p^k-c_p^k-1}.\,\upsilon^{\frac{c_p^k}{\alpha_p}-1}$$

and integrating both sides of integral equation [3.4] with respect to υ from 0 to x where $0 < x < \xi$ and applying Fox's Beta formula [2.4] in integral equation [3.4], we find

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(b_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(b_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p-1}\Gamma\left(\alpha_j^k-s\alpha_j\right)\Gamma\left(c_p^k-s\alpha_p\right)}{\prod_{j=1}^{p}\Gamma\left(\alpha_j^k-\alpha_j+s\alpha_j\right)} \qquad [3.7]$$

$$x^{-s}\sum_{h=1}^{n}a_{hk}\,F_h(1-s)ds = \frac{1}{\alpha_p\Gamma(\alpha_p^k-c_p^k)}(x)^{\frac{(1-a_p^k)}{\alpha_p}}$$

$$.\int_0^x (x^{1/\alpha_p}-\upsilon^{1/\alpha_p})^{\alpha_p^k-c_p^k-1}.\,\upsilon^{\frac{c_p^k}{\alpha_p}-1}L_k(\upsilon)d\upsilon$$

where $0<x<\xi$ and k=1,2,………….,n.

Using the Erdelyi-Kober Operator T from [2.6] in equation [3.7] for brevity, we write

$$T\left[a_j^k-c_j^k,\frac{c_j^k}{\alpha_j}-1\,;\frac{1}{\alpha_j}\right]\{\,L_k(x)\} = T_j^1\{\,L_k(x)\} \qquad [3.8]$$

where $0 < x < \xi$ and k=1,2,………….,n.

then

$$T\left[a_p^k-c_p^k,\frac{c_p^k}{\alpha_p}-1\,;\frac{1}{\alpha_p}\right]\{\,L_k(x)\} = T_p^1\{\,L_k(x)\} \qquad [3.9]$$

where $0<x<\xi$ and k=1,2,…………..,n.

Hence from [3.9] , the integral equation [3.7] can be written as

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(b_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(b_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p-1}\Gamma\left(\alpha_j^k-s\alpha_j\right)\Gamma\left(c_p^k-s\alpha_p\right)}{\prod_{j=1}^{p}\Gamma\left(\alpha_j^k-\alpha_j+s\alpha_j\right)}$$

$$x^{-s}\sum_{h=1}^{n}a_{hk}\,F_h(1-s)ds=T_p^1\{L_k(x)\} \qquad [3.10]$$

where $0<x<\xi$ and k=1,2,..,n

Now repeating the same process in integral equation [3.10] for j=p-1, p-2 ,……,3,2,1. Then the integral equation [3.10] takes the form

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(b_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(b_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p}\Gamma\left(c_j^k-s\alpha_j\right)}{\prod_{j=1}^{p}\Gamma\left(a_j^k-\alpha_j+s\alpha_j\right)}x^{-s}$$

$$\sum_{h=1}^{n}a_{hk}\,F_h(1-s)ds=\prod_{j=1}^{p}T_j^1\{L_k(x)\} \qquad [3.11]$$

where $0<x<\xi$ and k=1,2,..,n

Likewise in integral equation [3.11] we can transform the factor $(b_i^k+\beta_i-s\beta_i)$ to $\Gamma(d_i^k+\beta_i-s\beta_i)$ by using Fox's Beta formula [2.4] and the operator T_i^2 where

$$T[d_i^k-b_i^k,\frac{b_i^k}{\beta_i}:\ \frac{1}{\beta_i}]\{L_k(x)\}\ =\ T_i^2\{L_k(x)\} \qquad [3.12]$$

Accordingly [3.11] takes the form

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(b_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(d_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p}\Gamma\left(c_j^k-s\alpha_j\right)}{\prod_{j=1}^{p}\Gamma\left(a_j^k-\alpha_j+s\alpha_j\right)}x^{-s}$$

$$\sum_{h=1}^{n} a_{hk}\, F_h(1-s)ds = \left\{\prod_{i=1}^{q} T_i^2\right\}\left\{\prod_{j=1}^{p} T_j^1\right\}\{\, L_k(x)\} \qquad [3.13]$$

where 0<x<ξ and k=1, 2,………….,n.

Or

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(b_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(d_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p}\Gamma\left(c_j^k-s\alpha_j\right)}{\prod_{j=1}^{p}\Gamma\left(a_j^k-\alpha_j+s\alpha_j\right)}x^{-s}$$

$$\sum_{h=1}^{n} b_{hk}\, F_h(1-s)ds = \sum_{h=1}^{n} d_{hk}\left\{\prod_{i=1}^{q} T_i^2\right\}\left\{\prod_{j=1}^{p} T_j^1\right\}\{\, L_k(x)\} \qquad [3.13]$$

where 0<x<ξ and k=1, 2,………….,n, where d_{hk} are the elements of the matrix

$$[b_{hk}][a_{hk}]^{-1}$$

Similarly in integral equation [3.6] we can transform the factor (c_j^k-α_j+sα_j) to (a_j^k +sα_j) by using Fox's Beta formula [2.5] and the operator R_j^1 where

$$R\left[a_j^k-c_j^k,\frac{c_j^k}{\alpha_j}-1:\frac{1}{\alpha_j}\right]\{N_k(x)\} = R_j^1\{N_k(x)\} \qquad [3.14]$$

According [3.6] takes the form

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(d_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(d_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p}\Gamma\left(c_j^k-s\alpha_j\right)}{\prod_{j=1}^{p}\Gamma\left(a_j^k-\alpha_j+s\alpha_j\right)}x^{-s} \qquad [3.15]$$

$$\sum_{h=1}^{n} c_{hk}\, F_h(1-s)ds = \prod_{j=1}^{p} R_j^1\{N_k(x)\}$$

where η<x<∞ and k=1, 2,……….,n.

Likewise in integral equation [3.15] we can transform the factor $\boldsymbol{\Gamma}\left(d_i^k+\beta_i s\right)$ to $\boldsymbol{\Gamma}(b_i^k+\beta_i s)$ by using Fox's Beta formula [2.5] and the operator R_i^2 where

$$R\left[d_i^k-b_i^k,\frac{b_i^k}{\beta_i}:\frac{1}{\beta_i}\right]\{N_k(x)\} = R_i^2\{N_k(x)\} \qquad [3.16]$$

Accordingly [3.15] takes the form

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(b_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(d_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p}\Gamma\left(c_j^k-s\alpha_j\right)}{\prod_{j=1}^{p}\Gamma\left(a_j^k-\alpha_j+s\alpha_j\right)}x^{-s}$$

$$\sum_{h=1}^{n} c_{hk}\,F_h(1-s)ds = \left\{\prod_{i=1}^{q} R_i^2\right\}\left\{\prod_{j=1}^{p} R_j^1\right\}\{N_k(x)\} \qquad [3.17]$$

Where $\eta<x<\infty$ and k=1,2,……….,n.

Or

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma\left(b_i^k+s\beta_i\right)}{\prod_{i=1}^{q}\Gamma\left(d_i^k+\beta_i-s\beta_i\right)}\frac{\prod_{j=1}^{p}\Gamma\left(c_j^k-s\alpha_j\right)}{\prod_{j=1}^{p}\Gamma\left(a_j^k-\alpha_j+s\alpha_j\right)}x^{-s}$$

$$\sum_{h=1}^{n} b_{hk}\,F_h(1-s)ds = \sum_{h=1}^{n} e_{hk}\left\{\prod_{i=1}^{q} R_i^2\right\}\left\{\prod_{j=1}^{p} R_j^1\right\}\{N_k(x)\} \qquad [3.18]$$

Where e_{hk} are the elements of the matrix $[b_{hk}][c_{hk}]^{-1}$ where h=1,2,………..,n and k=1,2,………,n and $\eta<x<\infty$.

Hence Integral equations [3.4], [3.5] and [3.6] reduced to three corresponding integral equations [3.13], [3.5] and [3.18] having common kernel. If we now write

$$g_k(x) = \begin{cases} \sum_{h=1}^{n} d_{hk}\left\{\prod_{i=1}^{q} T_i^2\right\}\left\{\prod_{j=1}^{p} T_j^1\right\}\{L_k(x)\}, & \text{when } 0<x<\xi \\ M_k(x), \text{ when } \xi<x<\eta & \\ \sum_{h=1}^{n} e_{hk}\left\{\prod_{i=1}^{q} R_i^2\right\}\left\{\prod_{j=1}^{p} R_j^1\right\}\{N_k(x)\}, & \text{when } \eta<x<\infty \end{cases} \qquad [3.19]$$

where k=1, 2, ……….., n.

Then integral equations [3.13], [3.5] and [3.18] can be put into the compact form as

$$\frac{1}{2\Pi i}\int_L \frac{\prod_{i=1}^{q}\Gamma(b_i^k+s\beta_i)}{\prod_{i=1}^{q}\Gamma(d_i^k+\beta_i-s\beta_i)}\frac{\prod_{j=1}^{p}\Gamma(c_j^k-s\alpha_j)}{\prod_{j=1}^{p}\Gamma(a_j^k-\alpha_j+s\alpha_j)}x^{-s}$$

$$\sum_{h=1}^{n} b_{hk}\,F_h(1-s)ds = g_k(x) \quad [3.20]$$

where k=1, 2,……….., n & $x\in(0.\infty)$

Hence from the theorem of Mellin Transform [2.10] we can write

$$\frac{1}{2\Pi i}\int_L M\left\{H_{2p,2q}^{q,p}\left[ux\,\middle|\,\begin{matrix}(1-c_i^k,\alpha_i)\ ,(a_i^k-\alpha_i\ ,\alpha_i)\\(b_j^k,\beta_j)\qquad ,(1-d_j^k-\beta_j\ ,\beta_j)\end{matrix}\right]\right\}$$

$$\sum_{h=1}^{n} b_{hk}\,F_h(1-s)ds = g_k(x) \quad [3.21]$$

where k=1, 2,……….., n.

On applying the Parseval theorem [2.3] and treating $H_{2p,2q}^{q,p}[ux]$ as a symmetrical Fourier kernel, the formal solution of [3.21] is given by

$$f_h(u)=\sum_{k=1}^{n} f_{hk}\int_0^{\infty} H_{2p,2q}^{q,p}\left[ux\,\middle|\,\begin{matrix}(1-c_i^k,\alpha_i),(a_i^k-\alpha_i,\alpha_i)\\(b_j^k,\beta_j),(1-d_j^k-\beta_j,\beta_j)\end{matrix}\right]g_k(u)du$$

where h=1,2,…………,n. [3.22]

i.e.

$$f_h(u)=\sum_{k=1}^{n} f_{hk}\left[\begin{matrix}\int_0^{\xi} H_{2p,2q}^{q,p}\left[ux\,\middle|\,\begin{matrix}(1-c_i^k,\alpha_i)\ ,(a_i^k-\alpha_i\ ,\alpha_i)\\(b_j^k,\beta_j)\qquad ,(1-d_j^k-\beta_j\ ,\beta_j)\end{matrix}\right]\\ \sum_{h=1}^{n} d_{hk}\left\{\prod_{i=1}^{q}T_i^2\right\}\left\{\prod_{j=1}^{p}T_j^1\right\}\{L_k(u)\}\,du+\\ \int_{\xi}^{\eta} H_{2p,2q}^{q,p}\left[ux\,\middle|\,\begin{matrix}(1-c_i^k,\alpha_i)\ ,(a_i^k-\alpha_i\ ,\alpha_i)\\(b_j^k,\beta_j)\qquad ,(1-d_j^k-\beta_j\ ,\beta_j)\end{matrix}\right]\\ M_k(u)du+\\ \int_{\eta}^{\infty} H_{2p,2q}^{q,p}\left[ux\,\middle|\,\begin{matrix}(1-c_i^k,\alpha_i)\ ,(a_i^k-\alpha_i\ ,\alpha_i)\\(b_j^k,\beta_j)\qquad ,(1-d_j^k-\beta_j\ ,\beta_j)\end{matrix}\right]\\ \sum_{h=1}^{n} e_{hk}\left\{\prod_{i=1}^{q}R_i^2\right\}\left\{\prod_{j=1}^{p}R_j^1\right\}\{N_k(u)\}\,du\end{matrix}\right]$$

[3.23]

where h=1,2,…………,n.

Where f_{hk} are the elements of the matrix $[b_{hk}]^{-1}$. Since our method is purely formal.

It does not give the conditions for the validity of the solution. Also the formal solution of integral equations [3.1], [3.2] and [3.3] is given by

$$f_h(u)=\sum_{h=1}^{n} f_{hk}\left[\begin{matrix}\int_0^{\xi} H_{2p,2q}^{q,p}\left[ux\,\middle|\,\begin{matrix}(1-c_i^k,\alpha_i)\ ,(\alpha_i^k-\alpha_i\ ,\alpha_i)\\(b_j^k,\beta_j)\qquad ,(1-d_j^k-\beta_j\ ,\beta_j)\end{matrix}\right]\\ \sum_{h=1}^{n} d_{hk}\left\{\prod_{i=1}^{q}T_i^2\right\}\left\{\prod_{j=1}^{p}T_j^1\right\}\{L_k(u)\}\,du+\\ \int_{\xi}^{\eta} H_{2p,2q}^{q,p}\left[ux\,\middle|\,\begin{matrix}(1-c_i^k,\alpha_i)\ ,(\alpha_i^k-\alpha_i\ ,\alpha_i)\\(b_j^k,\beta_j)\qquad ,(1-d_j^k-\beta_j\ ,\beta_j)\end{matrix}\right]\\ M_k(u)du+\\ \int_{\eta}^{\infty} H_{2p,2q}^{q,p}\left[ux\,\middle|\,\begin{matrix}(1-c_i^k,\alpha_i)\ ,(\alpha_i^k-\alpha_i\ ,\alpha_i)\\(b_j^k,\beta_j)\qquad ,(1-d_j^k-\beta_j\ ,\beta_j)\end{matrix}\right]\\ \sum_{h=1}^{n} e_{hk}\left\{\prod_{i=1}^{q}R_i^2\right\}\left\{\prod_{j=1}^{p}R_j^1\right\}\{N_k(u)\}\,du\end{matrix}\right]$$

where h =1,2,……………..,n.

where d_{hk} and e_{hk} are the element of the matrices $[b_{hk}]\,[a_{hk}]^{-1}$ and $[b_{hk}]\,[c_{hk}]^{-1}$ respectively and f_{hk} are the elements of the matrix $[b_{hk}]^{-1}$.

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