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Inflationary Scenario in Spatially Homogeneous Bianchi Type IX Space-time with Flat Potential in General Relativity

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Abstract

Inflationary scenario in spatially homogeneous Bianchi Type IX space-time is investigated. To get the deterministic result, we have considered the potential $V(\phi)$ is constant and the scale factor $R \sim e^{Ht}$ where H is Hubble constant. We find that spatial volume (R^3) increases exponentially. Thus inflationary scenario exists in Bianchi Type IX space time. The Higgs field decreases slowly. The expansion (θ) is constant, shear (σ) is zero and deceleration parameter (q) = - 1 which indicate that the model represents de-Sitter universe. The model also gives accelerating phase of the universe.

1. Introduction

Bianchi Type IX space-time is interesting in the study because familiar solutions like FRW models[1] for positive curvature, the de-Sitter model[2], Taub-NUT solutions[3] are of Bianchi Type IX space-times. The models[2,3] allow not only expansion but also shear and rotation. Chakraborty[4] has investigated Bianchi Type IX inflationary and string cosmological models in general relativity. Chakraborty and Nandy[5] have investigated Bianchi Types II, VIII and IX string cosmological models in general relativity. Vaidya and Patel[6] have studied spatially homogeneous Bianchi Type IX space-time giving general scheme for getting exact solutions of Einstein Field equations corresponding to perfect fluid and pure radiation field. Bali and Upadhaya[7] have investigated Bianchi Type IX string dust cosmological model in general relativity. Bianchi Type IX viscous fluid cosmological model is investigated by Bali and Yadav[8]. Bali and Gupta[9] have investigated non-static barotropic perfect fluid cosmological model in general relativity. Bianchi Type IX tilted barotropic fluid cosmological model for perfect fluid distribution is investigated by Bali and Kumawat[10]. Bali and Goyal[11] have investigated Bianchi Type IX inflationary cosmological models for perfect fluid distribution in general relativity assuming the condition $\sigma \alpha \theta$ where σ is shear and θ the expansion in the model. Recently Bali and Singh[12] have investigated inflationary scenario in Bianchi Type IX space-time with massless scalar field.

In this paper, we have investigated Bianchi Type IX inflationary cosmological model with flat potential in general relativity. To get the deterministic solution in terms of cosmic time t, we have assumed that potential (V) is constant and scale factor $R \sim e^{Ht}$ considered by Kirzhnits and Linde[13] and Kirzhnits[14] where H is constant. We find that spatial volume (R^3) increases with time which indicates inflationary scenario of the universe. The deceleration parameter (q) < 0. Thus the model shows the accelerating phase of the universe. Since expansion (θ) is constant, shear (σ) is zero and deceleration parameter (q) = -1. Therefore the model also represents de-Sitter universe.

2. Metric and Field Equations

We consider Bianchi type IX metric in the form

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)\,dy^2 + (b^2\sin^2 y + a^2\cos^2 y)dz^2 - 2a^2\cos y\,dx\,dz \tag{2.1}$$

where a and b are metric potentials and the function of t-alone.

In case of gravity minimally coupled to a scalar field $V(\phi)$, we have

$$S = \int \sqrt{-g}\left(R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right)d^4x \tag{2.2}$$

which on variation of S with respect to dynamical fields leads to Einstein field equation

$$R_{ij} - \frac{1}{2}R\,g_{ij} = -8\pi T_{ij} \tag{2.3}$$

with

$$T_{ij} = \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_\ell\phi\partial^\ell\phi + V(\phi)\right]g_{ij} \tag{2.4}$$

and

$$\frac{1}{\sqrt{-g}}\partial_i\,[\sqrt{-g}\,\partial_i\phi] = -\frac{dV}{d\phi} \tag{2.5}$$

[in geometrized units where G = c = 1]

where

$$\partial_i\phi = \frac{\partial\phi}{\partial x^i}, \partial_j\phi = \frac{\partial\phi}{\partial x^j} \text{ and } \partial^\ell\phi = g^{\ell\nu}\partial_{\nu\phi} = g^{\ell\nu}\frac{\partial\phi}{\partial x^\nu}.$$

The Einstein field equations (2.3) for the metric (2.1) leads to

$$\frac{\dot{b}^2}{b^2} + \frac{2\ddot{b}}{b} + \frac{1}{b^2} - \frac{3}{4}\frac{a^2}{b^4} = \left[\frac{\dot{\phi}^2}{2} - V(\phi)\right] \tag{2.6}$$

$$\frac{a^2}{4b^4} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} + \frac{\ddot{a}}{a} = \left[\frac{\dot{\phi}^2}{2} - V(\phi)\right] \tag{2.7}$$

$$\frac{2\dot{a}\dot{b}}{ab} - \frac{a^2}{4b^4} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = -\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right] \tag{2.8}$$

$$\left[\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{a^2}{b^4} + \frac{\dot{a}\dot{b}}{ab} - \frac{1}{b^2} - \frac{\dot{b}^2}{b^2}\right]\cos y = 0 \tag{2.9}$$

The equation for the scalar field (2.5) for the line-element (2.1) leads to

$$\ddot{\phi}+\left(\frac{\dot{a}}{a}+\frac{2\dot{b}}{b}\right)\dot{\phi}=-\frac{dV}{d\phi} \qquad (2.10)$$

The dot over a,b denotes ordinary differentiation with respect to t.

3. Solution of the Field Equations :

We are interested in inflationary solutions, so flat region is considered, where potential $V(\phi)$ is constant. Thus

$V(\phi)$ = constant = V_0 (say) (3.1)

Using equation (3.1) in equation (2.10), we have

$$\ddot{\phi}+\left(\frac{\dot{a}}{a}+\frac{2\dot{b}}{b}\right)\dot{\phi}=0$$

The above equation leads to

$$\dot{\phi}=\frac{\lambda}{ab^2} \qquad \ldots (3.2)$$

λ being constant of integration.

To get deterministic solution in terms of cosmic time t, we assume that scale factor $R \sim e^{Ht}$, that is

$$R^3 = ab^2 = e^{3Ht}, \text{ H being Hubble constant} \qquad (3.3)$$

Using equation (3.3) in (3.2), we get

$$\dot{\phi}=\frac{\lambda}{e^{3Ht}} \qquad (3.4)$$

Equation (3.3) gives

$$\frac{\dot{a}}{a}+\frac{2\dot{b}}{b}=3H \qquad \ldots (3.5)$$

From equation (3.5), we have

$$\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}+2\left(\frac{\ddot{b}}{b}-\frac{\dot{b}^2}{b^2}\right)=0 \qquad (3.6)$$

Using equation (3.5), we have

$$\frac{\dot{a}}{a}=\left[3H-\frac{2\dot{b}}{b}\right] \qquad (3.7)$$

and equation (3.6) gives

$$\frac{\ddot{a}}{a}=-\frac{2\ddot{b}}{b}+\frac{6\dot{b}^2}{b^2}+9H^2-\frac{12\dot{b}}{b}H \qquad (3.8)$$

Now using equations (2.6), (2.7) and (2.8), we get

$$\frac{\dot{b}^2}{b^2}+\frac{2\ddot{b}}{b}+\frac{1}{b^2}-\frac{3}{4}\frac{a^2}{b^4}+\frac{2a^2}{4b^4}+\frac{2\dot{a}\dot{b}}{ab}+\frac{2\ddot{b}}{b}+\frac{2\ddot{a}}{a}$$

$$-\frac{2\dot{a}\dot{b}}{ab}+\frac{a^2}{4b^4}-\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}=\frac{\dot{\phi}^2}{2}-V(\phi)+\dot{\phi}^2-2V(\phi)+\frac{\dot{\phi}^2}{2}+V(\phi) \tag{3.9}$$

Equation (3.9) leads to

$$\frac{4\ddot{b}}{b}+\frac{2\ddot{a}}{a}=2\dot{\phi}^2-2V(\phi) \tag{3.10}$$

Using equation (3.8) in equation (3.10), we get

$$\frac{12\dot{b}^2}{b^2}-\frac{24\dot{b}}{b}H-2\lambda^2e^{-6Ht}-18H^2+L=0 \tag{3.11}$$

Using equation (3.4) and taking $2V(\phi) = L$, equation (3.11) leads to

$$\frac{\dot{b}}{b}=\frac{24H\pm\sqrt{576H^2+48(2\lambda^2e^{-6Ht}-L-18H^2)}}{24} \tag{3.12}$$

For simplicity, we assume that

$$576H^2-48(L+18H^2-2\lambda^2e^{-6Ht})=0 \tag{3.13}$$

which leads to

$$e^{6Ht}=\frac{2\lambda^2}{6H^2+L}=\text{ constant} \tag{3.14}$$

Using equation (3.14) in equation (3.12), we get

$$\frac{\dot{b}}{b}=H \tag{3.15}$$

which leads to

$$b=\ell e^{Ht} \tag{3.16}$$

which gives

$$b^2=\ell^2\ e^{2Ht} \tag{3.17}$$

ℓ being constant of integration.

Again from equation (3.3), we have

$$ab^2=e^{3Ht}$$

which gives

$$a=\frac{e^{3Ht}}{b^2} \tag{3.18}$$

Using equation (3.17) in equation (3.18), we get

$$a = \frac{1}{\ell^2} e^{Ht} \tag{3.19}$$

From equations (3.17) and (3.19), we have

$$\frac{b_4}{b} = H \text{ and } \frac{a_4}{a} = H \tag{3.20}$$

Now using equations (3.17) and (3.19) in metric (2.1), we get

$$ds^2 = -dt^2 + \frac{e^{2Ht}}{\ell^4} dx^2 + \ell^2 e^{2Ht} dy^2$$
$$+ \left(\ell^2 e^{2Ht} \sin^2 y + \frac{e^{2Ht}}{\ell^4} \cos^2 y \right) dz^2 - \frac{2e^{2Ht}}{\ell^4} \cos y \, dx \, dz \tag{3.21}$$

4. Physical and Geometrical Features :

From equation (3.2), we have

$$\dot{\phi} = \frac{\lambda}{ab^2} \tag{4.1}$$

Using equation (3.3) in (4.1), we get

$$\dot{\phi} = \frac{\lambda}{e^{3Ht}} \tag{4.2}$$

which leads to

$$\phi = -\frac{\lambda}{3H} e^{-3Ht} + \mu \tag{4.3}$$

whereμ being constant of integration.
The expansion (θ) is given by

$$\theta = \frac{\dot{a}}{a} + \frac{2\dot{b}}{b}$$
$$= H + 2H = 3H \tag{4.4}$$

Shear (σ) is given by

$$\sigma = \frac{1}{\sqrt{3}} \left[\frac{\dot{a}}{a} - \frac{\dot{b}}{b} \right]$$
$$= \frac{1}{\sqrt{3}} [H - H] = 0 \tag{4.5}$$

The spatial volume (R^3) is given by

$$R^3 = ab^2 = e^{3Ht} \tag{4.6}$$

The deceleration parameter (q) for the model (3.21) is given by

$$q = -\frac{R_{44}/R}{R_4^2/R^2} = -1 \tag{4.7}$$

5. Discussion and Conclusion

The spatial volume (R^3) increases exponentially representing inflationary scenario of the universe. The Higgs field (ϕ) decreases slowly. The expansion (θ) is constant but shear (σ) is zero. Since the deceleration parameter $q = -1$. Thus the model represents de-sitter universe and also shows accelerating phase of the universe. There is no initial singularity in the model (3.21).

References

1. Robertson, H.P., *Astrophys. J. 83*, 137 (1936).
2. de-Sitter, Proc. Akad. Wetensch Amsterdam, *19*, 1217 (1917).
3. Taub, A.H., *Ann. Math. 53*, 472 (1951).
4. Chakraborty, S., *Astrophys. and Space-Science, 180,* 293 (1991).
5. Chakraborty, S. and Nandy, G.C., *Astrophys and Space-Science*, 198, 299 (1992).
6. Vaidya, P.C. and Patel, L.K., Pramana *J. Phys. 27,* 63 (1986).
7. Bali, Raj and Upadhaya, R.D., Proc. Nat. Acad. of Science, India, 73(A) II, 239 (2003).
8. Bali, R. and Yadav, M.K., Pramana – *J. Phys., 64*, 187 (2005).
9. Bali, R. and Gupta, R., Proc.Nat.Academy. Sciences, India, 78, IIIA (2008).
10. Bali, R. and Kumawat, P., *Proc. Nat. Acad. Science, 78,* IVA (2008).
11. Bali, R. and Goyal, R., *J. Ultra Scientist of Physical Sciences* – an international J. Phys. Sciences, 21(1), 181 (2009).
12. Bali, R. and Singh, S., *Proc. Nat. Acad. of Sciences,* India, Section A, *83,* 115-117 (2013).
13. Kirzhnits, D.A. and Linde, A.D., *Ann. Phys. (N.Y.),* 101, 195 (1976).
14. Kirzhnits, D.A. *JETP Lett. 15,* 529 (1977).