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Multi-criteria Decision Making Models using Fuzzy TOPSIS Technique

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Abstract

In this paper we propose Multi-criteria decision making (MCDM) models using fuzzy technique for order performance by similarity to ideal solution (Fuzzy TOPSIS) is used to choose among a group of decision makers. Concerning the MCDM, the value of a fuzzy number is greater than or equal to another fuzzy number, a new distance measure. Here we described the TOPSIS technique and expansion of fuzzy TOPSIS techniques and lastly we discussed fuzzy TOPSIS for group decision making, which is applied to measure the distance of each fuzzy number from both fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). Then which is simultaneously closer to FPIS and farther from FNIS will be selected as the best choice.

Keywords: Multi-criteria decision making, fuzzy TOPSIS, group decision making, FPIS, FNIS.

Introduction

The multi-criteria decision making (MCDM) has been extensively used to select a finite number of alternatives, which is characterized by multiple conflicting criteria. The development of several MCDM approaches to solve different types of real-world problems. One of these techniques known as technique for order preference by similarity to ideal solution (TOPSIS), is used to evaluate the performance of alternative solution through the similarity with the ideal solution[23]. According to this technique, the best alternative would be one that is closest to the positive-ideal solution and farthest from the negative-ideal solution. The positive-ideal solution is one that maximizes the benefit criteria and minimizes the cost criteria. The negative-ideal solution maximizes the cost criteria and minimizes the benefit criteria.

The theory of fuzzy sets and fuzzy logic was developed by Zadeh[20]. He has demonstrated suitable models of uncertainty which were applied to a variety of problems in science and engineering. Bellman and

Zadeh[1] introduced the theory of fuzzy sets and problems of multi-criteria decision making as an effective approach to treat vagueness, lack of knowledge and ambiguity inherent in the human decision making process which are known as fuzzy multi-criteria decision making (FMCDM).

The process of building a model for multiple criteria decision making (MCDM) consists of alternatives and criterias which forms the decision matrix. For real world-problems the decision matrix is affected by uncertainty and may be modelled using fuzzy numbers. A fuzzy number can be seen as an extension of an interval with varied grade of membership. This means that each value in the interval is associated with a real number that indicates its compatibility with the vague statement associated with a fuzzy number. Fuzzy numbers have their own rules of operation. In the last decades many MCDM methods using fuzzy logic to describe uncertain data have been developed.

TOPSIS is widely used to treat real world decision making problems. Despite its popularity and simplicity in concept, this technique is often criticized because of its inability to deal adequately with uncertainty and imprecision inherent in the process of mapping the perceptions of decision-makers. In the traditional formulation of TOPSIS, the personal judgments are represented by numerical values. However, in many practical cases the human preference model is uncertain and the decision-makers might be unable to assign numerical values to the judgments of comparison. However, TOPSIS has been expanded to deal MCDM with an uncertain decision matrix resulting in fuzzy TOPSIS, which has successfully been applied to solve various MCDM problems[3, 4,5, 6, 7,14, 16,17,19,22].

Preliminaries :

Definition1 A fuzzy set $\tilde{A}$ in X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each point x a real number in the interval $[0,1]$ representing the grate of membership of x in $\tilde{A}$.

Definition 2 Let X be the universe of discourse. A fuzzy set $\tilde{A}$ of the universe of discourse X is said to be convex iff for all x_1 and x_2 in X there exists:

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq Min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$$

where $\mu_{\tilde{A}}$ is the membership function of the fuzzy set $\tilde{A}$ and $\lambda \in [0,1]$

Definition 3 A fuzzy number $\tilde{a}$ is defined by a triplet $\tilde{a} = (a_1, a_2, a_3)$. The membership function is defined by:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_2 \geq x \geq a_1 \\ \dfrac{x-a_2}{a_3-a_2}, & a_3 \geq x \geq a_2 \\ 0, & x > a_3 \end{cases}$$

where a_2 represents the value for which $\mu_a(a_2) = 1$ and a_1 and a_2 are the most extreme values on the left and on the right of the fuzzy number $\tilde{a}$ respectively with membership $\mu_{\tilde{a}}(a_1) = \mu_{\tilde{a}}(a_3) = 0$.

Definition 4 The $\tilde{a}$ -cut of fuzzy number $\tilde{A}$ is defined:

$$\tilde{A}^{\alpha} = \{x_i : \mu_{\tilde{A}}(x_i) \geq \alpha, x_i \in X\} \text{ where } \alpha \in [0,1].$$

Definition 5 If $\tilde{A}$ is a triangular fuzzy number and $\left[\tilde{A}\right]_\alpha^L > 0$ and $\left[\tilde{A}\right]_\alpha^U \le 1$ for $\alpha \in [0,1]$ then $\tilde{A}$ is called a normalized positive triangular fuzzy number.

Definition 6 $\tilde{A}$ is called a fuzzy matrix, if at least an entry in $\tilde{A}$ is a fuzzy number.

Definition 7 A linguistic variable is a variable whose values are linguistic terms. The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill defined to be reasonably described by Zadeh, 1965.

Definition 8 Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two positive triangular fuzzy numbers, then the operation with these fuzzy numbers are defined as follows:

$$\tilde{A}(+)\tilde{B} = (a_1,a_2,a_3)(+)(b_1,b_2,b_3) = (a_1+b_1, a_2+b_2, a_3+b_3)$$

$$\tilde{A}(-)\tilde{B} = (a_1,a_2,a_3)(-)(b_1,b_2,b_3) = (a_1-b_1, a_2-b_2, a_3-b_3)$$

$$\tilde{A}(\times)\tilde{B} = (a_1,a_2,a_3)(\times)(b_1,b_2,b_3) = (a_1\times b_1, a_2\times b_2, a_3\times b_3)$$

$$\tilde{A}(/)\tilde{B} = (a_1,a_2,a_3)(/)(b_1,b_2,b_3) = (a_1/b_1, a_2/b_2, a_3/b_3)$$

$$k\tilde{A} = k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$$

Definition 9 Let two triangular fuzzy numbers $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$, then the distance them is calculated by:

$$d\left(\tilde{a},\tilde{b}\right) = \sqrt{\frac{1}{3}\left[(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2\right]}$$

Multi-criteria Decision Making :

The continuing expansion of decision methods and their modifications,is important to have an understanding of their comparative value. Each of the methods uses numeric techniques and help decision makers to choose among a discrete set of alternative decisions. This is achieved on the basis of the impact of the alternatives on certain criteria and there by on the overall utility of the decision maker(s). Despite the various criticisms , some of the multi-criteria decision making are widely used. The weighted sum model (or WSM) is the earliest and probably the most widely used method. The weighted product model (or WPM) can be considered as a modification of the WSM, and has been proposed in order to overcome some of its weaknesses. The analytic hierarchy process (or AHP), as proposed by Saaty[12,13,24, 25], is a later development and it has recently become increasingly popular. Professors Belton and Gear[2] suggested a modification to the AHP that appears to be more powerful than the original approach. The other widely used method TOPSIS[23].

TOPSIS :

The decision matrix which is consists of alternative and criteria is described by

$$A = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}$$

where $A_1, A_2, \cdots, A_m$ are alternatives and $C_1, C_2, \cdots, C_n$ are criteria, x_{ij} are fuzzy numbers indicates the rating of the alternative A_i with respect to criteria C_j. TOPSIS is based upon the concept that the chosen alternative should have the shortest distance from the ideal solution and farthest from the negative ideal solution. Assume that each alternative takes the monotonically increasing (or decreasing) utility. It is then easy to locate the ideal solution, which is a combination of all the best benefit criteria value attainable, while the negative ideal solution is a combination of all the worst cost criteria values attainable. One approach is to take an alternative that has the minimum (weighted) Euclidean distance to the ideal solution of the TOPSIS method consists of the following steps:

Step 1 Calculate the normalized decision matrix. The normalized value N_{ij} is calculated as

$$N_{ij} = x_{ij} \Big/ \sqrt{\sum_{i=1}^{m} x_{ij}^2},$$
$$i = 1,2,\cdots,m, \quad j = 1,2,\cdots,n$$

Step 2 Calculate the weighted normalized decision matrix. The weighted normalized value $V_{ij} = W_j N_{ij}$, $i = 1,2,\cdots,m, \quad j = 1,2,\cdots,n$, where W_j is the *i th* attribute and $\sum_{j=1}^{n} W_j = 1$.

Step 3 Identify the positive ideal and negative ideal as follows:

$$A^+ = \{V_1^+, V_2^+, \cdots, V_n^+\}$$

and $$A^- = \{V_1^-, V_2^-, \cdots, V_n^-\}$$

where $V_j^+ = \left(\max_i V_{ij}, j \in J_1; \min_i V_{ij}, j \in J_2\right)$

and $$V_j^- = \left(\min_i V_{ij}, j \in J_1; \max_i V_{ij}, j \in J_2\right)$$

solution A^+ (benefits) and negative ideal solution A^- (costs)

where J_1 and J_2 represent the criteria benefits and cost respectively.

Step 4 Calculate the Euclidean distances from the positive ideal solution A^+ (benefits) and negative ideal solution A^- (costs) for each alternatives A_i respectively as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(D_{ij}^+\right)^2} \text{ and } D_i^- = \sqrt{\sum_{j=1}^{n} \left(D_{ij}^-\right)^2}$$

where $D_{ij}^+ = V_j^+ - V_{ij}$, with $i = 1,2,\cdots,m$

and $D_{ij}^- = V_j^- - V_{ij}$, with $i = 1,2,\cdots,m$

Step 5 Compute the relative closeness Ω_i for each alternative A_i with respect to positive ideal solution A^+ (benefits) as given by:

$$\Omega_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

Step 6 Rank the preference order. The best alternatives are those have higher value of Ω_i .

Fuzzy TOPSIS :

The decision matrix which is consists of alternative and criteria is described by

$$\tilde{A} = \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{pmatrix}$$

where $A_1, A_2, \cdots, A_m$ are alternatives and $C_1, C_2, \cdots, C_n$ are criteria, $\tilde{x}_{ij}$ are fuzzy numbers indicates the rating of the alternative A_i with respect to criteria C_j. The weight vector $W = (W_1, W_2, \cdots, W_n)$ composed of the individual weights W_j $(j = 1,2,\cdots,n)$ for each criteria C_j satisfying $\sum_{j=1}^{n} W_j = 1$. The weighted normalized fuzzy decision matrix $\tilde{M} = [\tilde{m}_{ij}]_{m\times n}$ with $i = 1,2,\cdots,m$ and $j = 1,2,\cdots,n$ is constructed by multiplying the normalized fuzzy decision matrix by its associated weights. The weights fuzzy normalized value $\tilde{V}_{ij}$ is calculated as:

$$\tilde{V}_{ij} = W_j \times \tilde{m}_{ij} \text{ with } i = 1,2,\cdots,m \text{ and } j = 1,2,\cdots,n \tag{1}$$

The fuzzy TOPSIS is described as follows:

Step 1 Identify the positive ideal solution A^+ (benefits) and negative ideal solution A^- (costs) as follows:

$$A^+ = \left(\tilde{V}_1^+, \tilde{V}_2^+, \cdots, \tilde{V}_m^+\right)$$

$$A^- = \left(\tilde{V}_1^-, \tilde{V}_2^-, \cdots, \tilde{V}_m^-\right)$$

where $\tilde{V}_j^+ = \left(\max_i \tilde{V}_{ij},\ j \in J_1; \min_i \tilde{V}_{ij},\ j \in J_2\right)$ $\tilde{V}_j^- = \left(\min_i \tilde{V}_{ij},\ j \in J_1; \max_i \tilde{V}_{ij},\ j \in J_2\right)$

where J_1 and J_2 represent the criteria benefits and cost respectively.

Step 2 Calculate the Euclidean distances from the positive ideal solution $\tilde{A}^+$ and negative ideal solution $\tilde{A}^-$. For each alternative A_i respectively as follows:

$$\tilde{D}_i^+ = \sum_{j=1}^{n} D\left(\tilde{V}_{ij}, \tilde{V}_j^+\right) \text{ with } i = 1,2,\cdots,m$$

$$\tilde{D}_i^- = \sum_{j=1}^{n} D\left(\tilde{V}_{ij}, \tilde{V}_j^-\right) \text{ with } i = 1,2,\cdots,m$$

where the distance $D\left(\tilde{V}_{ij}, \tilde{V}_j^+\right)$ between two fuzzy numbers.

Step 3 Calculate the relative closeness $\tilde{\Omega}_i$ for each alternative A_i with respect to positive ideal solution is

$$\tilde{\Omega}_i = \frac{\tilde{D}_i^-}{\tilde{D}_i^+ + \tilde{D}_i^-}$$

Proposed Methodology :

The steps for generating the group of best alternative are described as follows:

Step 1: Identify the positive ideal solution A^+ (benefits) and negative ideal solution A^- (costs) for each group member $r = 1,2,\cdots,R$ as follows:

$${}^rA^+ = \left({}^r\tilde{V}_1^+, {}^r\tilde{V}_2^+, \cdots, {}^r\tilde{V}_m^+\right)$$

$${}^rA^- = \left({}^r\tilde{V}_1^-, {}^r\tilde{V}_2^-, \cdots, {}^r\tilde{V}_m^-\right)$$

where $${}^r\tilde{V}_j^+ = \left(\max_i {}^r\tilde{V}_{ij}, j \in J_1; \min_i {}^r\tilde{V}_{ij}, j \in J_2\right)$$

$${}^r\tilde{V}_j^- = \left(\min_i {}^r\tilde{V}_{ij}, j \in J_1; \max_i {}^r\tilde{V}_{ij}, j \in J_2\right)$$

where J_1 and J_2 represent the criteria benefit and cost respectively.

Step2: Calculate the distance of each alternative with respect various members. The distance of alternative A_i from the positive ideal solution and the negative ideal solution of the group members S_r, ${}^r\tilde{D}_i^+$ and ${}^r\tilde{D}_i^-$ are given by:

$${}^r\tilde{D}_i^+ = \sum_{j=1}^n D\left({}^r\tilde{V}_{ij}, {}^r\tilde{V}_j^+\right)$$

$${}^r\tilde{D}_i^- = \sum_{j=1}^n D\left({}^r\tilde{V}_{ij}, {}^r\tilde{V}_j^-\right)$$

with $i = 1,2,\cdots,m; r = 1,2,\cdots,R$

where the distances $D\left({}^r\tilde{V}_{ij}, {}^r\tilde{V}_j^+\right)$ and $D\left({}^r\tilde{V}_{ij}, {}^r\tilde{V}_j^-\right)$ between two fuzzy numbers are calculated.

Step 3: Calculate the relative closeness for each alternative A_i of each member r, ${}^r\tilde{\Omega}(A_i)$ with respect to positive ideal solution as

$${}^r\tilde{\Omega}(A_i) = \frac{{}^r\tilde{D}_i^-}{{}^r\tilde{D}_i^+ + {}^r\tilde{D}_i^-}$$

with $i = 1,2,\cdots,m; r = 1,2,\cdots,R$

After calculating the ${}^r\tilde{\Omega}_i(A_i)$ for each member r we can form the relative-closeness matrix as given by:

$$Q = \begin{pmatrix} {}^1\tilde{\Omega}(A_1) & {}^2\tilde{\Omega}(A_1) & \cdots & {}^R\tilde{\Omega}(A_1) \\ {}^1\tilde{\Omega}(A_2) & {}^2\tilde{\Omega}(A_2) & \cdots & {}^R\tilde{\Omega}(A_2) \\ \vdots & \vdots & \ddots & \vdots \\ {}^1\tilde{\Omega}(A_m) & {}^2\tilde{\Omega}(A_m) & \cdots & {}^R\tilde{\Omega}(A_m) \end{pmatrix}$$

Now we can obtain the weighted relative closeness matrix by introducing the importance weights of group members into the relative closeness is given by:

$$Q\alpha = \begin{pmatrix} \alpha_1{}^1\tilde{\Omega}(A_1) & \alpha_2{}^2\tilde{\Omega}(A_1) & \cdots & \alpha_R{}^R\tilde{\Omega}(A_1) \\ \alpha_1{}^1\tilde{\Omega}(A_2) & \alpha_2{}^2\tilde{\Omega}(A_2) & \cdots & \alpha_R{}^R\tilde{\Omega}(A_2) \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1{}^1\tilde{\Omega}(A_m) & \alpha_2{}^2\tilde{\Omega}(A_m) & \cdots & \alpha_R{}^R\tilde{\Omega}(A_m) \end{pmatrix}$$

Step 4: Identify the groups, positive ideal solution and negative ideal solution

$$\begin{aligned} A_G^+ &= \left(V_{G1}^+, V_{G2}^+, \cdots\cdots, V_{GR}^+\right) \\ &= \left(\max_i \alpha_1{}^1\Omega(A_i), \max_i \alpha_2{}^2\Omega(A_i), \cdots\cdots, \max_i \alpha_R{}^R\Omega(A_i)\right) \end{aligned}$$

$$\begin{aligned} A_G^- &= \left(V_{G1}^-, V_{G2}^-, \cdots\cdots, V_{GR}^-\right) \\ &= \left(\min_i \alpha_1{}^1\Omega(A_i), \min_i \alpha_2{}^2\Omega(A_i), \cdots\cdots, \min_i \alpha_R{}^R\Omega(A_i)\right) \end{aligned}$$

Step 5: Calculate to each alternative A_i the distances from the group positive and negative ideal solutions A_G^+ and A_G^-, respectively as follows:

$$d_{Gi}^+ = \sqrt{\sum_{r=1}^{R}\left(\alpha_r{}^r\Omega(A_i) - V_{Gr}^+\right)^2}$$

$$d_{Gi}^- = \sqrt{\sum_{r=1}^{R}\left(\alpha_r{}^r\Omega(A_i) - V_{Gr}^-\right)^2}$$

with $i = 1, 2, \cdots, m$

Step 6: Calculate the group relative-closeness Ω_{Gi} for each alternative A_i with respect to group ideal solution as:

$$\Omega_G(A_i) = \frac{d_{Gi}^-}{d_{Gi}^- + d_{Gi}^+}$$

Conclusion

MCDM has found wide applications in the real world decision making problems. In this paper, we considered a MCDM method, when there is a group of decision makers; Consisting the value of a fuzzy number, greater than or equal to another fuzzy numbera new distance measure of each fuzzy number from both fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). Again we revise TOPSIS and fuzzy TOPSIS techniques. Also develop a fuzzy TOPSIS for a group decision making to tackle the multi-criteria decision making models. This method allows finding the best alternatives.

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