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Evolution of Flare Generated Magnetohydrodynamic Shock in Solar Wind

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Abstract

A self-similar spherically symmetric model is constructed to describe a blast wave in the solar wind produced by a solar flare. The shock wave is assumed to advance into a conducting gas streaming with a constant velocity ahead of the shock. Numerical solutions are obtained for the distribution of flow variables within the shocked gas for the special choice of parameters and for uniform and non-uniform distribution of density in ambient solar atmosphere. In particular, the time of transit of the shock at the earth's orbit is calculated. It is observed that the streaming ambient solar atmosphere reduces variations in flow variables in crossing the shock at earth's orbit.

Key words: solar wind, blast wave, magneto hydrodynamic shock

1. Introduction

It is well established fact that the solar corona undergoes a continuous dynamical expansion and produces a flow known as solar wind[1,2]. It is also well supported by observations that the travelling interplanetary shocks are associated with coronal mass ejections and flares[3], Holzer[4], Macqueen[5], Sheelay[6] *et al.*, Wu[7] *et al.*, Chao[8] etc. If the speed of the ejected material relative to the ambient solar wind exceeds the local magneto acoustic speeds magnetohydrodynamic shock front forms at the leading edge of the compressed ambient plasma. Many interesting theoretical ideas have been pursued to explain phenomena of the formation of interplanetary disturbances and their subsequent propagation through solar wind under different degrees of approximations (Parker[9], Simon and Axford[10], Lee and Chen[11], Korobeinikov[12], Tam and Yousofian[13], Hundhausen[14], Stenolfson[15,16], Summers[17], Low[18] etc).

Observations on the moving disturbances indicate that this process is purely magnetohydrodynamic phenomena and the expulsion of mass and magnetic field through the corona are the principal ingredient of the process. It is also evident from the typical velocity of the ejected material that the ensuing motion is highly energetic and non-linear and considerable amount of mass and magnetic field are ejected in the process.Comparing the motion with non-linear gas dynamics, Parker[9] has pointed out that the hydrodynamic blast wave theory can be used successfully to describe the flow due to sudden expansion of solar corona.By using similarity solution method of the hydrodynamic equations he has presented a number of numerical solutions and applied the results to the flow produced by sudden coronal expansion.

The first analytical treatment of the time dependent propagation of a flare generated disturbances through an ambient solar wind including the magnetic field was the similarily solution of Lee and Chen[11], which was extended to more general ambient atmosphere by Rosenau and Frankenthal[19] and Rosenau[20]. Constructing an idealized model of magnetohydrodynamic spherical blast wave, Summers[17] obtained complete analytical solutions by using the method of similarity transformations and applied the results to a flare generated shock.

Low[21] has derived a class of self-similar solutions of time dependent non-linear magnetohydrodynamic motion and describe the evolution of coronal transient phenomena. Ojha and Tiwari extended the analysis of Low[21] to include the rotation of solar wind and derived a class of self similar solutions of coronal transients in a rotating solar wind. In present study we have constructed a similarity solution following the method of Rogers and applied the results to a flare generated interplanetary hydrodynamic shock wave. To make the problem mathematically tractable the solar wind is assumed to behave as a polytrophic gas of infinite conductivity with negligible dissipative effects and the flow is spherically symmetric confined to the solar equatorial plane. Following Summers[17] we have omitted the effects due to gravity.The ambient solar wind velocity is taken into account in comparison with the velocity of blast wave (Lee and Chen[11]). It is assumed that the density of the ambient solar wind is decreasing as in case of Summers[17] and Lee and Chen[11] but not magnetic field.The present work extends this treatment to the case where the density behaves as r^{-l} with $0 \le l < 3$ in place of r^{-2}.

The similarity solutions obtained heredescribed the evolution of disturbances in the ambient solar wind between the leading shock and the trailing contact surface.The contact surface represents the trailing edge of the shocked ambient solar wind and the leading edge of the material originating in the solar event responsible for the disturbance. In particular, we have calculated time of transit of the resulting shock to the earth's orbit. It has been observed that the streaming solar wind reduces velocity and pressure across the shock wave on the earth's orbit.We are concerned ourselves mainly with illustrating basic physical process rather than to fit the theory to a comparison with specific observations.

2. Fundamental Equations and Shock Conditions :

The fundamental equations governing the motion of a spherically symmetric electrically conducting ideal gas are, Summers[17], Lee and Chen[11], Rosenau and Frankenthal[19]

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \tag{2.1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) + \frac{\partial p}{\partial r} + B\left(\frac{\partial B}{\partial r} + \frac{B}{r}\right) = 0 \tag{2.2}$$

$$\frac{\partial B}{\partial t} + \frac{1}{r}\frac{\partial (ruB)}{\partial r} = 0 \tag{2.3}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - \frac{\gamma p}{\rho}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \tag{2.4}$$

where r and t are the independent space and time co-ordinates,ρ the density,p the pressure,u the gas velocity,B the magnetic field and γ is the ratio of specific heats.

The conservation relations hold across a general hydro magnetic discontinuity. These relations become the appropriate boundary conditions for the interplanetary shock wave. The conservation of mass, momentum and energy for a perfect medium reduces to

$$\left[\rho(V-u)\right] = 0 \tag{2.5}$$

$$\left[\rho(V-u)^2 + p + \frac{B^2}{2}\right] = 0 \tag{2.6}$$

$$\left[(V-u)B\right] = 0 \tag{2.7}$$

$$\left[\frac{\gamma p}{\rho(\gamma-1)} + \frac{1}{2}(V-u)^2 + \frac{B^2}{\rho}\right] = 0 \tag{2.8}$$

Where V is the velocity of the shock and [ ] means the change of the enclosed quantity across the shock wave. Following Grib(1979), take

$$\frac{\rho_0}{\rho} = \frac{V-u}{V-u_0} = \frac{B_0}{B} = N \tag{2.9}$$

then

$$\rho = \frac{1}{N}\rho_0 \tag{2.10}$$

$$u = \left(1 - \frac{N}{\delta}\right)V \tag{2.11}$$

$$B = \frac{B_0}{N} \tag{2.12}$$

$$p = p_0 \left\{ 1 + \gamma M^2 (1 - N) - \frac{1}{\beta_0} \left(\frac{1}{N^2} - 1 \right) \right\} \quad (2.13)$$

$$\gamma M^2 (1 - N^2) + \frac{2\gamma}{\gamma - 1} \left(1 - \frac{Np}{p_0} \right) + \frac{4}{\beta_0} \left(1 - \frac{1}{N} \right) = 0 \quad (2.14)$$

$$N = \frac{\gamma - 1}{2(\gamma + 1)} + \frac{\left(1 + \frac{1}{\beta_0}\right)}{(\gamma + 1) M^2} + \left\{ \left(\frac{(\gamma - 1)}{2(\gamma + 1)} + \frac{\left(1 + \frac{1}{\beta_0}\right)}{(\gamma + 1) M^2} \right)^2 + \frac{2(2 - \gamma)}{\gamma (\gamma + 1) M^2 \beta_0} \right\}^{1/2} \quad (2.15)$$

$$M^2 = \frac{(V - u_0)^2 \rho_0}{\gamma p_0} \quad (2.16)$$

$$\beta_0 = \frac{2 p_0}{B_0^2} \quad (2.17)$$

$$\delta = \frac{V}{(V - u_0)} \quad (2.18)$$

where M, β_o and δ are defined as the Machnumber i.e. ratio of actual shock speed to the sound speed,the ratio of the fluid pressure to the magnetic pressure in the ambient solar atmosphere which is the parameter measuring strength of the magnetic field and the ratio of the apparent shock speed to the actual shock speed relative to the ambient streaming solar wind. u_0 represents the uniform solar wind streaming velocity in ambient atmosphere.

It is easy to see that in absence of magnetic field β_o tends to infinity and the above shock conditions reduce to ordinary shock conditions in a perfect gas.

For strong shocks, we have[11]

$$N = \frac{(\gamma - 1)}{(\gamma + 1)} \quad (2.19)$$

$$u = \left[\frac{\delta(\gamma + 1) - (\gamma - 1)}{\delta(\gamma + 1)} \right] V \quad (2.20)$$

$$p = \frac{2 \rho_0 V^2}{(\gamma + 1) \delta^2} \quad (2.21)$$

$$B = \frac{1}{N} \left(\frac{2 \rho_0}{\gamma \beta_0} \right)^{1/2} \frac{V}{M \delta} \quad (2.22)$$

3. Similarity Solutions :

Suppose that the motion of the disturbance is confined within the shock surface r=R(t) and the apparent velocity V of the shock surface moving outward isdR/dt. Consider a similarity transformation

$$u = \frac{r}{t} v(\eta) \quad (3.1)$$

$$\rho = \rho_0 r^k g(n) \quad (3.2)$$

$$p = \rho_0 r^{k+2} t^{-2} f(\eta) \quad (3.3)$$

$$B = \left(\rho_0 r^{k+2} t^{-2} \right)^{1/2} h(\eta) \quad (3.4)$$

where

$$\eta = r t^{-\alpha} \quad (3.5)$$

We choose the shock front to be determined by $\eta = \eta_0$ *where* η *is cons* tan *t and is given by*

$$\eta_0 = R t^{-\alpha} \quad (3.6)$$

and

$$V = \frac{dR}{dt} = \frac{\alpha R}{t} \quad (3.7)$$

Actual velocity of the shock wave $V - u_0 = \alpha . R/\delta . t$.

Let the distribution of density in the ambient solar wind be

$$\rho_0 = \rho_c r^{-l} \quad (3.8)$$

ρ_c and l are constants.

The total energy within the shock wave is given by,

$$E = 4\pi \int_0^R \left(\frac{1}{2} \rho u^2 + \frac{p}{\gamma - 1} + B^2 \right) r^2 dr \quad (3.9)$$

Writing (3.9) in similarity transformation form, we get

$$E = 4\pi\rho_c \int_0^{\eta} \left(\frac{1}{2} g v^2 + \frac{f}{\gamma - 1} + \frac{h^2}{2} \right) \eta^{k+4} t^{\alpha(k-l+5)-2} d\eta \tag{3.10}$$

Let the energy released by flare be instantaneous and therefore, total energy will depend only on □ . Thus, we have

$$\alpha\left(k - l + 5\right) - 2 = 0 \tag{3.11}$$

Introducing (3.1) – (3.5) in equation (2.1) –(2.4) we have

$$\left(v - \alpha\right)\frac{g'}{g} + \frac{v}{\eta}\left(3 - l + k\right) + v' = 0 \tag{3.12}$$

$$\eta\left(v - \alpha\right)v' + v\left(v - 1\right) + \frac{f}{g}\left(\eta\frac{f'}{f} + 1\right) + \frac{h}{g}\left(\eta\frac{h'}{h} + \frac{k - l + 4}{2}\right) = 0 \tag{3.13}$$

$$\left(v - \alpha\right)\frac{h'}{h} + v' + \frac{v}{\eta}\left(\frac{k - l + 6}{2}\right) - \frac{1}{\eta} = 0 \tag{3.14}$$

$$\eta\frac{f'}{f}\left(v - \alpha\right) + v\left(k - l + 2\right) - 2 - \gamma\left(\eta\frac{g'}{g}\left(v - \alpha\right) + v\left(k - l\right)\right) = 0 \tag{3.15}$$

Where dash (‘) denotes differentiation with respect to □.

Substituting (3.1) – (3.4) in shock conditions (2.10) – (2.14) we get

$$\rho_0 R^k g\left(\eta_0\right) = \frac{\rho_0}{N} \tag{3.16}$$

$$\frac{R}{t} v\left(\eta_0\right) = \left(1 - \frac{N}{\delta}\right)V \tag{3.17}$$

$$\rho_0 R^{k+2} t^{-2} f\left(\eta_0\right) = p_0\left\{1 + \gamma M^2\left(1 - N\right) - \frac{1}{\beta_0}\left(\frac{1}{N^2} - 1\right)\right\} \tag{3.18}$$

$$\left(\rho_0 R^{k+2} t^{-2}\right)^{1/2} h\left(\eta_0\right) = \frac{1}{N}\left(\frac{2\rho_0}{\gamma\beta_0}\right)^{1/2} \frac{V}{M\delta} \tag{3.19}$$

It is clear that for appropriate similarity transformation, it is necessary to take k = 0, Now the problem is to solve four ordinary differential equations

$$\left(\alpha - v\right)\frac{g'}{g} = \left(3 - l\right)\frac{v}{\eta} + v' \tag{3.20}$$

$$\left(\alpha - v\right)\frac{f'}{f} = \left(3\gamma + 2 - l\right)\frac{v}{\eta} - \frac{2}{\eta} + \gamma v' \tag{3.21}$$

$$\left(\alpha - v\right)\frac{h'}{h} = \frac{v}{\eta}\left(\frac{6 - l}{2}\right) - \frac{1}{\eta} + v' \tag{3.22}$$

$$v' = \frac{v\left(v-1\right)\left(\alpha - v\right) + \left(\left(3\gamma + 2 - l\right)v - 2\right)\frac{f}{g} + \left(\left(\frac{6-l}{2}\right)v - 1\right)\frac{h^2}{g} + \left\{\left(2 - l\right)\frac{f}{g} + \left(\frac{4-l}{2}\right)\frac{h^2}{g}\right\}\left(\alpha - v\right)}{\left(\alpha - v\right)^2 - \left(\frac{\gamma f}{g} + \frac{h^2}{g}\right)} \tag{3.23}$$

with boundary conditions

$$g\left(1\right) = \frac{1}{N} \tag{3.24}$$

$$v\left(1\right) = \left(1 - \frac{N}{\delta}\right)\alpha \tag{3.25}$$

$$f\left(1\right) = \frac{\alpha^2}{\delta^2}\left\{\frac{1}{\gamma M^2} + \left(1 - N\right) - \frac{1}{\gamma\beta_0 M^2}\left(\frac{1}{N^2} - 1\right)\right\} \tag{3.26}$$

$$h\left(1\right) = \frac{\alpha}{\delta N}\frac{1}{M}\left(\frac{2}{\gamma\beta_0}\right)^{1/2} \tag{3.27}$$

where N is given by equation (2.15).

4. Numerical Integration and Discussions:

Equations (3.20) – (3.23) with boundary conditions (3.24)–(3.27) are integrated numerically for the values of δ = 1, 1.5, l = 0, 1.5, 2.5 and β_0 = ∞, .05, .5, 1 and the variations of the flow variables *i.e.* velocity, density, pressure & magnetic field behind the shock are given in figs. 1 – 4 respectively in case of static and streaming ambient atmosphere ahead of the shock. The value of δ = 1 corresponds to the case where ambient solar atmosphere is static while δ = 3/2 shows ambient solar atmosphere is streaming ahead of the shock wave. If δ and apparent velocities of the shock are known, it is easy to calculate velocity of the ambient streaming atmosphere ahead of the shock. For example, let δ = 3/2 and apparent velocity ofthe shock be 1200 km./sec. then the actual velocity of the streaming atmosphere ahead of the shock is 400 km./sec.The variation in velocity behind the shock are plotted in fig.1 for the case of δ = 1 and δ = 3/2 separately. In both cases velocities decreases behind the shock wave. The presence of magnetic field in ambient atmosphere increases the decrease in velocity in both the above cases. In case of streaming atmosphere ahead of the shock, the decrease in velocity is sharp in narrow region in comparison to static ambient atmosphere.When l = 0 the value of density behind the shock decreases, which becomes sharp in case of streaming ambient atmosphere (see fig. 2 for δ = 1 and δ = 3/2). As l increases in presence of magnetic field the decrease in flow variables becomes slow in comparison to uniform atmosphere. For the case of l = 5/2, density increase behind the shock in both case δ = 1 and δ = 3/2. Presence of magnetic field slows down the increase in density behind the shock.

Fig. 3 shows that pressure also decreases behind the shock when δ = 1 and δ = 3/2 but in case of streaming atmosphere i.e. when δ = 3/2 the decrease in pressure looks faster than the case δ = 1. Presence of magnetic field slows down the decrease in pressure for both cases of the uniform and non-uniform distribution of density ahead of the shock.

The variation in magnetic field is plotted in fig. 4 when δ = 1 and δ = 3/2. When density distribution ahead of the shock is uniform, magnetic field decreases behind the shock. For non-uniform density distribution, given by l < 5/2, ahead of the shock magnetic field decreases behind the shock, in both the cases when δ = 1 and δ = 3/2, but when l = 5/2 magnetic field increases behind the shock.

Thus, we conclude that the presence of streaming ambient velocity, magnetic field, uniform and non-uniform density distribution all affect greatly the motion of the gas behind the shock wave.

The values of the density, particle velocity, pressure and magnetic field within the region of shocked gas at any point (r, t) in space time are obtained by our solutions given by

$$\rho(r,t) = \rho_0 g(\eta) \tag{4.1}$$

$$u(r,t) = \frac{V}{\alpha} v(\eta) \tag{4.2}$$

$$p(r,t) = \rho_0 \frac{V^2}{\alpha^2} f(\eta) \tag{4.3}$$

and

$$B(r,t) = \rho_0^{1/2} \frac{V}{\alpha} h(\eta) \tag{4.4}$$

where $\eta = \dfrac{r}{R}$

To find the time of transit of the resulting shock to the earth's orbit, take R = R_E = 1.4 × 10^{13} cm. The density of the solar wind at the earth's orbit is 10^{-23} gm./cm.^{3} (Summers[17]), For l = 5/2 the dimensional constant ρ_c takes the value ~ 6 × 10^9 gm./cm.^{3} The coronal gas density is 10^{-14} gm./cm.^{3} Thus, the distance R in which vicinity the solar flare is expected to erupt is of order 10^9 cm. Hence explosion takes place over this scale of distance and similarity analysis exists only when R > 10^9 cm. The observed speed of shock at the earth's orbit is 5 × 10^7 cm./sec.

For l = 5/2

$$V_E = \frac{4}{5}\frac{R_E}{T_E} \tag{4.5}$$

where V_E and T_E are the velocity of shock at the earth's orbit and time of transit at the earth's surface and is approximately equal to 2 ×10^5 sec. which is equal to approximately 2 days.

In case of strong shocks, the values of density, magnetic field are increased by a factor (γ+1/γ-1) in crossing the shock at earth's orbit where velocity and pressure are given by-

$$u_E = \frac{\delta(\gamma+1)-(\gamma-1)}{\delta(\gamma+1)} V_E \tag{4.6}$$

and

$$\rho_E = \frac{2\rho_0}{(\gamma+1)\delta^2} V_E^2 \tag{4.7}$$

Thus, streaming ambient atmosphere ahead of the shock reduces velocity and pressure in crossing shock at earth's surface.

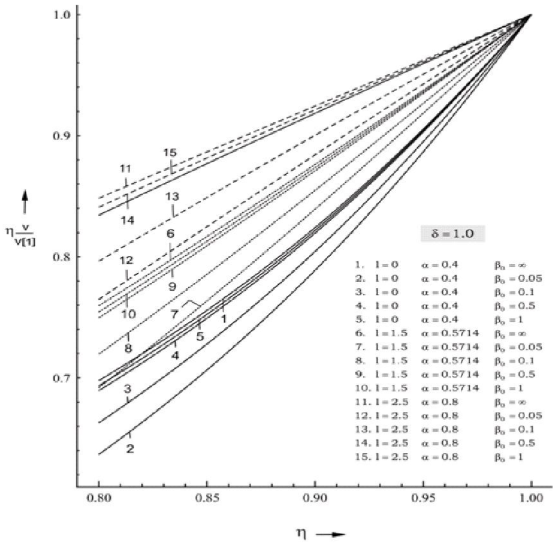

Figure 1: variation of velocity

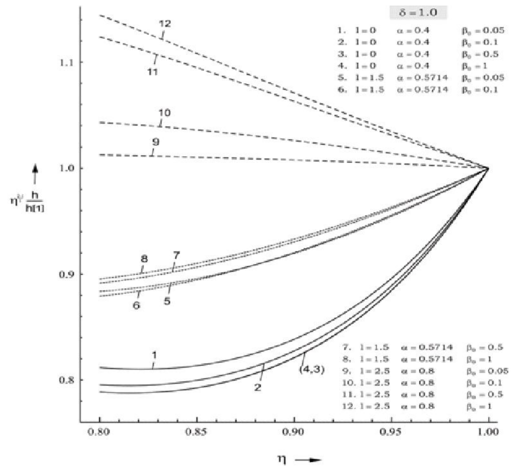

Figure 4: variation of magnetic field

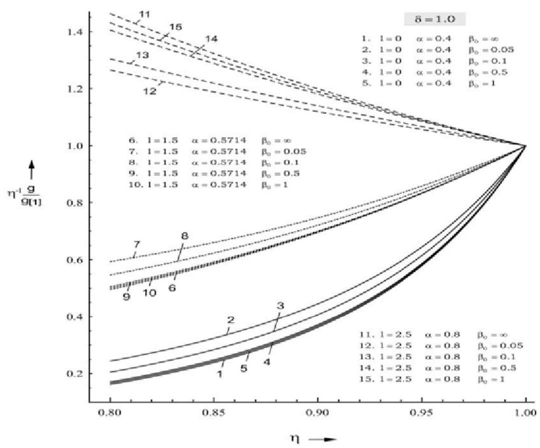

Figure 2: variation of density

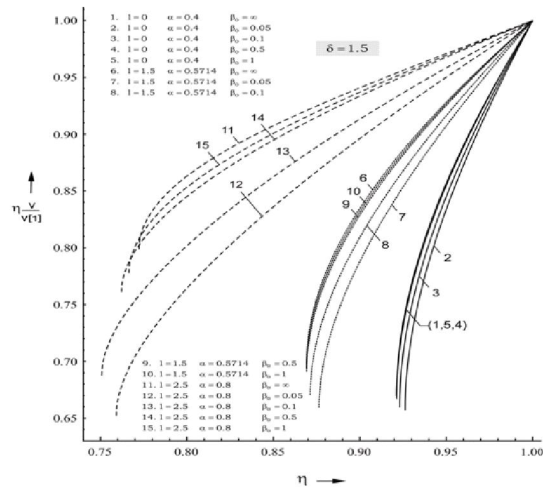

Figure 1: variation of velocity

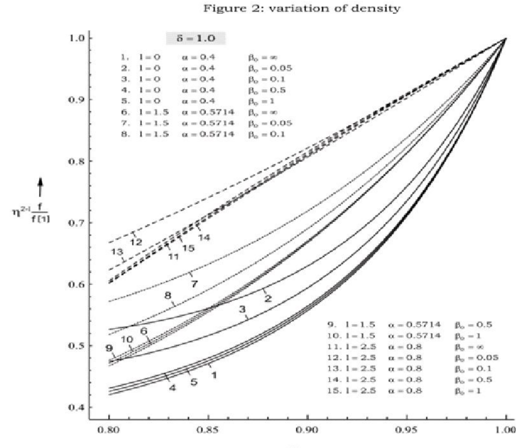

Figure 3: variation of pressure

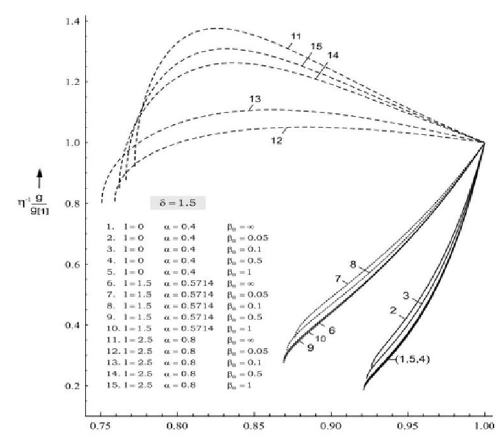

Figure 2: variation of density

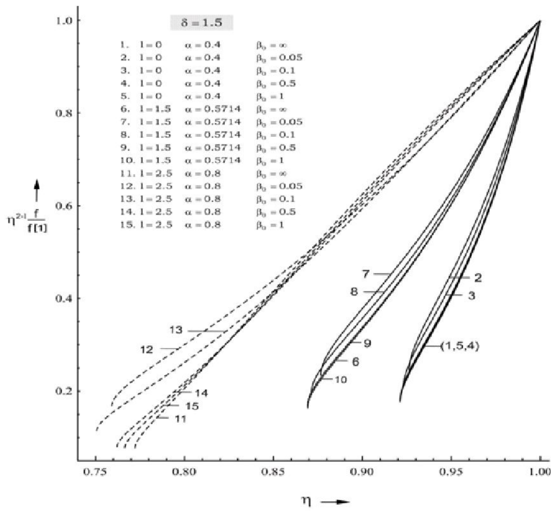

Figure 3: variation of pressure

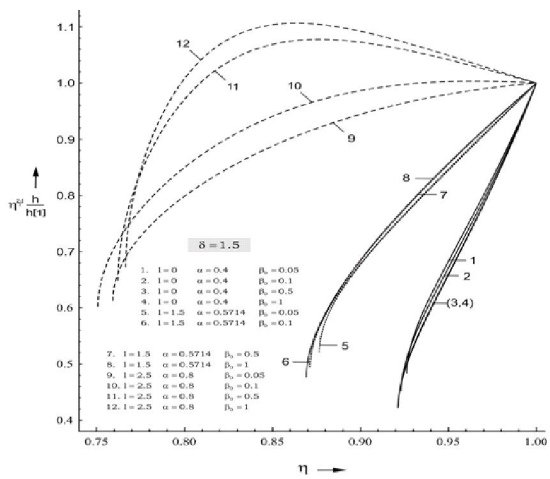

Figure 4: variation of magnetic field

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