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Equalityof Secure Domination and Inverse Secure Domination Numbers

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Abstract

Let $G = (V, E)$ be a graph. Let D be a minimum secure dominating set of G. If $V - D$ contains a secure total dominating set D' of G, then D' is called an inverse secure dominating set with respect to D. The smallest cardinality of inverse secure dominating set of G is the secure domination number $\gamma_s^{-1}(G)$ of G. In this paper, we obtain some graphs for which $\gamma_s(G) = \gamma_s^{-1}(G)$ and establish some results on this respect. Also we obtain some graphs for which $\gamma_s(G) = \gamma_s^{-1}(G) = p/2$. where p is the number of vertices of G.

Key words: dominating set, secure dominating set, inverse secure dominating set, inverse secure domination number..

Mathematics Subject Classification: 05*C*69, 05*C*78

1. Introduction

All graphs considered here are finite, undirected without isolated vertices, loops and multiple edges. For all further notation and terminology we refer the reader to[1]. Let $G = (V, E)$ be a graph. A set D of vertices in a graph G is a dominating set if every vertex in $V - D$ is adjacent to somevertexin D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. Recently severaldomination parameters are given in the books by Kulli in[2, 3, 4]. Let D be a minimum dominating set of G. If $V - D$ contains a dominating set D' of G, then D' is called an inverse dominating set of G with respect to D. The inverse domination number $\gamma^{-1}(G)$ of G is the minimum cardinality of an inverse dominating set of G. This concept was introduced by Kulliand Sigarkanti in[5]. Many other inverse domination parameters were studied, for example, in[6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 24].

A secure dominating set of a graph G is a dominating set $D \subseteq V$ with the property that for each $u \in V - D$, there exists $v \in D$ adjacent to u such that $(D - \{v\}) \cup \{u\}$ is a dominating set. The smallest cardinality of a secure dominating set is the secure domination number $\gamma_s(G)$ of G. This concept was studied, for example, in[21,22]. Let D be a minimum secure dominating set of G. If $V - D$ contains a secure dominating set D' of G, then D' is called an inverse secure dominating set with respect to D. The inverse secure domination number $\gamma_s^{-1}(G)$ of G is the minimum cardinality of an inverse secure dominating set of G. This concept was introduced by Enriquez in[22] and was studied by Kulli in[23].

A γ_s^{-1}-set is a minimum inverse secure dominating set. Similarly other sets can be expected. If $D = \{u\}$ is a secure dominating set of G, then u is called a secure dominating vertex of G. A vertex u of G is said to be a γ_s-required vertex of G if u lies in every γ_s-set of G.

An application of inverse secure domination is found in Computer Science. In the event that there is a need for all nodes in a system to have direct access to needed resources (for example, large database) a secure dominating set furnishes such a configuration. If a second important resource is needed, then a separatedisjoint secure dominating set provides duplication in case the first is corrupted in some way. We have $\gamma_s(G) \leq \gamma_s^{-1}(G)$. From the point of above, one may demand $\gamma_s(G) = \gamma_s^{-1}(G)$, where as many graphs do not enjoy such a property. For Example, we consider the graph G in Figure 1. Then $\gamma_s(G) = 2$ and $\gamma_s^{-1}(G) = p - 2$. In this case, if p is large, then $\gamma_s^{-1}(G)$ is sufficiently large compare to $\gamma_s(G)$.

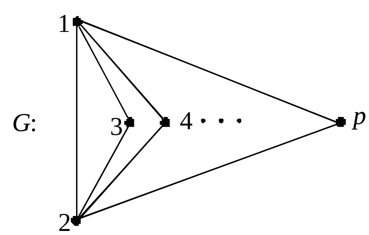

Figure 1

2. Graphs with $\gamma_s(G) = \gamma_s^{-1}(G)$

Proposition 1. If K_p is a complete graph with $p \geq 2$ vertices, then

$$\gamma_s(K_p) = \gamma_s^{-1}(K_p) = 1.$$

Proposition 2. If $K_{m,n}$ is a complete bipartite graph with $4 \leq m \leq n$, then

$$\gamma_s(K_{m,n}) = \gamma_s^{-1}(K_{m,n}) = 4.$$

Proposition 3. If $K_{m,n}$ is a complete bipartite graph with $4 \leq m \leq n$, then

$$\gamma_s\left(\overline{K_{m,n}}\right) = \gamma_s^{-1}\left(\overline{K_{m,n}}\right) = 2.$$

Proof : Clearly $\overline{K_{m,n}} = K_m \cup K_n$. Therefore

$$\gamma_s\left(\overline{K_{m,n}}\right) = \gamma_s(K_m) + \gamma_s(K_n) = 2. \qquad \gamma_s^{-1}\left(\overline{K_{m,n}}\right) = \gamma_s^{-1}(K_m) + \gamma_s^{-1}(K_n) = 2.$$

Hence the result follows.

Theorem 4: Let G be a graph with $\gamma_s(G) = \gamma_s^{-1}(G)$. Then G has no γ_s-required vertex.

Proof: Let G be a graph with $\gamma_s(G) = \gamma_s^{-1}(G)$. Let D be a γ_s-set and D' be a γ_s^{-1}-set of G. On the contrary, assume G contains a γ_s-required vertex u. Then u lies in every γ_s-set of G. Hence $u \in D$ and $u \in D'$, which is a contradiction to $D' \subseteq V - D$. Thus the result follows.

Theorem 5. Let u be a secure dominating vertex of a graph G. Then

$$\gamma_s^{-1}(G) = \gamma_s(G - u).$$

Proof: Let u be asecure dominating vertex of G. Then $\{u\}$ is a γ_s-set of G. Thus any γ_s^{-1}-set of G lies in $G - \{u\}$ and is a minimum dominating set of $G - \{u\}$. Hence $\gamma_s^{-1}(G) = \gamma_s(G - u)$.

Construct the graph G as follows:

Let $H_i = K_{m_i}$, i= 1, 2, ... ,r and $2 \leq m_1 \leq m_2 \leq \ldots \leq m_r$. Let $v_i \leq H_i$, i = 1, 2, ..., r. Consider the graph G obtained from joining the vertices$v_1 v_{i+1}$, i = 1, 2, ..., r – 1, see Figure 2. Consider the vertices$u_i \in H_i$ such that$u_i \neq v_i$,i =1, 2, ...,r.

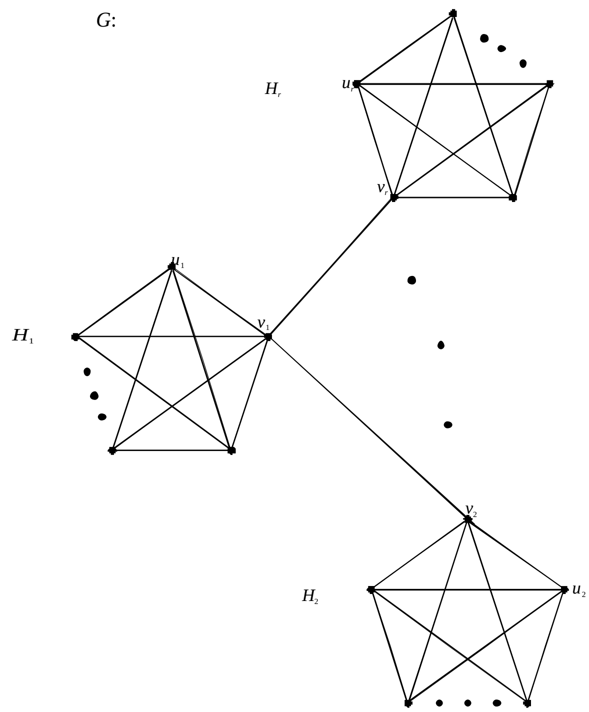

Figure 2

Proposition 6. Let G be a graph as shown in Figure 2. Then $\gamma_s(G)=\gamma_s^{-1}(G)=r$.

Proof: The set$D = \{v_1, v_2, \ldots, v_r\}$ is a γ_s-set in$G$. Then the set$D' = \{u_1, u_2, \ldots, u_r\}$ is a γ_s^{-1}-set in G for $u_1, u_2, \ldots, u_r \in V(G) - \{v_1, v_2, \ldots, v_r\}$. Thus

$$\gamma_s(G)=\gamma_s^{-1}(G)=r.$$

Corollary 7. Let G be a graph as shown in Figure 2 such that $m_1=m_2= \ldots =m_r=2$. Then $\gamma_s(G)=\gamma_s^{-1}(G)=\dfrac{p}{2}$, where $p=2r$ is the number of vertices of G.

Proposition A[22]*:* Let G be a connected non-complete graph with $p \geq 4$ vertices. If $\gamma_s^{-1}(G) = 2$, then $\gamma_s(G) = 2$.

Proposition 8. If G be a connected graph with $p \geq 4$ vertices such that $G \neq K_p$ and $\gamma_s^{-1}(G) = 2$, then $\gamma_s(G) = \gamma_s^{-1}(G) = 2$.

Proof: This follows from Proposition A.

Proposition 9. Let G and H becomplete graphs. Then $\gamma_s(G+H) = \gamma_s^{-1}(G+H)$=1.

Proof: IfG and H arecomplete graphs,then$G+H$ is complete. Thus $\gamma_s(G+H) = \gamma_s^{-1}(G+H) = 1$.

3. Graphs with $\gamma_s(G) = \gamma_s^{-1}(G) = \frac{p}{2}$

In this section, we establish some results for which $\gamma_s(G) = \gamma_s^{-1}(G) = \frac{p}{2}$.

Theorem 10. If $G = K_{4,4}, K_2$ or $K_4 - e$, then $\gamma_s(G) = \gamma_s^{-1}(G)=$, where p is the number of vertices of G.

Proof: If $G = K_{4,4}$, then by Proposition 2, $\gamma_s(G) = \gamma_s^{-1}(G) = \frac{p}{2}$. If $G = K_2$, then by Proposition 1, $\gamma_s(G) = \gamma_s^{-1}(G) = \frac{p}{2}$. If $G=K_4-e$, then we have $\gamma_s(G)=\gamma_s^{-1}(G)=\frac{p}{2}$, where p is the number of vertices of G.

Construct the graph G as follows: Let $e_i = u_iv_i$, $1 \le i \le m$ and $e_{i+1} = v_iu_{i+1}$ be the edges of a cycle C_{2m}. For each $e_i = u_iv_i$, join the vertices u_i, v_i to new vertices x_i, y_i to form the graph G, see Figure 3.

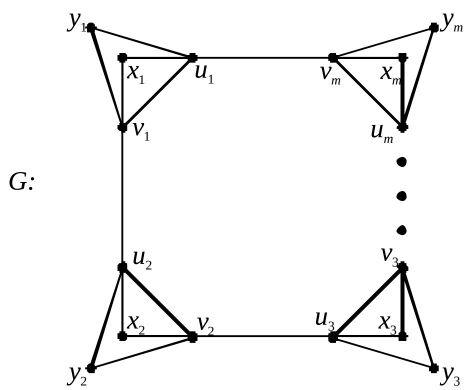

Figure 3

Theorem 11: Let G be a graph with $4m$ vertices as shown in Figure 3. Then

$$\gamma_s(G) = \gamma_s^{-1}(G) = 2m.$$

Proof: In the graph G of Figure 3, $V(G) = \{u_1, ..., u_m, v_1, ..., v_m, x_1, ..., x_m, y_1, ..., y_m\}$. Then the set $D = \{u_1, ... u_m, v_1, ..., v_m\}$ is a γ_s-set with $2m$ vertices and $D' = \{x_1, ..., x_m, y_1, ..., y_m\}$ is a γ_s^{-1}-set with $2m$ vertices. Thus $\gamma_s(G)=\gamma_s^{-1}(G)=2m$.

Remark 10. Let $G_1, G_2, \ldots, G_m$ be the m connected components of a graph G. Let D_i be a γ_s-set of G_i, and D'_i be a γ_s^{-1} -set of G_i, for $i = 1,2,\ldots,m$. Then $D_1 \cup D_2 \cup \ldots \cup D_m$ is a g_s-set of G and $D'_1 \cup D'_2 \cup \ldots \cup D'_m$ is a γ_s^{-1}-set of G. Thus $\gamma_s(G) = \sum_{i=1}^{m} \gamma_s(G_i)$ and

Theorem 13. Let $G_1, G_2, \ldots, G_m$ be the m components of a graph G. Then $\gamma_s(G) = \gamma_s^{-1}(G)$ if and only if $\gamma_s(G_i) = \gamma_s^{-1}(G_i)$, for $i = 1, 2, \ldots, m$.

Proof: Let $G_1, G_2, \ldots, G_m$ be the m connected components of graph G.

By Remark 12, $\gamma_s(G) = \sum_{i=1}^{m} \gamma_s(G_i)$ and $\gamma_s^{-1}(G) = \sum_{i=1}^{m} \gamma_s^{-1}(G_i)$. Therefore, $\gamma_s(G) = \gamma_s^{-1}(G)$ if $\gamma_s(G_i) = \gamma_s^{-1}(G_i)$ for $i=1, 2, \ldots, m$.

Conversely suppose $\gamma_s(G)=\gamma_s^{-1}(G)$. We have $\gamma_s(G_i) \le \gamma_s^{-1}(G_i)$, for $i = 1,2,\ldots, m$. We now prove that $\gamma_s(G_i)=\gamma_s^{-1}(G_i)$, for $i = 1, 2, \ldots, m$. On the contrary, assume $\gamma_s(G_i) < \gamma_s^{-1}(G_i)$ for some i. Then $\gamma_s(G_j) > \gamma_s^{-1}(G_j)$, for some $j, j \ne i$, which is a contradiction. Thus $\gamma_s(G_i) = \gamma_s^{-1}(G_i)$ for $i = 1, 2, \ldots, m$.

Corollary 14. If the connected components G_i of G are either K_2 or $K_4 - e$ or $K_{4,4}$ or G as shown in Figure 3, then $\gamma_s(G) = \gamma_s^{-1}(G) =$ where p is the number of vertices of G.

Proof: This follows from Theorems 10, 11, 13.

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