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MHD Flow of a Couple Stress fluid through A Porous Medium with Periodic Body Acceleration

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Abstract

In this paper, we have considered unsteady hydro dynamic poiseuille flow of an incompressible electrically conducting couple stress fluid through a porous medium between parallel plates, taking into account pulsation of the pressure gradient effect and periodic body acceleration with phase difference. The solution of the problem is obtained with the help of perturbation technique. Systematic expression is given for the velocity field, and the effects of the various governing parameters entering into the problem are discussed with the help of graphs. The shear stresses on the boundaries and the discharge between the plates are also obtained analytically and their behaviour computationally discussed with different variations in the governing parameters in detail.

Key words: Unsteady flows, rotating channels, couple stress fluids, periodic body acceleration and Brinkman's model.

1. Introduction

Many attempts have been made by several authors to describe blood as a simple model but failed to reach their attempts. In further investigation many authors have used one of the simplification is that they have assumed blood to be a suspension of spherical rigid particles (red cells), this suspension of spherical rigid particles will give rise to couple stresses in a fluid. Stokes[22] introduced the theory of couple stress fluid, which is a special case of micro-polar fluid. Valanis and sun[25] have proposed a mathematical model for blood flow by assuming blood as a couple stress fluid. It seems that their work contained some serious errors that have been corrected by Chaturani[2]. Further, Chaturani[3] has proposed a method to determine couple stresses parameters with the help of relative viscosity and velocity profiles. Chaturani and Upadhya[5] Investigated the pulsatile flow of couple stress fluid by using perturbation method. They have obtained the expressions for flow velocity, wall shear, flow rate and relative viscosity. They have suggested two methods for the determination of the value of pulsatile Reynolds's number. A critical study of poiseuille flow of couple stress fluid with application to blood flow has been carried out by Chaturani and Rathod[4]. The important conclusion of their analysis is a method (geometrical) that has been developed for studied a theoretical model for pulsatile flow of blood with varying cross sectional tube and its applications to cardiovascular diseases. It is observed that an increase in finding the precise value of non dimensional couple stress parameter. Rathod[16] studied a theoretical model for pulsatile flow of blood with carrying cross sectional tube and its applications to cardiovascular disease. It is observed that an increase in leads to an increase as the concentration decreases (*i.e.*, as increases). A simple mathematical model depicting blood flow through permeable tube by assuming blood as couple stress fluid has been studied by Pal *et al.*[12]. Sagayamary and Devanathan[17] have studied two dimensional flow of couple stress fluid through a rigid tube of varying cross section for low Reynolds numbers. Gopalan[7] discussed pulsatile blood flow in the lung alveolar sheets by idealized each of them as a channel covered by porous media.

To acquire a more complete understanding of the flow in small vessels, it is important, however, to consider flow not only in straight tubes, but also in other geometries. In cardiovascular system most of the blood vessels are not having uniform cross section[8]. Padmanabha[11] analyzed pulsatile flow of viscous fluid through a curved elastic tube. Batra and Jena[1] have studied the steady, laminar flow of a Casson fluid in a curved tube of circular cross section. Smith[21] has studied on flow through bends and branching. Schneck[18] has obtained an approximate analytical solution for a pulsatile flow through a diverging channel. Using perturbation method, Rao and Devanathan[14] have analyzed pulsatile flow of blood through varying cross sectional tube. Ramachandra Rao[15] has investigated oscillatory flow through an elastic tube with varying cross section. The pulsatile blood flow in an eccentric catheterized artery has been studied numerically by Prabir and Ranjan[13] by considering blood as Newtonian fluid. The axial pressure gradient and velocity distribution in the eccentric catheterized artery has been obtained as solutions of the problem. The flow of non Newtonian fluids in converging or diverging axi symmetric tubes has been investigated by Sutterby[23, 24]. A series solution for both converging and diverging axi-symmetric flow of an incompressible Newtonian fluid was developed by Forester and young[6]. Schneck and Ostrich[19] studied the pulsatile flow of blood in a channel of small expontical divergence. Schneck and Walburn[20] have investigated the pulsatile flow of blood with low Reynolds number assuming blood as a Newtonian fluid, through a channel of diverging cross section. They have observed a phase-lag between flow rate and pressure gradient. The steady flow of an incompressible micro polar fluid in a diverging channel has been studied by Kamel[9]. Misra and Ghosh[10] used a micro continuum approach to determine the velocity and pressure distributions in an exponentially diverging channel. Rathod[16] studied the pulsatile flow of couple stress fluid through slowly diverging tubes and its applications to cardiovascular diseases. It is observed that the point of inflection and back flow is observed in the axial velocity profiles for higher values of phase angles between flow rate and pressure gradient decreases as the concentration increases, *i.e.* $\overline{\alpha}$ decreases. The above-discussed models are related to steady and pulsatile flow of Newtonian and non-Newtonian fluids through curved, converging/diverging and exponentially diverging tubes. In this paper, we discuss an analytical study of unsteady hydro dynamic poiseuille flow of an incompressible electrically conducting couple stress fluid through a porous medium between parallel plates, taking into account pulsation of the pressure gradient effect and under the influence of periodic body acceleration with phase difference ϕ.

2. Formulation and Solution of the problem:

We consider the unsteady hydro flow of a couple stress fluid through a porous medium induced by the pulsation of the pressure gradient. The plates are assumed to be electrically insulated. The flow takes place under the influence of periodic body acceleration with phase difference ϕ.

We choose Cartesian co-ordinate system O(x, y, z) such that the boundary walls $y = 0$ and $y = h$. In such a way that the xz-plane is taken on the lower plate and this y-axis is normal to the plates. The induced magnetic field is assumed to be negligible and also the flow in the porous medium is assumed to be fully developed. The periodic body acceleration is assumed to be $G = g_0 \cos\phi$ where, g_0 is the amplitude of the body acceleration and ϕ is its phase difference. Under these assumptions the unsteady equations governing the couple stress fluid flow in the absence of body force f and body moment I are

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\eta}{\rho}\frac{\partial^4 u}{\partial y^4} - \frac{\mu}{k\rho}u + g_0 \cos\phi \qquad (2.1)$$

Where, u (y, t) is the velocity, the term $-\frac{\eta}{\rho}\frac{\partial^4 u}{\partial z^4}$ in the above equation gives the effect of couple stresses. All the physical quantities in the above equation have their usual meaning. The boundary conditions are

$$u = 0 \qquad at \qquad y = 0 \qquad (2.2)$$

$$u(1) = 0 \qquad at \qquad y = h \qquad (23)$$

Since the couple stresses vanish at both the plates which in turn, implies that

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad at \qquad y = 0 \tag{2.4}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad at \qquad y = h \tag{2.5}$$

Introducing non-dimensional variables are

$$x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}, \quad u^* = \frac{u\,h}{\nu},\ t^* = \frac{t\nu}{h^2}, \omega^* = \frac{\omega h^2}{\nu}, \quad p^* = \frac{p\,h^2}{\rho\upsilon^2}$$

Using the non-dimensional variables (dropping asterisks), The governing equation (2.1) reduces to

$$a^2\frac{\partial u}{\partial t} + \frac{\partial^4 u}{\partial y^4} - a^2\frac{\partial^2 u}{\partial y^2} + D^{-1}a^2\,u = -a^2\frac{\partial p}{\partial x} + a^2 G\cos\phi \tag{2.6}$$

Where

$a^2 = \dfrac{h^2\mu}{\eta}$ is the couple stress parameter

$D^{-1} = \dfrac{h^2}{k}$ is the inverse Darcy parameter

$G = \dfrac{g_0 h^3}{\upsilon^2}$ is the body acceleration parameter

Corresponding the non-dimensional boundary conditions are given by

$$u = 0 \qquad at \qquad y = 0 \tag{2.7}$$

$$u = 0 \qquad at \qquad y = 1 \tag{2.8}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad at \qquad y = 0 \tag{2.9}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad at \qquad y = 1 \tag{2.10}$$

For the pulsation pressure gradient

$$-\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_o e^{i\omega t} \tag{2.11}$$

The equation (2.6) reduces to the form

$$a^2\frac{\partial u}{\partial t} + \frac{\partial^4 u}{\partial y^4} - a^2\frac{\partial^2 u}{\partial y^2} + D^{-1}a^2\,u = -a^2\left\{\left(\frac{\partial p}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_o e^{i\omega t}\right\} + a^2 G\cos\phi \tag{2.12}$$

The equation (2.12) can be solved by using the following perturbation technique

$$u = u_s + u_o\,e^{i\omega t} \tag{2.13}$$

Substituting the equation (2.13) in (2.12) and equating like terms on both sides

$$\frac{d^4 u_s}{dy^4} - a^2\frac{d^2 u_s}{dy^2} + D^{-1}a^2\,u_s = -a^2\left(\frac{\partial p}{\partial x}\right)_s + a^2 G\cos\phi \tag{2.14}$$

$$\&\ \frac{d^4 u_o}{dy^4} - a^2\frac{d^2 u_o}{dy^2} + (D^{-1} + i\omega)a^2\,u_o = -a^2\left(\frac{\partial p}{\partial x}\right)_o \tag{2.15}$$

Subjected to the boundary conditions

$$u_s = 0 \qquad at \qquad y = 0 \tag{2.16}$$

$$u_s = 0 \qquad at \qquad y = 1 \tag{2.17}$$

$$\frac{\partial^2 u_s}{\partial y^2} = 0 \qquad at \qquad y = 0 \tag{2.18}$$

$$\frac{\partial^2 u_s}{\partial y^2} = 0 \qquad at \qquad y = 1 \tag{2.19}$$

$$u_o = 0 \qquad at \qquad y = 0 \tag{2.20}$$

$$u_o = 0 \qquad at \qquad y = 1 \tag{2.21}$$

$$\frac{\partial^2 u_o}{\partial y^2} = 0 \qquad at \qquad y = 0 \tag{2.22}$$

$$\frac{\partial^2 u_o}{\partial y^2} = 0 \qquad at \qquad y = 1 \tag{2.23}$$

Let

$$\left(\frac{\partial p}{\partial x}\right)_s = p_s \quad and \quad \left(\frac{\partial p}{\partial x}\right)_o = p_o$$

The solutions of the equations (2.14) and (2.15) subjected to the boundary conditions (2.16) to (2.23) give the velocity distribution of the fluid under consideration.

$$u = C_1 e^{m_1 y} + C_2 e^{m_2 y} + C_3 e^{-m_1 y} + C_4 e^{-m_2 y} + \frac{p_s + G\cos\phi}{D^{-1}} + \left(C_5 e^{m_5 y} + C_6 e^{m_6 y} + C_7 e^{-m_5 y} + C_8 e^{-m_6 y} + \frac{p_o}{D^{-1} + i\omega}\right)e^{i\omega t} \tag{2.24}$$

Where, the constants $C_1, C_2, \ldots\ldots\ldots C_8$ are given in appendix.

$$m_1 = \sqrt{\frac{a^2 + \sqrt{a^4 - 4a^2 D^{-1}}}{2}}, \; m_2 = \sqrt{\frac{a^2 - \sqrt{a^4 - 4a^2 D^{-1}}}{2}}$$

$$m_5 = \sqrt{\frac{a^2 + \sqrt{a^4 - 4a^2 (D^{-1} + i\omega)}}{2}}, \; m_6 = \sqrt{\frac{a^2 - \sqrt{a^4 - 4a^2 (D^{-1} + i\omega)}}{2}}$$

The shear stresses on the upper and lower plates are given in dimension less form as

$$\tau_U = \left(\frac{du}{dy}\right)_{y=1} \quad \& \quad \tau_L = \left(\frac{du}{dy}\right)_{y=0}$$

The non-dimensional discharge between the plates per unit depth is given by Q

$$Q = \int_0^1 u(y,t)\, dy$$

3. Results and Discussion

The motivation of this study is to investigate the effect of pulsation of pressure gradient as well as the influence of periodic body acceleration on the flow. The velocity field in the porous medium has been computationally evaluated for different variations in the governing parameters a, the couple stress parameters, D^{-1} the inverse Darcy Parameter and G the body acceleration parameter. From the linear momentum equations, we may note that if the magnitude of the body acceleration dominates over the axial pressure gradient then the velocity u is positive and the flow takes place from left to right. In case of the magnitude of pressure gradient is more then the body acceleration, then u is negative and the flow takes place from right to left. In general the magnitude of velocity µ increases from zero the state of rest on the lower boundary (y=0) to a maximum in the upper half region and later gradually reduces to rest on the upper boundary (y=1). The flow governing the non-dimensional parameters namely viz. a couple stress parameter, D^{-1} the inverse Darcy parameter, G body acceleration parameter, P_o the amplitude of pulsation pressure gradient. The fig (1-2) represent the velocity profiles for the pulsation pressure gradient dominates the body acceleration parameter and which corresponds to $\omega = \frac{\pi}{2}$ with variations in the governing parameters while fixing the other parameters and the figures (3-4) represents the reverse case with flow taking place from right to left. Fig. (1 and 3) illustrates the magnitude of the velocity u enhances with increasing the couple stress parameters "a" and fixing the other parameters. From figures (2 and 4), it is evident that the magnitude of the velocity u decreases with increasing the inverse Darcy parameter D^{-1}. Hence lesser the permeability of the porous medium greater the retardation experienced by the flow in the entire flow field. The velocity profile (5) exhibits how the velocity u influenced with the body acceleration parameter G. We may observe that the negative pressure gradient in the momentum equation balances the body acceleration term and hence in the absence of any other extraneous forces the fluid is at rest, since the channel walls are at rest. However, when the body acceleration dominates the pulsation pressure gradient, the magnitude of the velocity component u enhances with increase in G in the entire flow field. Likewise it is interesting to note that when the pulsation pressure gradient dominates the body acceleration, an increase in G the magnitude of the velocity u reduces in the entire flow field. The Fig. (6) illustrates the magnitude of the velocity u enhances with increase in the amplitude of pulsation of pressure gradient.

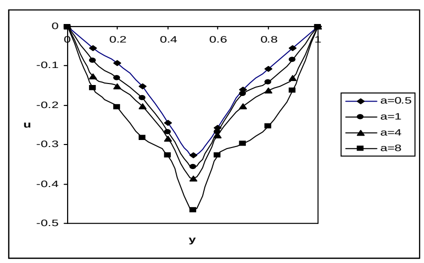

Fig. 1. The velocity profile for u with *a* G=1, D^{-1} =2000, P_o=P_s =10

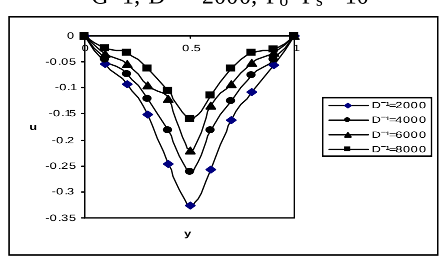

Fig. 2. The velocity profile for u with D^{-1} *a*=0.5, G=1, P_o=P_s =10.

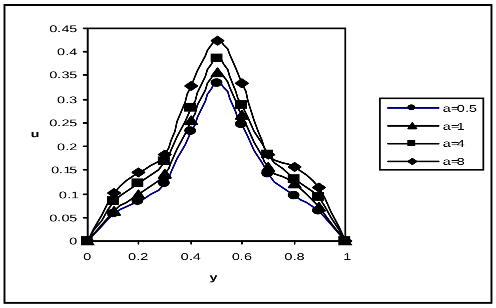

Fig. 3. The velocity profile for u with *a*
G=1, D^{-1} =2000, P_o=P_s =10

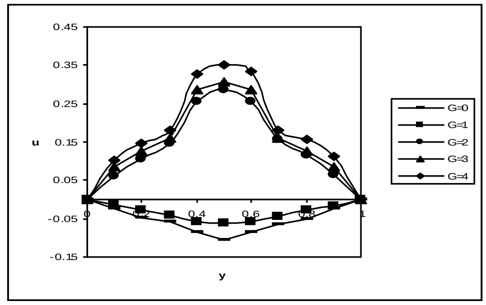

Fig. 5. The velocity profile for u with G
a=0.5, D^{-1} = 2000, P_o = P_s = 10,

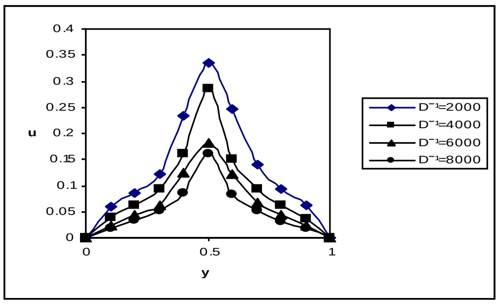

Fig. 4. The velocity profile for u with D^{-1}
G*a* = 0.5, G = 1, P_o = P_s =10

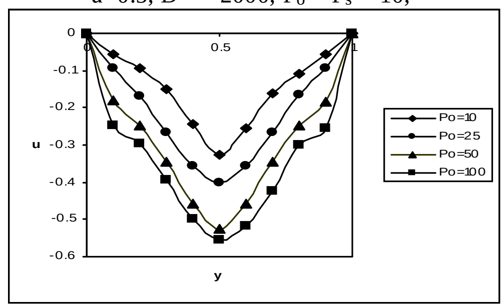

Fig. 6. The velocity profile for u with P_o.
a = 0.5, G = 1, D^{-1} = 2000, P_s = 10,

a	I	II	III	IV	V	VI	VII
0.5	0.052453	0.045215	0.038485	0.082458	0.121526	0.021152	0.010534
1	0.032145	0.030052	0.021566	0.056858	0.095652	0.015315	0.006536
4	0.015154	0.005266	0.002562	0.032565	0.068234	0.008245	0.004266

	I	II	III	IV	V	VI	VII
D^{-1}	2000	3000	4000	2000	2000	2000	2000
G	1	1	1	2	3	1	1
PO	10	10	10	10	10	25	50

Table I. The shear stresses on the upper plate.

P_s =10, t =1, $\omega = \frac{\pi}{2}$, $\phi = 60^0$

a	I	II	III	IV	V	VI	VII
0.5	-0.0082	-0.0075	-0.0068	-0.0153	-0.0453	-0.0005	-0.0004
1	-0.0065	-0.0053	-0.0043	-0.0105	-0.0163	-0.0002	-0.0001
4	-0.0032	-0.0015	-0.0005	-0.0052	-0.0831	-0.0001	-0.0001

I	II	III	IV	V	VI	VII	
D^{-1}	2000	3000	4000	2000	2000	2000	2000
G	1	1	1	2	3	1	1
PO	10	10	10	10	10	25	50

Table II: The shear stresses on the lower plate.

P_s = 10, t = 1, $\omega = \frac{\pi}{2}$, $\phi = 60^0$

a	I	II	III	IV	V	VI	VII
0.5	0.065245	0.034545	0.030834	0.052653	0.048343	0.153456	0.183453
1	0.095345	0.056536	0.048745	0.083759	0.065265	0.256534	0.534568
4	0.126546	0.084534	0.082463	0.096921	0.085665	0.356678	0.811579

	I	II	III	IV	V	VI	VII
D^{-1}	2000	3000	4000	2000	2000	2000	2000
G	1	1	1	2	3	1	1
Po	10	10	10	10	10	25	50

Table III. The Discharge Q

$P_s = 10$, t =1, $\omega = \frac{\pi}{2}$, $\phi = 60^0$

The shear stresses have been evaluated on the boundaries and tabulated in the tables I and II. The magnitude of the stresses on either plate enhances with increase in body acceleration parameter G, and it reduces with increase in the amplitude of pulsation pressure gradient and the inverse Darcy parameter D^{-1} fixing the other parameters. Thus lesser the permeability lower the stresses on the boundaries, also the magnitude of the stresses on the lower boundary is far lesser than the corresponding magnitudes on the upper boundary. We observe that the stresses reduces on the upper boundary while enhances on the lower boundary with increase in the couple stress parameter ‘a’. The discharge Q between the plates enhances with increase in the couple stress parameter ‘a’ and amplitude of pulsation pressure gradient P_o, and reducer with increase inverse Darcy parameter D^{-1} and body acceleration parameter G (table. III).

4. Conclusions

we have considered unsteady hydro dynamic poiseuille flow of an incompressible electrically conducting couple stress fluid through a porous medium between parallel plates, taking into account pulsation of the pressure gradient effect and periodic body acceleration

1. The magnitude of the velocity enhances with increase in the couple stress parameter ‘a’ and the amplitude of pulsation pressure gradient.
2. The magnitude of the velocity reduces with increase in the inverse Darcy parameter D^{-1}.
3. When the body acceleration dominates the pulsation pressure gradient, the magnitude of the velocity u enhances with increase in the body acceleration parameter G, while pulsation pressure gradient dominates body acceleration the magnitude of the velocity reduces with increase in G.
4. The magnitude of the stresses on either plate enhances with increase in body acceleration parameter G, and it reduces with increase in the amplitude pulsation pressure gradient and the inverse Darcy parameter. The stress reduces on the upper boundary and enhances on the lower boundary with increase in the couple stress parameter ‘a’.
5. The magnitude of the stresses on the lower boundary lesser than the corresponding values of the upper boundary.
6. The discharge Q between the plates enhances with increase in the couple stress parameter a and amplitude of pulsation pressure gradient P_o, and reduces with increase in the inverse Darcy parameter D^{-1} and the body acceleration parameter G.

References

1. Batra, R. L. and Bigyani Jena. “Flow of a Casson fluid in a slightly curved tube”, *Int. J. Engg. Sci.*, Vol. *29*, Issue. *10*, PP. 1245-1258 (1991).
2. Chaturani, P. “Some comments on Piseuille flow of a fluid with couple-stress, applications to blood flow”, *Biorheology*, Vol. *6*, PP. 85-97 (1979).
3. Chaturani, P “Viscosity of Poiseuille flow of a couple stress fluid with application to blood flow”, *Biorheology*, Vol. *15*, PP. 119-128 (1978).
4. Chaturani, P. and Rathod, V.P. “A critical study of Poiseuille flow of couple stress with applications to blood flow”, *Biorheology*, Vol. *18*, PP. 235-244 (1981).
5. Chaturani, P. and Upadhya, V.S. “Pulsatile flow of a couple stress fluid though circular tuves with applications to blood flow”, *Biorheology*, Vol. *18*, 235-244 (1981).
6. Foresster, J. H. and Young, D.F “Flow through a converging-diverging tube and its implications in occlusive vascular disease”. *Int. J. Bio-Mechanics*, Vol. *3*, P. 297 (1970).
7. Gopalan. N.P., *Bulletin of mathematical Biology*, Vol. 45, Issue. No. *5*, PP. 563-577(1981).
8. Jefford, J.V. and Knisely, M.H. “Concerning the geometric shape of arteries and arteriols”. *Angelology*, Vol. *7*, P. 105 (1956).

9. Kamel, M.T. “Flow of micro polar fluid in a diverging channel”, *Int. J. Engg. Sci*. Vol. 25, Issue. 6, PP. 759-768 (1987).
10. Misra, J. C. and Ghosh, S.K. “A mathematical model for the study of blood flow through a channel with permeable walls”. *ActaMechanica,* Vol. *122,* PP. 137-153 (1997).
11. Padmanabhan, N. “Pulsatile flow of a viscous fluid of a viscous fluid through curved elastic pipes”, *Proc. Ind. Nat. Sci. Acad A. Part A*. Vol. *53,* P. 208 (1987).
12. Pal L., Rudraiah, N. and Devanathan R. “A couple stress model of blood flow in the Microcirculation”, *Bull. Math.Biol*. Vol. *50,* P. 329 (1988).
13. Prabir Daripa and Ranjan, K.D. “A numerical study of pulsatile blood flow in an eccentric catheterized artery using a fast algorithm”. *Journal of Engineering Mathematics*, Vol. *42,* PP. 1-22 (2002).
14. Ram Chandra Rao and Devanathan, R. “Pulsatile flow in tuves of varying cross sections”. *ZAMP,* Vol. *24,* PP. 203-213 (1973).
15. Ram Chandra Rao, A. “Oscillatory flow in elastic tube of carrying cross section”, *Acta Mech.,* Vol. *46,* P. 155 (1983).
16. Rathod, V.P. “Mathematical modeling of blood flow through tubes with various cross sections and its applications, Ph.D. Thesis, IIT, Bombay(1984).
17. Sagayamary, R.V. and Devanathan, R. “Steady flow of couple stress fluid through tubes of slowly varying cross sections, Applications to blood flow”, *Biorheology,* Vol. *26,* PP. 753-769 (1956).
18. Schneck, D.J. “Pulsatile blood flow in a diverging circular channel”, Ph.D. Thesis, *Case Western Reserve University, Cleveland, Ohio, USA* (1973).
19. Schneck, D.J. and Walburn, F.J. “Pulsatile blood flow in a channel of small exponential divergence-I. The linear approximations for low mean Reynolds number”. *J. Fluids. Engg.* Vol. *97,* PP. 353-360 (1975).
20. Schneck, D.J. and Walburn, F.J. “Pulsatile blood flow in a channel of small exponential divergence Part-II: Steady streaming due to the interaction of viscous effects with convected inertia”. *J. Fluids Engg.,* Vol. *98,* PP. 707-714 (1976).
21. Smith, F. T. “On flow trough bends and branchings”. *Biorheology*, Vol. *39,* PP. 373-378 (2002).
22. Stokes, V.K. “Couple stresses in fluid”, *The Physics of Fluids*, Vol. *9,* 1709-1715 (1966).
23. Sutterby, J. L. “Laminar converging flow of dilute polymer solutions in conical section-Part-I, Viscosity data, New viscosity model, tube solutions”, *AICHE J.* Vol. *12,* P. 63 (1966).
24. Sutterby, J.L. “Laminar converging flow of dilute polymer solutions in conical sections-II”, *Trans, Soc. Rheo*. Vol. *9,* P. 227 (1965).
25. Valanis, K.C. and Sun, C.T., “Poiseulle flow of fluid with couple stress with applications to blood flow”, *Biorheology*, Vol. *6,* PP. 85-97 (1979).