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Neutrosophic Optimization Technique and its Application on Structural DesignMRIDULA SARKAR^{1*} and TAPAN KUMAR ROY²^{1,2}Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur.

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Abstract

In this paper, we develop a neutrosophic optimization (NSO) approach for optimizing the design of plane truss structure with single objective subject to a specified set of constraints. In this optimum design formulation, weight of truss and deflection of loaded joint are the objective functions. The design variables and constraints are the cross-sectional areas and the stresses in members respectively. A classical truss optimization example is presented herein to demonstrate the efficiency of the neutrosophic optimization approach. The test problem includes a two-bar planar truss subjected to a single load condition. This single-objective structural optimization model is solved by fuzzy as well as neutrosophic optimization approach. Numerical example is given to illustrate our NSO approach. The result shows that the NSO technique plays a significant role in finding the best ever optimal solutions.

Key word : Neutrosophic Set, Single Valued Neutrosophic Set, Neutrosophic Optimization, Single-Objective Structural optimization.

1. Introduction

The research area of optimal structural design has been receiving increasing attention from both academia and industry over the past four decades in order to improve structural performance and to reduce design costs. However, in the real world, uncertainty or vagueness is prevalent in the Engineering Computations. In the context of structural design the uncertainty is connected with lack of accurate data of design factors. This problem has been solving by use of fuzzy mathematical algorithm for dealing with this class of problems. Fuzzy set (FS) theory has long been introduced to deal with inexact and imprecise resources by Zadeh¹, As an application Bellman and Zadeh² used the fuzzy set theory to the decision making problem. In such extension, Atanassov³ introduced Intuitionistic fuzzy set (IFS) which is one of the generalizations of fuzzy set theory and

is characterized by a membership function, a non membership function and a hesitancy function. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a degree of acceptance and degree of rejection so that their sum is less than one. As a generalization of fuzzy set and intuitionistic fuzzy set F. Smrandache⁴ introduced a new notion which is known as neutrosophic set (NS in short) in 1995. NS is characterized by degree of truth membership, degree of indeterminacy membership and degree of falsity membership. The concept of NS generates the theory of neutrosophic sets by expressing indeterminacy of imprecise information. This theory is considered as complete representation of structural design problems like other decision making problems. Therefore, if uncertainty is involved in a structural model we use fuzzy theory while dealing indeterminacy, we need neutrosophic theory.

This is the first time neutrosophic optimization techniques are applied in structural design. Several researchers like Wang *et al.*⁵ first applied α -cut method to structural designs where the non-linear problems were solved with various design levels α , and then a sequence of solutions were obtained by setting different level-cut value of α . To design a four-bar mechanism for function generating problem Rao⁶ used the same α -cut method. Structural optimization with fuzzy parameters was developed by Yeh *et al.*⁷. Xu⁸ used two-phase method for fuzzy optimization of structures. A level-cut of the first and second kind approach used by Shih *et al.*⁹ for structural design optimization problems with fuzzy resources. Shih *et al.*¹⁰ developed an alternative α -level-cuts methods for optimum structural design with fuzzy resources. Dey *et al.*¹¹ used generalized fuzzy number in context of a structural design. Dey *et al.*¹² developed parameterized t-norm based fuzzy optimization method for optimum structural design. Also, a parametric geometric programming is introduced by Dey *et al.*¹³ to Optimize shape design of structural model with imprecise coefficient.

A transportation model was solved by Jana *et al.*¹⁴ using multi-objective intuitionistic fuzzy linear programming. Dey *et al.*¹⁵ solved two-bar truss non linear problem by using intuitionistic fuzzy optimization problem. Dey *et al.*¹⁶ used intuitionistic fuzzy optimization technique for multi objective optimum structural design.

The present study investigates computational algorithm for solving single-objective structural problem by single valued NSO approach. The impact of truth, indeterminacy and falsity membership function in such optimization process also has been studied here. A comparison is made numerically between fuzzy optimization technique and neutrosophic optimization technique. From our numerical result, it is clear that neutrosophic optimization technique provides better results than fuzzy optimization.

2. Single-objective Structural Model :

In sizing optimization problems, the aim is to minimize single objective function, usually the weight of the structure under certain behavioural constraints on constraint and displacement. The design variables are most frequently chosen to be dimensions of the cross sectional areas of the members of the structures. Due to fabrications limitations the design variables are not continuous but discrete for belongingness of cross-sections to a certain set. A discrete structural optimization problem can be formulated in the following form

$$\text{Minimize } WT(A) \tag{1}$$

$$\text{subject to } \sigma_i(A) \leq [\sigma_i(A)], i = 1, 2, \dots, m$$

$$A_j \in R^d, \quad j = 1, 2, \dots, n$$

where $WT(A)$ represents objective function, $\sigma_i(A)$ is the behavioural constraints and $[\sigma_i(A)]$ denotes the maximum allowable value, m and n are the number of constraints and design variables respectively. A given

set of discrete value is expressed by R^d and in this paper objective function is taken as

$$WT(A) = \sum_{i=1}^m \rho_i l_i A_i$$

and constraint are chosen to be stress of structures as follows

$$\sigma_i(A) \leq \sigma_i^0 \text{ with allowable tolerance } \sigma_i^0 \text{ for } i = 1, 2, \dots, m$$

Where ρ_i and l_i are weight of unit volume and length of i^{th} element respectively, m is the number of structural element, σ_i and σ_i^0 are the i^{th} stress, allowable stress respectively.

3. Mathematical preliminaries

3.1. Fuzzy Set

Let X be a fixed set. A fuzzy set A set of X is an object having the form $\tilde{A} = \{(x, T_A(x)) : x \in X\}$ where the function $T_A : X \rightarrow [0, 1]$ defined the truth membership of the element $x \in X$ to the set A .

3.2. Intuitionistic Fuzzy Set :

Let a set X be fixed. An intuitionistic fuzzy set or IFS \tilde{A}^i in X is an object of the form $\tilde{A}^i = \{ \langle x, T_A(x), F_A(x) \rangle \mid x \in X \}$ where $T_A : X \rightarrow [0, 1]$ and $F_A : X \rightarrow [0, 1]$ define the truth membership and falsity membership respectively, for every element of $x \in X$, $0 \leq T_A(x) + F_A(x) \leq 1$.

3.3. NeutrosophicSet

Let a set X be a space of points (objects) and $x \in X$. A neutrosophic set \tilde{A}^n in X is defined by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity membership function $F_A(x)$ and having the form $\tilde{A}^n = \{ \langle x, \mu_A(x), I_A(x), F_A(x) \rangle \mid x \in X \}$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is

$$T_A(x) : X \rightarrow]0^-, 1^+[$$

$$I_A(x) : X \rightarrow]0^-, 1^+[$$

$$F_A(x) : X \rightarrow]0^-, 1^+[$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ so

$$0^- \leq \sup T_A(x) + I_A(x) + \sup F_A(x) \leq 3^+.$$

3.4. Single Valued Neutrosophic Set :

Let a set X be the universe of discourse. A single valued neutrosophic set \tilde{A}^n over X is an object having the form $\tilde{A}^n = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \}$ where $T_A : X \rightarrow [0, 1]$, $I_A : X \rightarrow [0, 1]$ and $F_A : X \rightarrow [0, 1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$.

3.5. Complement of Neutrosophic Set :

Complement of a single valued neutrosophic set A is denoted by $c(A)$ and is defined by

$$T_{c(A)}(x) = F_A(x), I_{c(A)}(x) = 1 - F_A(x), F_{c(A)}(x) = T_A(x)$$

3.6. Union of Neutrosophic Sets :

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C , written as $C = A \cup B$, whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$$T_{c(A)}(x) = \max(T_A(x), T_B(x)),$$

$$I_{c(A)}(x) = \max(I_A(x), I_B(x))$$

$$F_{c(A)}(x) = \min(F_A(x), F_B(x)) \text{ for all } x \in X$$

3.7. Intersection of Neutrosophic Sets :

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C , written as $C = A \cap B$, whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$$T_{c(A)}(x) = \min(T_A(x), T_B(x))$$

$$I_{c(A)}(x) = \min(I_A(x), I_B(x))$$

$$F_{c(A)}(x) = \max(F_A(x), F_B(x)) \text{ for all } x \in X$$

4. Mathematical Analysis :

4.1. Neutrosophic Optimization Technique to Solve Minimization Type Single-Objective :

Let a nonlinear single-objective optimization problem be

Minimize $f(x)$

$$g_j(x) \leq b_j \quad j = 1, 2, \dots, m \tag{2}$$

$$x \geq 0$$

Usually constraints goals are considered as fixed quantity .But in real life problem ,the constraint goal can not be always exact. So we can consider the constraint goal for less than type constraints at least b_j and it may possible to extend to $b_j + b_j^0$. This fact seems to take the constraint goal as a neutrosophic fuzzy set and which will be more realistic descriptions than others. Then the NLP becomes NSO problem with neutrosophic resources, which can be described as follows

Minimize $f(x)$ (3)

$$g_j(x) \leq \tilde{b}_j^n \quad j = 1, 2, \dots, m$$

$$x \geq 0$$

To solve the NSO (3), following warner's (1987) and Angelov (1995) we are presenting a solution procedure for single-objective NSO problem (3) as follows

Step-1: Following warner's approach solve the single objective non-linear programming problem without tolerance in constraints (i.e $g_j(x) \leq b_j$), with tolerance of acceptance in constraints (i.e $g_j(x) \leq b_j + b_j^0$) by appropriate non-linear programming technique

Here they are

Sub-problem-1

$$\text{Minimize } f(x) \tag{4}$$

$$g_j(x) \leq b_j \quad j = 1, 2, \dots, m$$

$$x \geq 0$$

Sub-problem-2

$$\text{Minimize } f(x) \tag{5}$$

$$g_j(x) \leq b_j + b_j^0, \quad j = 1, 2, \dots, m$$

$$x \geq 0$$

we may get optimal solutions $x^* = x^1, f(x^*) = f(x^1)$ and $x^* = x^1, f(x^*) = f(x^1)$.

Step-2: From the result of step 1 we now find the lower bound and upper bound of objective functions. If

$U_{f(x)}^T, U_{f(x)}^I, U_{f(x)}^F$ be the upper bounds of truth, indeterminacy, falsity function for the objective respectively and $L_{f(x)}^T, L_{f(x)}^I, L_{f(x)}^F$ be the lower bound of truth, indeterminacy, falsity membership functions of objective respectively then

$$U_{f(x)}^T = \max \{f(x^1), f(x^2)\}, L_{f(x)}^T = \min \{f(x^1), f(x^2)\},$$

$$U_{f(x)}^F = U_{f(x)}^T, L_{f(x)}^F = L_{f(x)}^T + \varepsilon_{f(x)} \text{ where } 0 < \varepsilon_{f(x)} < (U_{f(x)}^T - L_{f(x)}^T)$$

$$L_{f(x)}^I = L_{f(x)}^T, U_{f(x)}^I = L_{f(x)}^T + \xi_{f(x)} \text{ where } 0 < \xi_{f(x)} < (U_{f(x)}^T - L_{f(x)}^T)$$

Step-3: In this step we calculate membership for truth, indeterminacy and falsity membership function of objectives as follows

$$T_{f(x)}(f(x)) = \begin{cases} 1 & \text{if } f(x) \leq L_{f(x)}^T \\ \left(\frac{U_{f(x)}^T - f(x)}{U_{f(x)}^T - L_{f(x)}^T} \right) & \text{if } L_{f(x)}^T \leq f(x) \leq U_{f(x)}^T \\ 0 & \text{if } f(x) \geq U_{f(x)}^T \end{cases}$$

$$I_{f(x)}(f(x)) = \begin{cases} 1 & \text{if } f(x) \leq L_{f(x)}^I \\ \left(\frac{U_{f(x)}^I - f(x)}{U_{f(x)}^I - L_{f(x)}^I} \right) & \text{if } L_{f(x)}^I \leq f(x) \leq U_{f(x)}^I \\ 0 & \text{if } f(x) \geq U_{f(x)}^I \end{cases}$$

$$F_{f(x)}(f(x)) = \begin{cases} 0 & \text{if } f(x) \leq L_{f(x)}^F \\ \frac{f(x) - L_{f(x)}^F}{U_{f(x)}^F - L_{f(x)}^F} & \text{if } L_{f(x)}^F \leq f(x) \leq U_{f(x)}^F \\ 1 & \text{if } f(x) \geq U_{f(x)}^F \end{cases}$$

where ψ, τ are non-zero parameters prescribed by the decision maker.

Step-4: In this step using linear membership function we calculate, indeterminacy and falsity membership function for constraints as follows

$$T_{g_j(x)}(g_j(x)) = \begin{cases} 1 & \text{if } g_j(x) \leq b_j \\ \left(\frac{b_j + b_j^0 - g_j(x)}{b_j^0} \right) & \text{if } b_j \leq g_j(x) \leq b_j + b_j^0 \\ 0 & \text{if } g_j(x) \geq b_j^0 \end{cases}$$

$$I_{g_j(x)}(g_j(x)) = \begin{cases} 1 & \text{if } g_j(x) \leq b_j \\ \left(\frac{b_j + \xi_{g_j(x)} - g_j(x)}{\xi_{g_j(x)}} \right) & \text{if } b_j \leq g_j(x) \leq b_j + \xi_{g_j(x)} \\ 0 & \text{if } g_j(x) \geq b_j + \xi_{g_j(x)} \end{cases}$$

$$F_{g_j(x)}(g_j(x)) = \begin{cases} 0 & \text{if } g_j(x) \leq b_j + \varepsilon_{g_j(x)} \\ \left(\frac{g_j(x) - b_j - \varepsilon_{g_j(x)}}{b_j^0 - \varepsilon_{g_j(x)}} \right) & \text{if } b_j + \varepsilon_{g_j(x)} \leq g_j(x) \leq b_j + b_j^0 \\ 1 & \text{if } g_j(x) \geq b_j + b_j^0 \end{cases}$$

where ψ, τ are non-zero parameters prescribed by the decision maker and for $j = 1, 2, \dots, m$ $0 < \varepsilon_{g_j(x)}, \xi_{g_j(x)} < b_j^0$.

Step-5: Now using NSO for single objective optimization technique the optimization problem (2) can be formulated as

$$\text{Maximize } (\alpha + \gamma - \beta)$$

Such that

$$T_{f(x)}(x) \geq \alpha; T_{g_j}(x) \geq \alpha; \tag{6}$$

$$I_{f(x)}(x) \geq \gamma; I_{g_j}(x) \geq \gamma;$$

$$F_{f(x)}(x) \leq \beta; F_{g_j}(x) \leq \beta;$$

$$\alpha + \beta + \gamma \leq 3; \alpha \geq \beta; \alpha \geq \gamma;$$

$$\alpha, \beta, \nu \in [0, 1]$$

Where $\alpha = \mu_{\tilde{D}^n}(x) = \min \{T_{f(x)}(f(x)), T_{g_j(x)}(g_j(x))\}$ for $j = 1, 2, \dots, m$

$\gamma = I_{\tilde{D}^n}(x) = \min \{I_{f(x)}(f(x)), I_{g_j(x)}(g_j(x))\}$ for $j = 1, 2, \dots, m$ and

$\beta = \nu_{\tilde{D}^n}(x) = \min \{F_{f(x)}(f(x)), F_{g_j(x)}(g_j(x))\}$ for $j = 1, 2, \dots, m$

are the truth, in determinacy and falsity membership function of decision set

$$\tilde{D}^n = f^n(x) \bigcap_{j=1}^m g_j^n(x).$$

5. Solution of Single-objective Structural Optimization Problem (SOSOP) by Neutrosophic Optimization Technique :

To solve the SOSOP (1), step 1 of 4 is used and we will get optimum solutions of two sub problem as A^1 and A^2 . After that according to step 2 we find upper and lower bound of membership function of objective function

as $U_{WT(A)}^T, U_{WT(A)}^I, U_{WT(A)}^F$ and $U_{WT(A)}^T, U_{WT(A)}^I, U_{WT(A)}^F$ where $U_{WT(A)}^T = \max \{WT(A^1), WT(A^2)\}$,

$L_{WT(A)}^T = \min \{WT(A^1), WT(A^2)\}$, $U_{WT(A)}^F = U_{WT(A)}^T$, $L_{WT(A)}^F = L_{WT(A)}^T + \varepsilon_{WT(A)}$ where

$0 < \varepsilon_{WT(A)} < (U_{WT(A)}^T - L_{WT(A)}^T)$; $L_{WT(A)}^I = L_{WT(A)}^T$, $U_{WT(A)}^I = L_{WT(A)}^T + \xi_{WT(A)}$ where

$$0 < \xi_{WT(A)} < (U_{WT(A)}^T - L_{WT(A)}^T)$$

Let the linear membership function for objective be

$$T_{WT(A)}(WT(A)) = \begin{cases} 1 & \text{if } WT(A) \leq L_{WT(A)}^T \\ \left(\frac{U_{WT(A)}^T - WT(A)}{U_{WT(A)}^T - L_{WT(A)}^T} \right) & \text{if } L_{WT(A)}^T \leq WT(A) \leq U_{WT(A)}^T \\ 0 & \text{if } WT(A) \geq U_{WT(A)}^T \end{cases}$$

$$I_{WT(A)}(WT(A)) = \begin{cases} 1 & \text{if } WT(A) \leq L_{WT(A)}^T \\ \left(\frac{(L_{WT(A)}^T + \xi_{WT(A)}) - WT(A)}{\xi_{WT(A)}} \right) & \text{if } L_{WT(A)}^T \leq WT(A) \leq L_{WT(A)}^T + \xi_{WT(A)} \\ 0 & \text{if } WT(A) \geq L_{WT(A)}^T + \xi_{WT(A)} \end{cases}$$

$$v_{WT(A)}(WT(A)) = \begin{cases} 0 & \text{if } WT(A) \leq L_{WT(A)}^T + \varepsilon_{WT(A)} \\ \left(\frac{WT(A) - (L_{WT(A)}^T + \varepsilon_{WT(A)})}{U_{WT(A)}^T - L_{WT(A)}^T - \varepsilon_{WT(A)}} \right) & \text{if } L_{WT(A)}^T + \varepsilon_{WT(A)} \leq WT(A) \leq U_{WT(A)}^T \\ 1 & \text{if } WT(A) \geq U_{WT(A)}^T \end{cases}$$

and constraints be

$$T_{\sigma_i(A)}(\sigma_i(A)) = \begin{cases} 1 & \text{if } \sigma_i(A) \leq \sigma_i \\ \left(\frac{(\sigma_i + \sigma_i^0) - \sigma_i(A)}{\sigma_i^0} \right) & \text{if } \sigma_i \leq \sigma_i(A) \leq \sigma_i + \sigma_i^0 \\ 0 & \text{if } \sigma_i(A) \geq \sigma_i + \sigma_i^0 \end{cases}$$

$$I_{\sigma_i(A)}(\sigma_i(A)) = \begin{cases} 1 & \text{if } \sigma_i(A) \leq \sigma_i \\ \left(\frac{(\sigma_i + \xi_{\sigma_i(x)}) - \sigma_i(A)}{\xi_{\sigma_i(x)}} \right) & \text{if } \sigma_i \leq \sigma_i(A) \leq \sigma_i + \xi_{\sigma_i(x)} \\ 0 & \text{if } \sigma_i(A) \geq \sigma_i + \xi_{\sigma_i(x)} \end{cases}$$

$$F_{\sigma_i(A)}(\sigma_i(A)) = \begin{cases} 0 & \text{if } \sigma_i(A) \leq \sigma_i + \varepsilon_{\sigma_i(x)} \\ \left(\frac{\sigma_i(A) - \sigma_i - \varepsilon_{\sigma_i(x)}}{\sigma_i^0 - \varepsilon_{\sigma_i(x)}} \right) & \text{if } \sigma_i + \varepsilon_{\sigma_i(x)} \leq \sigma_i(A) \leq \sigma_i + \sigma_i^0 \\ 1 & \text{if } \sigma_i(A) \geq \sigma_i + \sigma_i^0 \end{cases}$$

where ψ, τ are non-zero parameters prescribed by the decision maker and for $j = 1, 2, \dots, m$ $0 < \varepsilon_{\sigma_i(x)}, \xi_{\sigma_i(A)} < \sigma_i^0$

then neutrosophic optimization problem can be formulated as

Maximize $(\alpha + \beta - \gamma)$

such that

$$T_{WT(A)}(WT(A)) \geq \alpha; T_{\sigma_i(A)}(\sigma_i(A)) \geq \alpha;$$

$$I_{WT(A)}(WT(A)) \geq \gamma; I_{\sigma_i(A)}(\sigma_i(A)) \geq \gamma;$$

$$F_{WT(A)}(WT(A)) \leq \beta; F_{\sigma_i(A)}(\sigma_i(A)) \leq \beta$$

$$\sigma_i(x) \leq [\sigma_i];$$

$$\alpha + \beta + \gamma \leq 3; \alpha \geq \beta; \alpha \geq \gamma;$$

$$\alpha, \beta, \gamma \in [0, 1]$$

The above problem can be reduced to following crisp linear programming problem, whenever linear membership are considered, as

$$\text{Maximize } (\alpha - \beta + \gamma) \tag{7}$$

Such that

$$WT(A) + \alpha(U_{WT(A)}^T - L_{WT(A)}^T) \leq U_{WT(A)}^T;$$

$$WT(A) + \gamma(U_{WT(A)}^T - L_{WT(A)}^T - \xi_{WT(A)}) \leq L_{WT(A)}^T + \xi_{WT(A)};$$

$$WT(A) - \beta(U_{WT(A)}^T - L_{WT(A)}^T - \varepsilon_{WT(A)}) \leq L_{WT(A)}^T + \varepsilon_{WT(A)};$$

$$\sigma_T(A) + \alpha(U_{\sigma_T(A)}^T - L_{\sigma_T(A)}^T) \leq U_{\sigma_T(A)}^T;$$

$$\sigma_T(A) + \gamma(U_{\sigma_T(A)}^T - L_{\sigma_T(A)}^T - \xi_{\sigma_T(A)}) \leq U_{\sigma_T(A)}^T + \xi_{\sigma_T(A)};$$

$$\sigma_T(A) - \beta(U_{\sigma_T(A)}^T - L_{\sigma_T(A)}^T - \varepsilon_{\sigma_T(A)}) \leq L_{\sigma_T(A)}^T + \varepsilon_{\sigma_T(A)};$$

$$\sigma_C(A) + \alpha(U_{\sigma_C(A)}^T - L_{\sigma_C(A)}^T) \leq U_{\sigma_C(A)}^T;$$

$$\sigma_C(A) + \gamma(U_{\sigma_C(A)}^T - L_{\sigma_C(A)}^T - \xi_{\sigma_C(A)}) \leq U_{\sigma_C(A)}^T + \xi_{\sigma_C(A)};$$

$$\alpha + \beta + \gamma \leq 3;$$

$$\alpha \geq \beta; \alpha \geq \gamma;$$

$$\alpha, \beta, \gamma \in [0, 1]$$

This crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

6. Numerical Illustration :

A well-known two-bar planar truss structure (Fig. 1) is considered. The design objective is to minimize weight of the structural $WT(A_1, A_2, y_B)$ of a statistically loaded two-bar planar truss subjected to stress $\sigma_i(A_1, A_2, y_B)$ constraints on each of the truss members $i = 1, 2$.

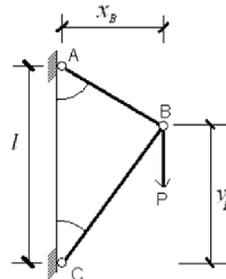


Fig. 1. Design of the two-bar planar truss

The multi-objective optimization problem can be stated as follows

$$\text{Minimize } WT(A_1, A_2, y_B) = \rho \left(A_1 \sqrt{x_B^2 + (l - y_B)^2} + A_2 \sqrt{x_B^2 + y_B^2} \right) \tag{8}$$

Such that

$$\sigma_{AB}(A_1, A_2, y_B) \equiv \frac{P \sqrt{x_B^2 + (l - y_B)^2}}{l A_1} \leq [\sigma_{AB}^T];$$

$$\sigma_{BC}(A_1, A_2, y_B) \equiv \frac{P \sqrt{x_B^2 + y_B^2}}{l A_2} \leq [\sigma_{BC}^C];$$

$$0.5 \leq y_B \leq 1.5$$

$$A_1 > 0, A_2 > 0;$$

where P = nodal load ; ρ = volume density ; l = length of AC ; x_B = perpendicular distance from AC to point B . A_1 = Cross section of bar- AB ; A_2 = Cross section of bar- BC . $[\sigma_T]$ = maximum allowable tensile stress, $[\sigma_C]$ = maximum allowable compressive stress and y_B = y -co-ordinate of node B . Input data are given in table 1.

Table 1. Input data for crisp model (7)

Applied load P (KN)	Volume density ρ (KN / m ³)	Length l (m)	Maximum allowable tensile stress $[\sigma_T]$ (Mpa)	Maximum allowable compressive stress $[\sigma_T]$ (Mpa)	Distance of x_B from AC (m)
100	7.7	2	130 with fuzzy region 20	90 with fuzzy region 10	1

Solution : According to step 2 of 4, we find upper and lower bound of membership function of objective function as $U_{WT(A)}^T, U_{WT(A)}^I, U_{WT(A)}^F$ and $U_{WT(A)}^T, U_{WT(A)}^I, U_{WT(A)}^F$ where $U_{WT(A)}^T = 14.23932 = U_{WT(A)}^F$, $L_{WT(A)}^T = 12.57667 = L_{WT(A)}^I$, $L_{WT(A)}^F = 12.57667 + \varepsilon_{WT(A)}$ where $0 < \varepsilon_{WT(A)} < 1.66265$; and $U_{WT(A)}^\sigma = L_{WT(A)}^\mu + \xi_{WT(A)}$ where $0 < \xi_{WT(A)} < 1.66265$

Now using the bounds we calculate the membership functions for objective as follows

$$T_{WT(A_1, A_2, y_B)}(WT(A_1, A_2, y_B)) = \begin{cases} 1 & \text{if } WT(A_1, A_2, y_B) \leq 12.57667 \\ \left(\frac{14.23932 - WT(A_1, A_2, y_B)}{1.66265} \right) & \text{if } 12.57667 \leq WT(A_1, A_2, y_B) \leq 14.23932 \\ 0 & \text{if } WT(A_1, A_2, y_B) \geq 14.23932 \end{cases}$$

$$I_{WT(A_1, A_2, y_B)}(WT(A_1, A_2, y_B)) = \begin{cases} 1 & \text{if } WT(A_1, A_2, y_B) \leq 12.57667 \\ \left(\frac{(12.57667 + \xi_{WT}) - WT(A_1, A_2, y_B)}{\xi_{WT}} \right) & \text{if } 12.57667 \leq WT(A_1, A_2, y_B) \leq 12.57667 + \xi_{WT(A)} \\ 0 & \text{if } WT(A_1, A_2, y_B) \geq 12.57667 + \xi_{WT(A)} \end{cases}$$

$$F_{WT(A_1, A_2, y_B)}(WT(A_1, A_2, y_B)) = \begin{cases} 0 & \text{if } WT(A_1, A_2, y_B) \leq 12.57667 + \varepsilon_{WT(A)} \\ \left(\frac{WT(A_1, A_2, y_B) - 12.57667 - \varepsilon_{WT(A)}}{1.66265 - \varepsilon_{WT(A)}} \right) & \text{if } 12.57667 + \varepsilon_{WT(A)} \leq WT(A_1, A_2, y_B) \leq 14.23932 \\ 1 & \text{if } WT(A_1, A_2, y_B) \geq 14.23932 \end{cases}$$

Similarly the membership functions for tensile stress are

$$T_{\sigma_T(A_1, A_2, y_B)}(\sigma_T(A_1, A_2, y_B)) = \begin{cases} 1 & \text{if } \sigma_T(A_1, A_2, y_B) \leq 130 \\ \left(\frac{150 - \sigma_T(A_1, A_2, y_B)}{20} \right) & \text{if } 130 \leq \sigma_T(A_1, A_2, y_B) \leq 150 \\ 0 & \text{if } \sigma_T(A_1, A_2, y_B) \geq 150 \end{cases}$$

$$I_{\sigma_T(A_1, A_2, y_B)}(\sigma_T(A_1, A_2, y_B)) = \begin{cases} 1 & \text{if } \sigma_T(A_1, A_2, y_B) \leq 130 \\ \left(\frac{(130 + \xi_{\sigma_T}) - \sigma_T(A_1, A_2, y_B)}{\xi_{\sigma_T}} \right) & \text{if } 130 \leq \sigma_T(A_1, A_2, y_B) \leq 130 + \xi_{\sigma_T} \\ 0 & \text{if } \sigma_T(A_1, A_2, y_B) \geq 130 + \xi_{\sigma_T} \end{cases}$$

$$F_{\sigma_T(A_1, A_2, y_B)}(\sigma_T(A_1, A_2, y_B)) = \begin{cases} 0 & \text{if } \sigma_T(A_1, A_2, y_B) \leq 130 + \varepsilon_{\sigma_T} \\ \left(\frac{\sigma_T(A_1, A_2, y_B) - 130 - \varepsilon_{\sigma_T}}{20 - \varepsilon_{\sigma_T}} \right) & \text{if } 130 + \varepsilon_{\sigma_T} \leq \sigma_T(A_1, A_2, y_B) \leq 150 \\ 1 & \text{if } \sigma_T(A_1, A_2, y_B) \geq 150 \end{cases}$$

where $0 < \varepsilon_{\sigma_T}, \xi_{\sigma_T} < 20$

and the membership functions for compressive stress constraint are

$$T_{\sigma_C(A_1, A_2, y_B)}(\sigma_C(A_1, A_2, y_B)) = \begin{cases} 1 & \text{if } \sigma_C(A_1, A_2, y_B) \leq 90 \\ \left(\frac{100 - \sigma_C(A_1, A_2, y_B)}{10} \right) & \text{if } 90 \leq \sigma_C(A_1, A_2, y_B) \leq 100 \\ 0 & \text{if } \sigma_C(A_1, A_2, y_B) \geq 100 \end{cases}$$

$$I_{\sigma_C(A_1, A_2, y_B)}(\sigma_C(A_1, A_2, y_B)) = \begin{cases} 1 & \text{if } \sigma_C(A_1, A_2, y_B) \leq 90 \\ \left(\frac{(90 + \xi_{\sigma_C}) - \sigma_C(A_1, A_2, y_B)}{\xi_{\sigma_C}} \right) & \text{if } 90 \leq \sigma_C(A_1, A_2, y_B) \leq 90 + \xi_{\sigma_C} \\ 0 & \text{if } \sigma_C(A_1, A_2, y_B) \geq 90 + \xi_{\sigma_C} \end{cases}$$

$$F_{\sigma_C(A_1, A_2, y_B)}(\sigma_C(A_1, A_2, y_B)) = \begin{cases} 0 & \text{if } \sigma_C(A_1, A_2, y_B) \leq 90 + \varepsilon_{\sigma_C} \\ \left(\frac{\sigma_C(A_1, A_2, y_B) - 90 - \varepsilon_{\sigma_C}}{10 - \varepsilon_{\sigma_C}} \right) & \text{if } 90 + \varepsilon_{\sigma_C} \leq \sigma_C(A_1, A_2, y_B) \leq 100 \\ 1 & \text{if } \sigma_C(A_1, A_2, y_B) \geq 100 \end{cases}$$

where $0 < \varepsilon_{\sigma_C}, \xi_{\sigma_C} < 10$

Now, using above mentioned truth, indeterminacy and falsity membership function in (7) NLP(8) can be solved by NSO technique for different values of $\varepsilon_{WT}, \varepsilon_{\sigma_T}, \varepsilon_{\sigma_C}$ and $\xi_{WT}, \xi_{\sigma_T}, \xi_{\sigma_C}$. The optimum solution of SOSOP(7) is given in table 2 and the solution is compared with fuzzy problem.

Table 2: Comparison of Optimal solution of SOSOP (7) based on different method

Methods	A_1 (m^2)	A_2 (m^2)	$WT(A_1, A_2)$ (KN)	y_B (m)
Fuzzy single-objective non-linear programming (FSONLP)	.5883491	.7183381	14.23932	1.013955
Neutrosophic optimization (NSO) $\varepsilon_{WT} = 0.33253, \varepsilon_{\sigma_T} = 4, \varepsilon_{\sigma_C} = 2$ $\xi_{WT} = .498795, \xi_{\sigma_T} = 6, \xi_{\sigma_C} = 3$.5954331	.7178116	13.07546	.818181

Here we get best solutions for the different tolerance $\xi_{WT(A)}$, $\xi_{\sigma T}$ and $\xi_{\sigma C}$ for indeterminacy membership function of objective functions for this structural optimization problem. From table 2, it is shown that NSO technique gives better Pareto optimal result in the perspective of Structural Optimization.

7. Conclusions

The main objective of this work is to illustrate how neutrosophic optimization technique using linear membership function can be utilized to solve a single objective-nonlinear structural problem. The concept of neutrosophic optimization technique allows one to define a degree of truth membership, which is not a complement of degree of falsity; rather, they are independent with degree of indeterminacy. The numerical illustration shows the superiority of neutrosophic optimization over fuzzy optimization. The results of this study may lead to the development of effective neutrosophic technique for solving other models of single objective nonlinear programming problem in other engineering field.

Conflict of interests: The authors declare that there is no conflict of interests.

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