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A Vacation Model $\left(M^{X}/G/1\right)$: (E, MV) with General Service Bulk Queue

SANDEEP DIXIT

Department of Mathematics, V.S.S.D. College, Kanpur (U.P.) India

Email of Corresponding Author: - sd0408@rediffmail.com

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Abstract

In this paper we discuss the batch arrival vacation model appear in many situations such as computer communication systems. The common method of studying the batch arrival queueing system with vacations is by using supplementary variables.

Key words: General Service, Multiple Vacation, Bulk Queue.

Introduction

Consider an $\left(M^{X}/G/1\right)$ queue where customers arrive in batches according to a passion process with rate λ. The batch size X is a random variable, with the distribution function and p.g.f.

$$P(X=j)=g_j \qquad j=1,2,3.......... \qquad G(Z)=\sum_{j=1}^{\infty} g_j Z^j \tag{1}$$

Respectively, the mean of g= E (X); and the second moment of $g^{(2)}=E(X^2)$. The service time i.i.d. random variable denoted by B, with general distribution B (x) and probability density b(x). The vacation time also i.i.d. random variables denoted by v, with general distribution V (x) and probability density n(x). In addition the service time and the vacation time are independent. To study the queue length distribution, we use the residual service time or the residual vacation time as the supplementary variable. At an arbitrary time, the study state of the system can be described by the following random variables:[1-5]

$$\xi=\begin{cases} 0 \text{ if the server is no vacation,} \\ 1 \text{ if the server is busy,} \qquad , \end{cases}$$

$L_{\vee}$ = The number of customers present.

$\hat{B}$ = The residual service time for customer in service.

$\hat{V}$ = The residual service time for the server on vacation

Now

$$\pi_n (x)\ dx = P(L_V = n, \chi < \hat{B} \leq \chi + d\chi,\ \xi = 1),\ n = 1,2,\ldots$$

$$w_n (x)\ dx = P(L_V = n, x < \hat{B} < \hat{V} \leq x + dx,\ \xi = 0),\ n = 1,2,\ldots$$

And the L S T

$$\pi_n^*(s) = \int_0^\infty e^{-xs} \pi_n (x)\ dx,\ w_n^*(s) = \int_0^\infty e^{-xs}\ w_n(x)\ dx \ldots.(2)$$

By the Steady state transitions, we obtain the following differential difference equations:-

$$-\frac{d\,\pi, (x)}{dx} = -\lambda \pi_1 (x) + \pi_2(o)\, b(x) + w_1(o)\, b(x),$$

$$-\frac{d\,\pi_n (x)}{dx} = -\lambda \pi_n (x) + \sum_{j=1}^{\eta-1} \lambda g_j\, \pi_{n-j}(x) + \pi_{n+1}(o)\, b(x) + w_n(o)\, b(x),\ n > 2,$$

$$-\frac{dw_0(x)}{dx} = -\lambda w_0(x) + \pi_1(o)\, \upsilon(x) + w_0(o)\, \upsilon(x)$$

$$-\frac{dw_n(x)}{dx} = -\lambda w_n(x) + \sum_{j=1}^{n} \lambda\, g_j w_{n-j}(x)\ \ n \geq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Taking the LST on both sides of the equations (3) we get

$$-s\pi_1^*(s) + \pi_1(o) = -\lambda\, \pi_1^*(s) + \pi_2(o)\, B^*(s) + w_1(o) B^*(s),$$

$$-s\pi_n^*(s) + \pi_n(o) = -\lambda\, \pi_n^*(s) + \sum_{j=1}^{n-1} \lambda\, g_j \pi_{n-j}^*(s) + \pi_{n+1}(o) B^*(s) + w_n(o) B^{*(s)},$$

$$-sw_0^*(s) + w_0(o) = -\lambda\, w_0^*(s) + \hat{\lambda}_1(o) V^*(s) + w_0(o) V^*(s),$$

$$-s\pi_n^*(s) + w_n(o) = -\lambda\, w_n^*(s) + \sum_{j=1}^{n} \lambda\, g_j w_{n-j}^*(s),\ n \geq 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

also we define

$$\pi(Z,0)=\sum_{n=1}^{\infty}\lambda_n(o)\,Z^n, \qquad w\,(Z,0)=\sum_{n=0}^{\infty}w_n(o)\,Z^n,$$

$$\pi^*(z,s)=\sum_{n=1}^{\infty}\pi_n^*(s)\,z^n, \qquad w^*(z,s)=\sum_{n=0}^{\infty}w_n^*(s)\,z^n \text{..................(5)}$$

Multiplying the second equation by z^n, summing over n, and using the first equation of (4) and G (z), we have

$$\left[s-\lambda-\lambda\,G\,(Z)\right]\pi^*\,(Z,S)=B^*(s)\left[\pi\,(Z,0)-\pi_1\,(o)\,Z\right]/\,Z-\left[w\,(Z,0)-w_0(o)\right]B^*(s)+\pi\,(Z,0)\text{...(6)}$$

Similarly, multiplying the fourth equation by Z^n, summing over n, and using the third equation of (4), we have[6-8]

$$\left[s-\lambda-\lambda\,G\,(Z)\right]w^*(Z,s)=w(Z,0)-\pi,(o)\,v^*(s)-w_0(o)\,V^*(s)\text{..................(7)}$$

Putting $s=\lambda-\lambda\,G\,(Z)$ into equation (6) and (7), we have

$$-B^*(\lambda-\lambda\,G\,(z)\,)\left[\pi\,(Z,0)-\pi_1(o)\,Z\right]/Z-\left[w\,(Z,0)-w_0(o)\right]B^*\,(\lambda-\lambda\,G(z)+\pi\,(Z,0)=0$$

$$w\,(Z,0)-\pi_1(o)\,V^*(\lambda-\lambda\,G\,(z))-W_0(o)\,V^*(\lambda-\lambda\,G\,(z)\,)=0\text{............(8)}$$

Next, inserting z=0 in the second equation of (8) and using w (0,0) = w_0 (o) we get

$$w_0(o)=V^*(\lambda\,)\;\;\pi_1(o)/\left[1-V^*\;(\,\lambda\,)\right]\text{....................................(9)}$$

substituting (9) into the second equation of (8) we get

$$w\,(Z,0)=V^*(\lambda-\lambda\,G\,(Z)]\,\pi_1(o)/\left[1-V^*(\,\lambda\,)\;\right]\text{..........................(10)}$$

from the first equation of (8) and (10), we obtain

$$\lambda\,(Z,0)=\frac{Z\,B^*\,(\lambda-\lambda\,G\,(z)\,)\left[V^*\,(\lambda-\lambda\,G\,(z))-1\right]\pi_1(o)}{\left[1-V^*\;(\lambda)\right]\left[Z-B^*\,(\lambda-\lambda G\,(z)\;)\right]}\qquad\text{..........(11)}$$

substituting (9), (10) and (11) into (6) we get

$$\pi^*\,(z,s)=\frac{Z\left[V^*\,(\lambda-\lambda\,G\,(z)-1)\right]\left[B^*\,(\lambda-\lambda\,G\,(z)\,)-B^*\,(s)\right]\pi_1\,(o)}{\left[1-V^*\,(\,\lambda\,)\right]\left[Z-B^*\,(\lambda-\lambda\,G\,(z)\,)\;\right]\left[s-\lambda+\lambda\,G\,(z)\right]}\text{.................(12)}$$

substituting (9) and (10) into (7) we get

$$w^*(Z,S) = \frac{\left[V^*(\lambda-\lambda G(z)) - V^*(s)\right]\pi_1,(o)}{\left[1-V^*(\lambda)\right]\left[s-\lambda+\lambda G(z)\right]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

since $\pi^*(1,0) + w^*(1,0) = 1$ using the L' Hospital rule on (12) and (13) we get

$$\pi,(o) = (1-\ell)\left[1-V^*(\lambda)\right] / E(V)$$

Therefore the expected number of customers in the system is

$$E(L) = \left.\frac{\partial \pi^*(Z,s)}{\partial Z}\right]_{Z=1,\, s=0} + \left.\frac{\partial w^*(Z,s)}{\partial Z}\right]_{Z=1,\, s=0}$$

$$= \ell + \frac{\lambda g\, E(V^2)}{2E(V)} + \frac{\lambda\left[\lambda g^2 b^2 + g^2\, E(B)\right]}{2(1-\ell)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Now we give the waiting time and the busy-period analysis for this model. The stationary waiting time W_v of an arbitrary or test customer in an arriving batch can be decomposed into the sum of independent random variables. We first write $W_v = W_1 + W_2$, where W_1 is the waiting time of the first customers in the test customers batch and W_2 is the waiting time for the service of the batch-mates who are served before the test customer under consideration. The LST of W_1 can be written as[9-11]

$$W_1^*(s) = \sum_{k=1}^{\infty} \pi_k^*(s)\left[B^*(s)\right]^{k-1} + \sum_{K=1}^{\infty} w_K^*(s)\left[B^*(s)\right]^{K}$$

$$= \pi^*(B^*(s), s)/B^*(s) + w^*(B^*(s), s)$$

$$= \frac{(1-\ell)\left[1-V^*(s)\right]}{E(V)\left[s-\lambda+\lambda G(B^*(s))\right]} \quad \ldots\ldots\ldots\ldots\ldots(15)$$

Let r_n (n=1,2....) be the probability of the test customer being in the n[th] position of the arriving batch. Using the result of renewal theory, we have

$$r_n = \frac{1}{g}\sum_{k=n}^{\infty} g_n$$

Hence we have

$$W_2^*(s) = \sum_{K=1}^{\infty} r_k \left[B^*(s)\right]^{k-1} = \frac{1-G\left(B^*(s)\right)}{g\,(1-B^*(s))} \text{.......................(16)}$$

Using (15) (16) and the fact of W_1 and W_2 are independent, It follows that-

$$W^*(s) = W_1^*(s)\, W_2^*(s)$$

$$= \frac{(1-\ell)\, s\left[1-G\,(B^*(s)\right]}{g\left[s-\lambda+\lambda\, G\left(B^*(s)\right)\right]\left[1-B^*(s)\right]} \; \frac{1-V^*}{s\,E\,(V)} \text{ ....(17)}$$

This expression gives the property of the stationary waiting time[1-11].

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