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Prime Labeling of Some Special Class of graphs

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Abstract

Prime labeling originated with Entringer and was introduced by Tout, Dabboucy and Howalla[5]. A Graph G(V,E) is said to have a **prime labeling** if its vertices are labeled with distinct integers 1,2,3,…,|V(G)| such that for each edge xy, the labels assigned to x and y are relatively prime. A graph admits a prime labeling is called a prime graph. In this paper, we prove that $K_n^c+K_2$, $(K_n^c+K_2) \odot K_1$, $F_m^{(n)}$, F_m @ $2P_n$, C_m @ $2P_n$ and $P_{n_{(m)}}$ are prime graphs.

1. Introduction

A simple graph G(V,E) is said to have a prime labeling (or called prime) if its vertices are labeled with distinct integers 1,2,3,…,|V(G)|, such that for each edge xy∈E(G), the labels assigned to x and y are relatively prime[2].

We begin with listing a few definitions\notations that are used.

- A graph G= (V,E) is said to have order |V| and size |E|.
- A vertex v∈V(G) of degree 1 is called pendant vertex.
- P_n is a path of length n-1.
- The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which has p_1 points) and p_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2[2].
- The graph $P_{n_{(m)}}$ = G (V, E) such that V (G)=$\{v_{ij}/1\le i\le m$ and and $1\le j\le n\}$ E (G)=$\{v_{ij}v_{i(j+1)}/1\le i\le m$ and $1\le j\le n-1\}\cup\{v_{i(n-1)} v_{(i+1)(n-1)}/ 1\le i\le m-1\}$
- The graph $G^{(t)}$ denotes the one point union of t copies of the graph G.
- G_1 @ G_2 is the one point union of G_1 and G_2.

2. Main Results

Theorem-2.1

$K_n^c+K_2$ is prime

Proof:

Let G= $K_n^c+K_2$

Let u and w be the vertices of K_2 and $v_1, v_2,...,v_n$ be the vertices of K_n^c.

Let V (G) = $\{u, w, v_i/1 \leq i \leq n\}$
Let E (G) = $\{uw, uv_i, wv_i/1 \leq i \leq n\}$

There are n+2 vertices and 2n+1 edges.
Define f: V$\rightarrow$ {1, 2,..., n+2} by
f (u)=1
f (v_i)= i+1, i=1,2,3,...n

Suppose n+2 is a prime number then f (w)= n+2
Suppose n+2 is not a prime

Let j be the greatest prime number less than n+2.

Then there exists a k such that f (v_k)=j, $1 \leq k \leq n$
f (v_i)= i+1, i=1,2,3,...k-1
f (v_k) = n+2
f (v_i) = i+1, i= k+1, k+2,...,n
f (w) = j

If n+2 is a prime, then gcd (f(w), f(v_i)) = gcd (n+2, i+1) =1
If n+2 is not a prime, thengcd (f(w), f(v_i)) = gcd (j, i+1) =1
Hence, $K_n^c+K_2$ is prime graph.

Example 2.2

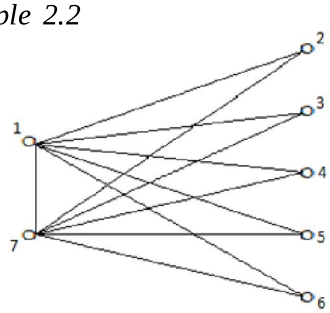

Fig 1: $K_5^c+K_2$

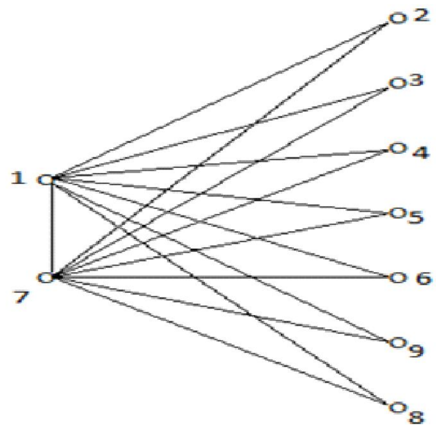

Fig 2:$K_7^c+K_2$

Theorem-2.3

$(K_n^c+K_2) \odot K_1$ is a prime (or) $K_{2,n} \odot K_1$ is a prime.

Proof:

Let G = $(K_n^c+K_2) \odot K_1$

Let u, w be the vertices of K_2 and $u_1, u_2,..., u_n$ be the vertices of K_n^c. Let $v_1, v_2,..., v_{n+2}$ be the pendant vertices.

Let V (G) = $\{u, w, u_i, v_j/i=1, 2,..., n$ and $j=1,2,...,n+2\}$

Let E (G)=$\{uw, uu_i, wu_i/ i=1, 2,..., n\} \cup \{u_iv_i/i= 1,2,...,n\} \cup \{wv_{n+1}, uv_{n+2}\}$

There are 2n+4 vertices and 3n+3 edges.

Define f: V $\rightarrow$ {1, 2,..., 2n+4} as follows

Case 1: If 2n+3 is a prime
f (u)=1
f (u_i)=2i+1, i= 1,2,...,n

$f(w) = 2n+3$

$f(v_j)=2j,\ j=1,2,\ldots,n$

$f(v_{n+1}) = 2n+2$

$f(v_{n+2}) = 2n+4$

$\gcd(f(w), f(v_{n+1})) = \gcd(2n+3, 2n+2) = 1$

$\gcd(f(w), f(u_i)) = \gcd(2n+3, 2i+1) = 1,\ i=1,2,\ldots,n$

$\gcd(f(u_i), f(v_i)) = \gcd(2i+1, 2i) = 1,\ i=1,2,\ldots,n$

Case 2: If 2n+3 is not a prime

Let j be the greatest prime number less than 2n+3.

$f(u_i)= 2i+1,\ i=1,2,3,\ldots k-1$

$f(u_k) = 2n+3$

$f(u_i) = 2i+1,\ i= k+1, k+2,\ldots,n$

$f(v_i) = 2i,\ i= 1,2,\ldots, k-1, k+1, k+2,\ldots,n$

$f(v_k) = 2n+2$

$f(w) = j$

$f(v_{n+1}) = j-1.$

$\gcd(f(w), f(v_{n+1})) = \gcd(j, j-1) = 1$

$\gcd(f(w), f(u_i)) = \gcd(j, 2i+1) = 1,\ i=1,2,\ldots,k-1$

$\gcd(f(w), f(u_k)) = \gcd(j, 2n+3) = 1$

$\gcd(f(w), f(u_i)) = \gcd(j, 2i+1) = 1,\ i= k+1,k+2,\ldots, n$

$\gcd(f(u_i), f(v_i)) = \gcd(2i+1, 2i) = 1,\ i= 1,2,\ldots, k-1, k+1, k+2,\ldots, n$

$\gcd(f(u_k), f(v_k)) = \gcd(2n+3, 2n+2) = 1$

Hence, $(K_n^c+K_2) \odot K_1$ is a prime graph.

Example 2.4

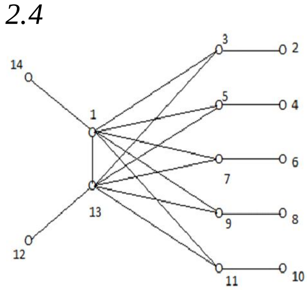

Fig 3 Case 1: $(K_5^c+K_2) \odot K_1$

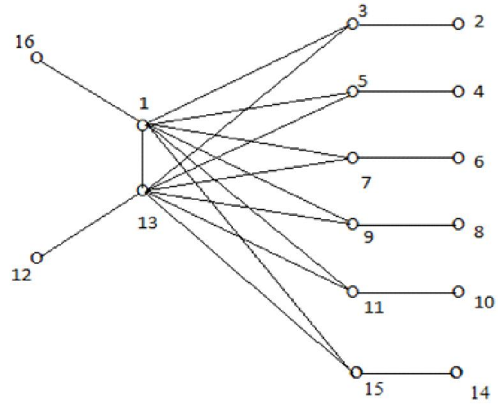

Fig 4 Case 2: $(K_6^c+K_2) \odot K_1$

Theorem-2.5

$F_m^{(n)}$ is a prime

Proof:

Let G = $F_m^{(n)}$

Let $v_1, v_2,\ldots,v_{mn}$ be the vertices of the fan . Let w be the central vertex.

Let $V(G) = \{v_i, w / i = 1, 2,\ldots,mn\}$

Let $E(G) = \{wv_i / i = 1, 2,\ldots,mn\}$

There are mn+1 vertices and 9n edges.

Define $f: V(G) \rightarrow \{1,2,\ldots,mn+1\}$ by

$f(w)= 1$

$f(v_i)= i+1,\ i= 1,2,\ldots,mn$

$\gcd(f(w), f(v_i)) = \gcd(1, i+1) = 1,\ i=1,2,\ldots,mn$

$\gcd(f(v_i), f(v_{i+1})) = \gcd(i+1, i+2) = 1,$ $i=1,2,\ldots,m-1$

Hence, $F_m^{(n)}$ is a prime graph.

Example 2.6

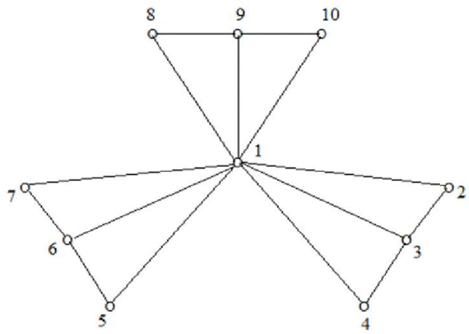

Fig 5: F_m

Theorem-2.7

F_m @ $2P_n$ is a prime

Proof:

Let G= F_m @ $2P_n$

Let V (G)={u, v_i, u_j, w_j/$1\leq i\leq m$, $1\leq j\leq n-1$}

Let E (G)={uv_i,v_mu_1, u_ju_{j+1}, uw_1,w_jw_{j+1}/ $1\leq i\leq m$, $1\leq j\leq n-2$}

There are m+2n-1 vertices and 2m+2n-3 edges.

Define f: V (G) $\rightarrow$ {1, 2,..., m+2n-1} by

f (u)= 1

f (v_i) = i+1, i= 1,2,...,m

f (u_j) = m+j+1, j= 1,2,...,n-1

f (w_j) = m+n+j, j=1,2,...,n-1

gcd(f(u), f(v_i)) = gcd (1, i+1) = 1, i=1,2,...,m

gcd(f(v_m), f(u_1)) = gcd (m+1, m+2) =1

gcd (f(u_j), f(u_{j+1})) = gcd (m+j+1, m+j+2) = 1, j= 1,2,...,n-2

gcd (f(u), f(w_1)) = gcd (1, m+n+1) = 1

gcd (f(w_j), f(w_{j+1})) = gcd (m+n+j, m+n+j+1) = 1, j=1,2,...,n-2

Hence, F_m @ $2P_n$ is a prime graph.

Example 2.8

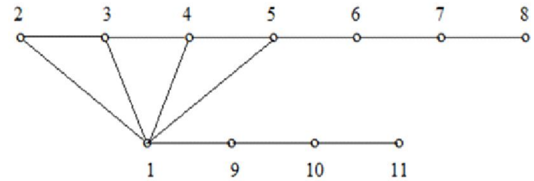

Fig 6: F_4 @ $2P_4$

Theorem-2.9

C_m @ $2P_n$ is a prime graph.

Proof:

Let G= C_m @ $2P_n$

Let u_1, u_2,..., u_m be the vertices of the cycle C_m. Let v_1, v_2,..., v_{n-1} and w_1, w_2,..., w_{n-1} be the path vertices.

Let V (G)={u_i, v_j, w_j/$1\leq i\leq m$, $1\leq j\leq n-1$}

Let E (G) = {u_iu_{i+1}, u_mu_1, u_mv_1, u_1w_1, v_jv_{j+1}, w_jw_{j+1}/$1\leq i\leq m-1$, $1\leq j\leq n-2$}

There are m+2n-2 vertices and m+2n-2 edges.

Define f: V (G) $\rightarrow$ {1,2,...,m+2n-2} by

f (u_i)= i, i= 1,2,...,m

f (v_j) = m + j, j= 1,2,...,n-1

f (w_j)= m + n + j-1, j=1,2,...,n-1

gcd (f(u_i), f(u_{i+1}))=gcd (i, i+1)=1, i=1,2,...,m-1

gcd (f(u_m), f(u_1)) = gcd (m,1) =1

gcd (f(u_m), f(v_1)) = gcd (m, m+1) = 1

gcd (f(u_1), f(w_1)) = gcd (1, m+n) = 1

gcd (f(v_j), f(v_{j+1}))=gcd (m+j, m+j+1)=1, j= 1,2,...,n-2 gcd (f(w_j), f(w_{j+1})) = gcd (m+n+j-1, m+n+j) = 1, j=1,2,...,n-2

Hence, C_m @ $2P_n$ is a prime graph.

Example 2.10

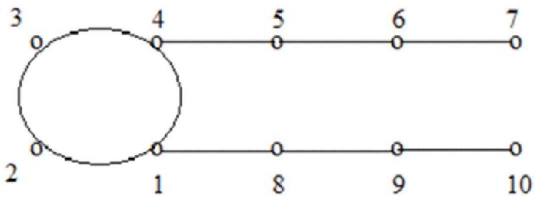

Fig 7: C_4 @ $2P_4$

Theorem-2.11

The graph $P_{n_{(m)}}$ is a prime for all n, m ≥ 2.

Proof:

Let G be the graph $P_{n_{(m)}}$.

Let V (G) = {v_{ij}/ 1≤i≤m and 1≤j≤n}

E (G)={$v_{ij}v_{i(j+1)}$/ 1≤i≤m and 1≤j≤ n-1} ∪ {$v_{i(n-1)}$ $v_{(i+1)(n-1)}$/ 1≤i≤m-1}

Then = mn and $|E(G)|$ = mn-1.

Let f: V (G) → {1,2,...,mn} be defined as follows

f (v_{ij}) = n(i-1)+j, 1≤i≤m and 1≤j≤n

gcd (f (v_{ij}), f($v_{i(j+1)}$)) = gcd (n(i-1)+j, n(i-1)+j+1) =1, 1≤i≤m and 1≤j≤n-1

gcd (f($v_{i(n-1)}$), f($v_{(i+1)(n-1)}$)) = gcd (n(i-1)+n-1, n(i+1-1)+n-1), 1≤i≤m-1

= gcd (n (i-1) + n-1, ni + n-1)

= gcd (ni-1, ni + n-1)

= gcd (ni-1, n (i+1)-1)

Hence, the graph $P_{n_{(m)}}$ is a prime for all n, m ≥2.

Example 2.12

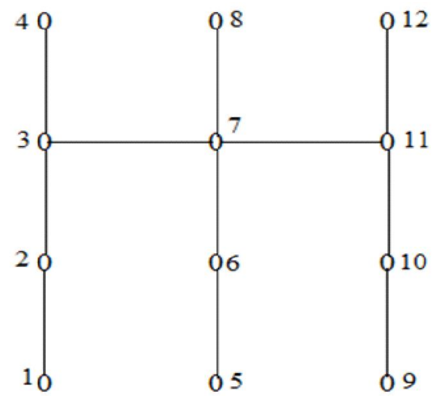

Fig 8: $P_{4_{(3)}}$

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