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On b-Chromatic Number for Certain Triangular Line graphs

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Abstract

The b-Chromatic number of a graph is the maximum number of k of colors that can be used to color the vertices of G . Let us consider the color classes for different line graphs, where every line graph with i has a representative (the color i dominating vertex) that is able to communicate with all the other line graphs. Further we investigate the b-chromatic number for certain triangular graphs such as triangular snake graphs, triangular fan graphs, triangular cylindrical graphs and we also generalized some of the results based on these graphs.

Key words : b-chromatic, triangular line graph, cycles, coloring,

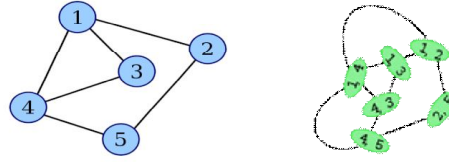
Introduction

A b-coloring by k colors is a proper coloring of the vertices of G such that in each color classes there exists a vertex has neighbors in all the other $k-1$ color classes. In other words each color class contains a vertex which has at least one neighbor in all the other color classes. Such vertex is called color dominating vertex. The b-chromatic number $\phi(G)$ is the largest integer k such that G admits a b-coloring with k -colors. The basic concept of b-chromatic number was introduced Irving and Manlove by considering proper coloring that are minimal with respect to a partial order defined on the set of all partition of $V(G)$. Naturally, a proper coloring of G with $\chi(G)$ colors is a b-coloring of G since it cannot be improved so that $\chi(G) \leq b(G)$. The NP completeness results have incited researchers to establish bounds on the b-chromatic number in general or to find its exact values for classes of graphs.

The b-chromatic number of certain product of some families of graphs is examined by Balakrishnan *et al.* and b-coloring for square of Cartesian product of two cycles is discovered by Chandra Kumar and Nicholas. Some results on b-coloring and b-chromatic are introduced by Alkhteeb. The b-chromatic of some cycle related graph has examined by Vaidhya and Shukla and also they established the b-chromatic number of certain wheel graphs.

Definition 1.1 : A line graph $L(G)$ of a simple graph G is obtained by associating a vertex with each edge of the graph and the connecting two vertices with an edge iff the corresponding edges of G have a common vertex.

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**Graph G****Line Graph G**

Definition 1.2 : The triangle graph, $T(G)$ of a graph G has as its vertices the triangles of G and two vertices of $T(G)$ are adjacent if the corresponding triangles in G have a common edge.

Definition 1.3 : A triangular snake is a triangular cactus whose block cut point graph path $P = a_1 a_2 \dots a_{n+1}$ by joining a_i and a_{i+1} to a new vertex b_i for $i = 1, 2, 3, \dots, n$.

We denote this snake graph as by $S(n)$.

Line graphs of triangular snakes :

The line graph $L(G)$ of an undirected graph G is another graph $L(G)$ that represents the adjacencies between edges of G . In other words, given graph G , its line graph $L(G)$ is a graph such that (1) each vertex of $L(G)$ represents an edge of G and (2). Two vertices of $L(G)$ are adjacent iff their corresponding edges adjacent in G .

Figure 1. A graph G and its line graph L

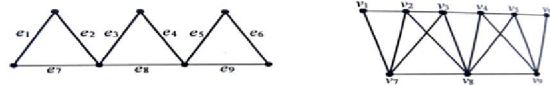
The name linegraph comes from a paper Harary *et. al*⁵ although Whitney¹⁴ used the construction before Hemminger *et. al*⁶. This concept has been rediscovered in various contexts and thus many other names, adjointgraph, derived graph and edge-to-vertex dual are few. Moon⁹ studied onn the line graph as a complete bigraph. Beineke³ proposed anew characterizations of such graphs.

Definition: The block cut point of a graph G is a bipartite graph H in which one partite set consists of the cut vertices of G an the other has a vertex b_i For each block B_i of G .

Structural properties of a Triangular snake graph :

1. The number of vertices in $L(T(n))=3n$.
2. The number of edges inn $L[T(n)]=7n-4$.
3. Maximum degree =6.
4. Minimum degree =2.

The following fig shows a triangular snake graph and its line graph

Figure 4: The graph $T(3)$ and its line graph $L[T(3)]$

Theorem 1 : Let $L[S(n)]$ be a line graph of a triangular snake graph $S(n)$, $n \geq 9$. Then $b(L[S(n)]) = \delta(L[s(n)]) + 1$.

Proof : Let us consider that V_i be the set of vertices of $S(n)$. we also assign the colors as $v_i = l$.

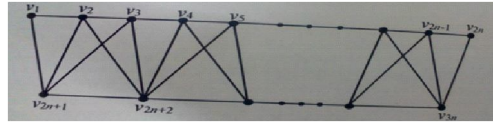


Fig 1.1

Net us assign the coloring C to the vertices as follows

$$Let \ c(v_{2i}) = \begin{cases} (i+2) \bmod 7 & 1 \leq i \leq n \\ (i+2) \bmod 7 + 7 & \text{otherwise} \end{cases} \quad c(v_{2n+i}) = \begin{cases} (i+6) \bmod 7 & (i+6) \bmod 7 \neq 0 \\ (i+3) \bmod 7 + 7 & \text{otherwise} \end{cases}$$

$$c(v_{2i-1}) = \begin{cases} (i+3) \bmod 7 & 1 \leq i \leq n \\ (i+3) \bmod 7 + 7 & \text{otherwise} \end{cases}$$

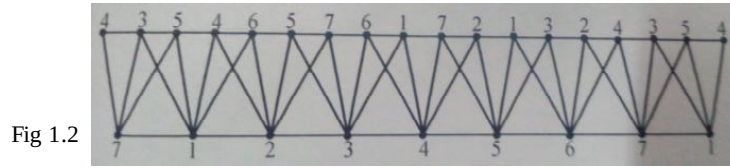


Fig 1.2

That clearly shows that C is a 7-coloring.

Case:1 C is a b-coloring

It is sufficient that to consider the vertices $v_{2n+1}, v_{2n+2}, \dots, v_{3n}$ of $L[S(n)]$. It is clear that all the vertices except the odd degree vertices have maximum degree 6. As we specified that for 7 coloring of any vertex of maximum degree is adjacent to all the vertices.

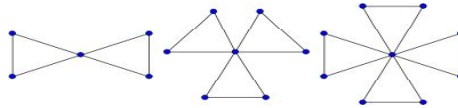
Case 2 : The b-chromatic number of $L[S(n)]$ is 7.

If $C(v_{3n}) = 8$ then v_{3n-1} are not has adjacent to 3 different colors. Thus the vertices $L(G)$ maximum degree 6 minimum degree 2.

Definition 1.3 : A fan graph $F_{m,n}$ is defined as the graph joining $K_m + P_n$, Where K_m is the empty graph on m nodes and P_n is the path graph on n nodes. The case of $m=1$ corresponds to the usual fan graph. The graph in fig 1.3 shows a triangular fan graph and its line graph.

Fig. The graph $F(1,2)$ and its line

Definition 1.4 : A fan graph or a friendship graph $F(nC3)$ is defined as the following connected graph containing n copies of circuits of each length 3.



Theorem 1.1: If F_n be the line graph of the fan graph, the $\phi(L(F_n)) = (n-1)$

Proof : Let us consider the F_n be the fan graph with n -vertices with set of $V(F_n) = \{f_1, f_2, f_3, \dots, f_n\}$ with $L(F_n)$ as a complete graph K_{n-1} with $V(K_{n-1}) = \{u_1, u_2, \dots, u_{n-1}\}$ and a cycle C with $V(C) = \{v_1, v_2, \dots, v_{n-1}\}$.

Now to assign the proper coloring and the b-chromatic for the set of cycles $C = \{1, 2, 3, \dots, n\}$

And define the coloring for as function $f: V \rightarrow C$. Assign the color C to the vertex set $V(K_{n-1}) = u_i$ for $i = 1, 2, 3, \dots, (n-1)$

Now we will color the vertices of a cycle. If one more color is introduced, we say C_n to be colored for any one of the vertex of the cycle C . In the outer layer of the cycle C are adjacent with $v_2, v_{n-1}, u_1, u_2, v_{n-1}$ is adjacent with $v_1, v_{n-2}, u_{n-1}, u_1$. In general each vertex v_i is adjacent to v_{i+1}, v_{i-1} for $i = 1, 2, 3, \dots, (n-1)$. Hence we can assign the $(n-1)$ colors which are assigned to the complete graph. Now the vertices in the cycle v_i for $i = 3, 4, \dots, (n-1)$ should be the color C_i for $i = \{1, 2, 3, \dots, n-3\}$ and the vertex v_i should be assigned by the color C_{n-2} by v_2 by C_{n-1} .

Hence $\phi(L(F_n)) = n$.

Definition 1.3 : A Cylindrical graph C_n is a graph with n vertices ($n \geq 6$), formed by connecting with a multi vertex to all the vertices of an $(n+1)$ cycle.

Line graph of a Cylindrical graph : In the following figure 1.4 illustrates a Cylindrical graph and Line graph of cylindrical graph.

Theorem 1.3 : If is a complete cylindrical graph, then $\chi(C_n) = n$, for any value of n .

Proof : Let us consider C_n be the cylindrical graph with n vertices. Let $V(C_n) = \{v_1, v_2, v_3, \dots, v_n\}$

be the set of vertex with cylindrical graph C_n Then $|V(G)| = n$ and $|E(G)| = n(n-1)/2$.

To determine the proper coloring we consider the following condition.

Case 1: C_n is a proper coloring with $n=4$

$$V(C_4) = \{v_1, v_2, v_3, v_4\}, V(G) = 4, E(G) = 4. \chi(C_n) = n$$

In this case, G is a three vertices of degree 3. Maximum degree 3. Using one the proposition we have $\chi(C_4) \leq 4$.

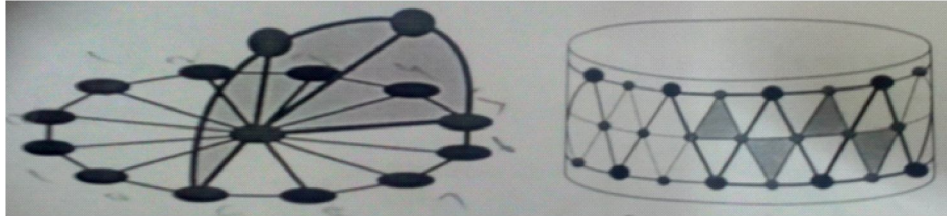


Fig 1.4

To assign a proper coloring and b-coloring consider the color of set $C = \{1, 2, 3, 4\}$ and define the color function $f: V \rightarrow C$ so that $f(V_1) = 1, f(V_2) = 2, f(V_3) = 3, f(V_4) = 4$. Therefore

$$\chi(C_4) = 4.$$

Case 2: C_n is a b-coloring

It is observed in the above case that G has four vertices of degree three. Max degree is 3. In the graph G must have three vertices of a degree 3 which is possible. To assign the proper coloring and for b-coloring consider the color set $C = \{1, 2, 3, 4\}$ and define the function as we above and color the graph. Hence we get the required result as $\chi(C_4) = 4$.

From these two conditions we say that the b-chromatic number of the given cylindrical triangular line graph with n vertices is n .

$$\text{Hence } \chi(C_n) = n.$$

Conclusion

Graph theory has many applications in biochemistry, electrical networks engineering computer science and operational research. Graph coloring is also many practical applications in bandwidth allocation and matching of such patterns. Here we have obtained some the b-chromatic number of line graphs and certain triangular line graphs such as snake triangular graphs, triangular fan graphs, cylindrical triangular pattern line graphs.

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