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Weak Isothermal Shock Wave in Heat Radiating Atmosphere

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Abstract

The motion of strong spherical shock in non-uniform atmosphere has been investigated by Chester-Chesnell-Whitham method. This method is often used to study the freely propagation of shock waves in uniform and non-uniform medium. Though the effect of overtaking disturbances plays a significant role but for the sake of simplicity, its effect on the shock has neglected. The expressions for shock velocity, shock strength, particle velocity, and pressure are obtained using Bhowmick conditions.

Key words : Isothermal propagation, Weak shock, Uniform and non-uniform density, CCWmethod.

Introduction

The study of propagation[1-3] of weak isothermal shock in uniform density media is very important due to its applications in different fields of physics. Thomas[4] discussed the propagation of weak discontinuities through uniform media. Frankel[5] has studied the problem of a sound pulse *i.e*. weak shock in rotating gas. Elliot[6] has studied self-similar solutions for spherical blast waves in air using Rosseland's diffusion approximations under the assumption of non-existance of heat flux at the centre of symmetry. Severa[l7-10,15] workers developed the similar surface theory to cover up some complicated cases of real gas. Ram and Gaur[16] discussed the propagation of sonic discontinuities in a uniform medium of dissociating and thermally conducting fluids. Elcrat[11] studied the non-uniform propagation of sonic discontinuities in an unsteady flow of perfect gas. Ray and Benerjee[13] have obtained similarity solutions for a strong wave in a transparent grey gas of uniform density at very locations. Singh and Shrivastava[12,14] have adopted the self-similar model of a weak shock wave with varying energy. Vishwakarma[18] have studied the problem of non-uniform propagation of weak waves through thermally conducting and dissociation gases using similarity method. Propagation of converging and diverging shock waves under isothermal condition is discussed by Levin and Zhuravskaya[17]. Yadav and Rathore[19] studied the effect of variable magnetic field on strong cylindrical shock in rotating gas. Isothermally shock propagation in uniform medium having radiative heat flux is discussed by Gangwar[20]. Vishwakarma and Arvind[21] obtained self-similar solutions for the shock propagation in a non-uniform gaseous atmosphere. Nath[22] evaluated the propagation of a strong cylindrical shock wave in a rotational axisymmetric dusty gas with exponentially varying density by using the non-similarity method. Singh *et al.*[23] have investigated the problem of entropy change of non-uniform medium due to isothermal propagation of strong spherical shock waves. Vishwakarma and Nath[24] studied the magnetogas dynamic shock waves in rotating gas with exponentially varying density. The problem self-similar cylindrical ionizing shock waves in a rotational axisymmetric non-ideal gas with radiation heat flux is investigated by Singh and Nath[25]. Vishwakarma and Patel[26] have discussed magneto gas dynamic cylindrical shock waves in rotation non-ideal gas with radiation heat flux. Flow behind an exponential shock wave in a rotational axisymmetricperfect gas with magnetic field and variable density is discussed by Sahu and Nath[27].

The aim of the present paper is to study the propagation of weak cylindrical shock waves for two cases (i) when the medium has constant density and (ii) when medium has density distribution $\rho_0 \propto _r{}^{\omega}$ where ω is density parameter.

2. Equations of Motion and Boundary Conditions:

The basic equation of motion in cylindrical symmetry are

$$\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \qquad \text{(i)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \qquad \text{(ii)}$$

$$\frac{\partial T}{\partial r} = 0 \qquad \text{(iii)}$$

The weak shock conditions are given by :

$$\left.\begin{aligned} u &= (1-\beta)(1+\varepsilon)a_0 \\ P &= (1-\beta)a_0^2\rho_0(1+2\varepsilon) \end{aligned}\right\} \qquad \text{(iv)}$$

3. Solution :

The characteristic equation is –

$$dp + \rho a du + \rho a^2 \frac{u}{u+a} \quad \frac{dr}{r} = 0$$

(A) When the medium has constant density :

from equation (iv)

$$du = (1-\beta)a_0 d\varepsilon \qquad \text{(vi)}$$

$$dp = (1-b)2r_0 a_0^2 de \qquad \text{(vii)}$$

putting these values in equation (v) and solving we get

$$\left[-(2\beta+1)[1-\beta] + \frac{1}{(1+\varepsilon)}\right]d\varepsilon = \frac{dr}{r}$$

taking integration and solving we get

$$\varepsilon = \frac{1-kr}{[(2\beta+1)[(1-\beta)+kr]}$$

this equation is used to compute the shock velocity U, shock strength U/a_0, particle velocity u and pressure P/P_0.

(B) When medium has density distribution $\rho_0 \propto _r{}^{\omega}$ where ω is density parameter :

$$du = (1-\beta)a_0 d\varepsilon$$

$$dP = (1-\beta)2\rho' a_0^2 [2d\varepsilon r^{\omega} + (1+2\varepsilon)\,\omega r^{\omega-1}dr]$$

Putting these values in equation (v) and solving we get

$$\frac{(2\beta+1)}{\beta^2\omega^2}\left[\frac{\beta\omega}{(1+2\varepsilon)} - \frac{1}{(1+4\varepsilon)} - \frac{\varepsilon}{(1+4\varepsilon)} + \frac{(1-\beta)}{(1+4\varepsilon)} + \frac{2\varepsilon(1-\beta)}{(1+4\varepsilon)}\right]d\varepsilon = -\frac{dr}{r}$$

taking integration and simplifying we get the following expression

$$\varepsilon = \frac{4 - 4\left[\frac{r}{k}\right]^{\frac{\beta^2\omega^2}{(2\beta+1)}} + \frac{(2\beta-1)}{4}}{\left[\{4\beta\omega - (1+2\beta)\}\left(\frac{r}{k}\right)^{\frac{\beta^2\omega^2}{(2\beta+1)}} - (2\beta-1)\right]}$$

from this expression, the expression for shock velocity U, shock strength $\frac{U}{a_0}$, particle velocity u and pressure $\frac{P}{P_0}$ are computed.

4. Results and Discussion

Results are obtained in the following forms:

$$U = (1+\varepsilon)a_0, \frac{U}{a_0} = (1+\varepsilon)$$

$$u = (1+\varepsilon)(1-\beta)a_0$$

$$\frac{P}{P_0} = (1-\beta)(1+2\varepsilon)\gamma$$

(A) When the medium has constant density :

The nature of flow variables are illustrated in table 1, 2. In the table 1 the variation of shock velocity U, shock strength U/a_0, particle velocity u and pressure P/P_0 are obtained with r at constant β. It is observed that as the distance from the shock source increases shock velocity U, shock strength U/a_0, particle velocity u and pressure P/P_0 decreases.

In the table 2 the variation of shock velocity U, shock strength U/a_0, particle velocity u and pressure P/P_0 with temperature parameter β at constant propagation distance r. As β increases shock velocity U, shock strength U/a_0, particle velocity u and pressure P/P_0 decreases.

(B) When medium has density distribution $\rho_0 \propto r^{\omega}$ where ω is density parameter:

The nature of flow variables are illustrated through tables 3, 4 and 5.

In the table 3 the variation of shock velocity U, shock strength U/a_0, particle velocity u and pressure P/P_0 with temperature parameter β at constant r and ω. It is found that as β increases shock velocity U, shock strength, U/a_0, particle velocity u and pressure P/P_0 decreases.

In the table 4 the variation of shock velocity U, shock strength U/a_0, particle velocity u and pressure P/P_0 with propagation distance r at constant β and ω. As r increases shock velocity U, shock strength U/a_0, particle velocity u and pressure P/P_0 decreases.

In the table 5 the variation of shock velocity U, shock strength the U/a_0, particle velocity u and pressure P/P_0 with density parameter w at constant r & β. It is observed that as ω increases shock velocity U, shock strength U/a_0, particle velocity u, and pressure P/P_0 decreases.

Table 1. Variation of shock velocity shock strength, particle velocity and pressure with propagation distance r (Initially taking β=0.1, r=11, U=517.4271, a_0 = 330)

r	U	U/a_0	u	P/P_0
11	517.4271	1.5679	465.6844	3.6949
12	509.8249	1.5444	458.8424	3.5871
13	502.9338	1.5240	452.6404	3.4908
14	496.6396	1.5049	446.9756	3.4040
15	490.8527	1.4874	441.7674	3.3251
16	485.5022	1.4712	436.9519	3.2530
17	480.5306	1.4561	432.477	3.1867
18	475.8909	1.4420	428.3018	3.1255

Table 2. Variation of shock velocity shock strength, particle velocity and pressure with temperature parameter β (Initially taking β=0.2, U=523.7876, r=11, a_0 = 330)

β	U	U/a_0	u	P/P_0
0.2	523.7876	1.5872	471.4089	3.3657
0.3	522.0392	1.5819	469.8353	2.9252
0.4	519.8865	1.5754	467.8979	2.4868
0.5	517.4204	1.5679	465.6783	2.0526
0.6	514.7059	1.5597	463.2353	1.6250
0.7	511.7911	1.5508	460.6120	1.2048
0.8	508.7116	1.5415	457.8404	0.7937

Table 3. Variation of shock velocity shock strength, particle velocity and pressure with temperature parameter β (Initially taking β=0.2, ω=1, U=523.7967, r=11, a_0 = 330)

β	U	U/a_0	u	P/P_0
0.2	523.7967	1.5872	471.4170	3.3658
0.3	522.0571	1.5819	469.8514	2.9254
0.4	519.9134	1.5754	467.9220	2.4871
0.5	517.4566	1.5680	465.7189	2.0529
0.6	514.7524	1.5598	463.2772	1.6253
0.7	511.8498	1.5510	460.6648	1.2051
0.8	508.7860	1.5417	457.9074	0.7939
0.9	505.5903	1.5320	455.0312	0.3918

Table 4. Variation of shock velocity shock strength, particle velocity and pressure with propagation distance r (Initially taking r=11,β=0.1, ω=1, U=517.5062, a_0=330)

r	U	U/a_0	u	P/P_0
11	517.5062	1.5682	465.7555	3.6960
12	509.9832	1.5454	458.9849	3.5894
13	503.1642	1.5247	452.8477	3.4940
14	496.9356	1.5058	447.2420	3.4080
15	491.2092	1.4885	442.0882	3.3299
16	485.9145	1.4727	437.3230	3.2585
17	480.9948	1.4575	432.8953	3.1918
18	476.4035	1.4436	428.7631	3.1322
19	472.4035	1.4315	425.1631	3.0799
20	468.0585	1.4183	421.2527	3.0235

Table 5. Variation of shock velocity shock strength, particle velocity and pressure with density parameter ω (Initially taking r=11, β=0.1, U=517.3481, a_0=330)

ω	U	U/a_0	u	P/P_0
2	517.3481	1.5677	465.6132	3.6938
3	509.6664	1.5444	458.6998	3.5849
4	502.7035	1.5233	452.4331	3.4876
5	496.3435	1.5040	446.7092	3.3999
6	490.4962	1.4863	441.4466	3.3203
7	485.0899	1.4699	436.5809	3.2475
8	480.0664	1.4547	432.0597	3.1806
9	475.3783	1.4405	427.8404	3.1188
10	470.9824	1.4272	423.8841	3.0613
11	466.8572	1.4147	420.1715	3.0080

4. Conclusion

In the present paper, flow variables behind the weak isothermal shock wave in heat radiating constant and variable density atmosphere are obtained, neglecting the effect of overtaking disturbances. It is concluded that shock velocity, shock strength, particle velocity and pressure decreases as shock advances isothermally in heat radiating atmosphere. Results may be changed if the effect of overtaking disturbances (EOD) may be taken into account.

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