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Flow of A Visco-Elastic Rivlin-Ericksen (1955) Fluid Through Porous Medium in a Long Uniform Rectangular Duct Due to an Impulsive Pressure Gradient Acting at the Central Part of a Section

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Abstract

The aim of the present paper is to study the unsteady flow of a visco-elastic Rivlin-Ericksen (1955) type fluid through porous medium in a long uniform rectangular duct under the influence of an impulsive pressure gradient acting at the central part of a section. The analytical expressions for velocity profile and flux have been determined by the application of transform technique. Some particular cases have been discussed in detail. The results for the flow of ordinary viscous fluid have also been deduced by taking parameter $\mu_{1 \to 0}$.

Key words: visco-elastic, Rivlin-Ericksen (1955), fluid, unsteady flow

Introduction

Ghosh[6] discussed the viscous fluid flow through a long rectangular channel under the influence of an impulsive pressure gradient acting at the central part of a section. The problem of viscous fluid flow subjected to uniform and periodic body force for a finite time has been studied by Dutta[5]. Roy, Sen and Lahiri[13] investigated the problem of unsteady flow of Rivlin-Ericksen visco-elastic fluid through a rectangular duct due to an impulsive pressure gradient acting at the central part of a section. Many researchers have paid their attention towards the application of visco-elastic fluid flow through porous medium such as Ahmad and Taqui[1]; Thingarajan, Ramamurthi and Rahim[18];Pundir and pundir[12]; Sharma and Sharma[14]; Prakash and Aggarwal[11]; Kumar, Sharma and Singh[8]; Singh, Mishra and Sharma[17]; Singh, Kumar and Sharma[16]; Singh[15] and Kumar, Singh and Sharma[9]; Kumar, Gupta and Jain[7]; Panda and Mohapatra[10]; have studied the flow problems of visco-elastic fluid through porous medium under the influence of magneticfield applied perpendicularly to the flow of fluid. Alle *et. al.*[2] have studeady flow of a dusty visco-elastic fluid through a inclined channel. Banayal[4] stuied a characterization of Rivlin-Ericksen visco-elastic fluid in the presence of a magnetic field and rotation. Arya, Kumar and Singh[3] have discussed the unsteady flow of visco-elastic Rivlin-Ericksen fluid through porous medium in a long uniform rectangular channel. Venkateswarlu and Narayana[20] studed MHD visco-elastic fluid over a continuously moving vertical surface with chemical reaction. Tripathi[19] studied flow of a visco-elastic Rivlin-Ericksen type fluid through a long uniform rectangular duct when an impulsive pressure gradient acting at the central part of a section.

In the present paper, the flow of visco-elastic Rivlin-Ericksen type fluid through porous medium in a rectangular duct due to an impulsive pressure gradient acting at the central part of a section has been studied. The analytical expressions for the velocity profiles and the flux have been determined by using Laplace integral transform. Some particular cases and deduction are also considered.

Basic Theory :

For slow motion, the rheological equations for Rivlin-Ericksen (1955) visco-elastic fluid are:

$$\tau_{ij} = -p\delta_{ij} + \tau'_{ij} \tag{1}$$

$$\tau'_{ij} = 2\mu\left(1 + \mu_1 \frac{\partial}{\partial t}\right) e_{ij} \tag{2}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

where τ_{ij} is the stress tensor, τ'_{ij} the deviatoric stress tensor, e_{ij} the rate of strain tensor, μ the coefficient of visco elasticity, p the fluid presser, u_i the velocity components of the fluid and δ_{ij} is the kronecker delta.

The Navier-Stokes equation of motion for Rivlin-Ericksen fluid is

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\nabla^2\vec{q} + \vec{F} \tag{4}$$

where $\vec{q}$ is the velocity, $\vec{F}$ the body force, $\nu\left(=\frac{\mu}{\rho}\right)$ the coefficent of viscosityand ρ is the density of fluid.

Formulation of the problem :

Consider the rectangular Cartesian coordinates system (x,y,z); z-axis is taken towards the direction of motion of the fluid. The wall of the rectangular duct are taken to be the planes x=0, x=a and y=0, y=b. Let u, v, w be the components of the velocity of the fluid along positive direction of x-axis, y-axis and z- axis respectively. Also consider rectangular duct is long and the fluid velocity w in the direction of z-axis only, therefore

u =0,v=0, w=W (x,y,t)

In view of the above assumptions the equation of visco-elastic Rivlin-Ericksen fluid through porous medium is given by

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) - \frac{\nu}{K}W \tag{5}$$

where W is the velocity of the fluid in z-direction, t the time, K the permeability of porous media.

Introducing the following non-dimensional quantities:

$$x^* = \frac{x}{a},\ y^* = \frac{y}{a},\ z^* = \frac{z}{a},\ t^* = \frac{\nu}{a^2}t,\ p^* = \frac{a^2}{\rho\nu^2}p,$$

$$,\ \mu_1^* = \frac{\nu}{a^2}\mu_1,\ K^* = \frac{1}{a^2}K$$

in equation (5), there is found (after dropping stars)

$$\frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) - \frac{1}{K}W \tag{6}$$

Let $-\dfrac{\partial p}{\partial z} = f(x,y)\delta t,$ δt is Dirac delta function, be an impulsive pressure gradient acting at the central part of any section such that :
Boundary conditions are

$$\left.\begin{aligned} f(x,y) &= p \quad \text{for} \begin{cases} \frac{1}{2} - l_1 \le x \le \frac{1}{2} + l_1 \\ \frac{c}{2} - l_2 \le y \le \frac{c}{2} + l_2 \end{cases} \\ &= 0 \quad \text{for} \begin{cases} 0 \le x < \dfrac{1}{2}, \dfrac{1}{2} + l_1 < x \le 1 \\ 0 \le y < \dfrac{c}{2}, \dfrac{c}{2} + l_2 < y \le c \end{cases} \end{aligned}\right\} \tag{7}$$

$$= 0 \text{ for } t \le 0$$

where c (= b/a) <1, a non dimensional quantity.

To find the solution of equation (6), it is defined as

$$f(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \tag{8}$$

Multiplying both sides of equation (8) by $\sin\dfrac{m\pi x}{1}\sin\dfrac{n\pi y}{c}$ and Integrating, it is found

$$B_{mn}\int_0^c\int_0^1 \sin^2\frac{m\pi x}{1}\sin^2\frac{n\pi y}{c}\,dxdy = P\int_{\frac{c}{2}-l_2}^{\frac{c}{2}+l_2}\int_{\frac{1}{2}-l_1}^{\frac{1}{2}+l_1} \sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c}\,dxdy$$

$$\Rightarrow B_{mn} = \frac{16P}{mn\pi^2}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c} \tag{9}$$

Now let it be assumed that the velocity of fluid be

$$W = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn(t)} \sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{10}$$

Substituting equations (8) and (10) in equation(6), it is found

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\partial}{\partial t}\{A_{mn}(t)\}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} +$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}(t)\left\{\left(-\frac{m^2\pi^2}{1}\right) + \left(-\frac{n^2\pi^2}{c^2}\right)\right\}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} +$$

$$\mu_1 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\partial}{\partial t}\{A_{mn}(t)\}\left\{\left(-\frac{m^2\pi^2}{1}\right)+\left(-\frac{n^2\pi^2}{c^2}\right)\right\} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} -$$

$$\frac{1}{K}\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t) \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \tag{11}$$

Taking Laplace transform of equation (11), it is found

$$s\overline{A}_{mn}(s) = \frac{B_{mn}}{s} - \overline{A}_{mn}(s)C_{mn} - \mu_1 s\overline{A}_{mn}(s)C_{mn} - \frac{1}{K}\overline{A}_{mn}$$

$$\text{or}\quad \left\{(1+\mu_1 C_{mn})s + \left(\frac{1}{K} + C_{mn}\right)\right\}\overline{A}_{mn} = \frac{B_{mn}}{s}$$

$$\text{or}\quad \overline{A}_{mn}(s) = \frac{B_{mn}}{s\left\{\left((1+\mu_1 C_{mn})s + \left(\frac{1}{K}+C_{mn}\right)\right)\right\}}$$

$$\text{or}\quad \overline{A}_{mn}(s) = \frac{B_{mn}}{(1+\mu_1 C_{mn})s}\frac{1}{\left\{s + \left(\frac{\frac{1}{K}+C_{mn}}{1+\mu_1 C_{mn}}\right)\right\}}$$

$$\text{or}\ \overline{A}_{mn}(s) = \frac{B_{mn}}{\left(\frac{1}{K}+C_{mn}\right)}\left[\frac{1}{s} - \frac{1}{\left\{s + \left(\frac{\frac{1}{K}+C_{mn}}{1+\mu_1 C_{mn}}\right)\right\}}\right] \tag{12}$$

$$\text{where}\overline{A}_{mn}(s) = \int_0^{\infty} e^{-st} A_{mn}(t)dt \quad (\text{Laplace transform of function } A_{mn}(t))$$

$$\text{and}\qquad C_{mn} = \frac{m^2\pi^2}{1} + \frac{n^2\pi^2}{c^2} = \left(m^2 + \frac{n^2}{c^2}\right)\pi^2$$

Now applying the inverse Laplace transform of (12), we get

$$L^{-1}\{\overline{A}_{mn}(s)\} = A_{mn}(t)$$

$$= \frac{B_{mn}}{\left(\frac{1}{K}+C_{mn}\right)}\left[L^{-1}\left(\frac{1}{s}\right) - L^{-1}\frac{1}{\left\{s + \left(\frac{\frac{1}{K}+C_{mn}}{1+\mu_1 C_{mn}}\right)\right\}}\right]$$

$$= \frac{B_{mn}}{\left(\frac{1}{K} + C_{mn}\right)}\left[1 - e^{-\left(\frac{\frac{1}{K}+C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right] \tag{13}$$

From equations (9), (10) and (13), the velocity of visco-elastic Rivlin-Ericksen fluid is obtained as

$$W = \frac{16P}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\left[1 - e^{-\left(\frac{\frac{1}{K}+C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{mn\left(\frac{1}{K}+C_{mn}\right)}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{14}$$

The flux Q across any section at any instant is given by

$$Q = \int_0^1\int_0^c w\,dx\,dy$$

$$= \frac{64cP}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\left[1 - e^{-\left(\frac{\frac{1}{K}+C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{m^2n^2\left(\frac{1}{K}+C_{mn}\right)}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c} \tag{15}$$

when m and n both are odd.

Particular cases :

Case I: When $\mu_{1\to 0}$ *i.e.* the coefficient of visco-elasticity becomes zero, the velocity of purely viscous fluid through porous medium is given from equation (14) is

$$W = \frac{16P}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\left[1 - e^{-\left(\frac{1}{K}+C_{mn}\right)t}\right]}{mn\left(\frac{1}{K}+C_{mn}\right)}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c}\sin\frac{m\pi x}{1}\sin\frac{n\pi y}{c} \tag{16}$$

and flux from equation (15) is

$$Q = \frac{64cP}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\left[1 - e^{-\left(\frac{1}{K}+C_{mn}\right)t}\right]}{m^2n^2\left(\frac{1}{K}+C_{mn}\right)}\sin\frac{m\pi l_1}{2}\sin\frac{n\pi l_2}{2c} \tag{17}$$

Case II: When $K \to \infty$ *i.e.* porous medium is withdrawn, the velocity of visco-elastic Rivlin-Ericksen fluid is given from equation (14) is

$$W = \frac{16P}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{mnC_{mn}} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \tag{18}$$

and flux from (15) is

$$Q = \frac{64cP}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[1 - e^{-\left(\frac{C_{mn}}{1+\mu_1 C_{mn}}\right)t}\right]}{m^2 n^2 C_{mn}} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \tag{19}$$

which are the same result obtained by Roy, Sen and Lahiri[13] with slight change of notations.

Case III : When $\mu_1 \to 0, K \to \infty$ then velocity of purely incompressible viscous fluid is given from equation (14) is

$$W = \frac{16P}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \frac{[1 - e^{-C_{mn}t}]}{C_{mn}} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \sin\frac{m\pi x}{1} \sin\frac{n\pi y}{c} \tag{20}$$

and flux from (15) is

$$Q = \frac{64cP}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{[1 - e^{-C_{mn}t}]}{m^2 n^2 C_{mn}} \sin\frac{m\pi l_1}{2} \sin\frac{n\pi l_2}{2c} \tag{21}$$

which are the same result obtained by Gosh[6] with slight change of notations.

References

1. Ahmad, N. and Taqvi, Z. A., Jnanabha, *26*, p. 61 (1996).
2. Alle, G. *et. al.*, Advance in Pure Mathematics, Vol. *1*, p. 187 (2011).
3. Arya, M.K.; Kumar, R. and Singh, K.K., International Journal of Mathematical Archive, Vol. *4*, No. 1, p-261 (2013).
4. Banyal, A.S., Engineering Transactions, Vol. *60*, No. 4, p-355 (2012).
5. Dutta, S., Jour. of Tech. *3*, p.-58 (1998).
6. Ghosh, A. K., Bull. Cal. Math. Soc. *60(4)* p.-163 (1968).
7. Kumar, N.,Gupta, S. and Jain, T., *Ultra Scientist of Physical Sci*. Vol. *22*, No. 1(M), p.-191 (2010).
8. Kumar, R., Sharma, A. K. and Singh, K. K., *Ultra Scientist of Physical Sci*. Vol. *20*, No. 2(M), p.-545 (2008).
9. Kumar, R., Singh, K. K. and Sharma A. K., *Ultra Scientist of Physical Sci*. Vol.*21*, No. 2(M), p.-571 (2009).
10. Panda, J. P. andMohapatra, J. K., *Ultra Scientist of Physical Sci.* Vol. *22*, No. 1(M), p.-155 (2010).
11. Prakash, K. and Aggarwal, A. K., Proc. Nat. Acad. Sci. India, *77*(A) IV, p.-373 (2007).
12. Pundir, S. and Pundir, R., Acta Ciencia Indica, Vol. XXIXM No. 3, p.-593 (2003).
13. Roy, A.K.; Sen, S. and Lahiri, S., *Ind. Jour. of Theo. Phys.*, Vol. *38*, No. 1, p-11 (1990).
14. Sharma, P. R. and Sharma, S., *Bull. Cal. Math. Soc., 97(6)*, p.-477 (2005).
15. Singh, N., Ph.D. Thesis, Agra Univ., p.-75 (2009).
16. Singh, K. K., Kumar, R. and Sharma, A. K., Acta Ciencia Indica, Vol. XXXIV M No. 1, p.-351 (2008).
17. Singh, K. K., Mishra G. and Sharma A .K., Acta Ciencia Indica, Vol. XXXIV M No. 1, p.-421 (2008).
18. Thingarajan, S. Ramamurthy, M. V. and Rahim, Mohd. A., *Bull. Pure Appl. Sci., 21*E No. 1, p.-155 (2002).
19. Tripathi, A., *International Journal of Mathematical Archive*, Vol. *7*, No. 11, p.-1 (2016).
20. Venkateswarlu, B. and Narayana. P.V.S., Walailak *J. Sci. & Tech.*, Vol. *12*, No. 9, p-775 (2015).