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Results on Accurate Edge Domination Number in Graphs

VENKATESH S.H[1], V.R. KULLI[2], VENKANAGOUDA M. GOUDAR[3] and VENKATESHA[4]

[1]Research Scholar, Sri Gauthama Research Centre, (Affiliated to Kuvempu University), Sri Siddhartha Institute of Technology, Tumkur, Karnataka, India

[2] Department of Mathematics, Gulbarga University, Gulbarga, India

[3] Department of Mathematics, Sri Gauthama Research Centre, (Affiliated to Kuvempu University), Sri Siddhartha Institute of Technology, Tumkur, karnataka, India

[4] Department of Mathematics, Kuvempu University Shankarghatta, Shimoga, Karnataka, India.

Email address of Corresponding Author: sh1.venkatesh@gmail.com

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Abstract

A edge dominating set F of a graph $G=(V,E)$ is an accurate edge dominating set, if $<E-F>$ has no edge dominating set of cardinality $|F|$. The accurate edge domination number $\gamma_{ae}(G)$ is the minimum cardinality of an accurate edge dominating set. We study the graph theoretic properties of $\gamma_{ae}(G)$ and its exact values for some standard graphs. The relation between $\gamma_{ae}(G)$ with other parameters are also investigated.

Key words: Accurate edge dominating set, Accurate edge Dominating set.

Mathematics Subject Classification: 05C,05C05, 05C70.

1 Introduction

In this paper we follow the notations of [3], Let G be a finite, simple, non-trivial, undirected and connected (p,q) graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex v in a graph G is the number of edges of G incident with v and is denoted by $deg(v)$ and $N(v)(N[v])$ denotes the open (closed) neighborhoods of a vertex v. A vertex of degree one is called an pendent vertex. A vertex adjacent to pendent vertex is called the support vertex. As usual P_p, C_p, W_p, K_p and $K_{1,p}$ are respectively the path, cycle, wheel, complete graph and star.

The notation $\alpha_0(G)(\alpha_1(G))$ is the minimum number of vertices (edges) in a vertex (edge) cover of G. Also $\beta_0(G)(\beta_1(G))$ is the maximum number of vertices (edges) in a maximum independent set of vertex (edge) of G. The greatest distance between any two vertices of a connected graph G is called the diameter of G and is denoted by $diam(G)$. The maximum (minimum) degree of a vertex v is denoted by $\Delta(G)(\delta(G))$. For any real number x, $\lceil x \rceil$ denotes the smallest integer not less than x and $\lfloor x \rfloor$ denotes the greatest integer not greater than x. A cut vertex of a graph is one whose removal increases the number of components. In general $< X >$ to denote the subgraph induced by the set of vertices X.

Spider is a tree with one vertex of degree at least three and all others with degree at most one. Caterpillar tree is a tree in which all the vertices are within distance one of a central path.

A set $F \subseteq E(G)$ is said to be an edge dominating set if every edge in $E(G)-F$ is adjacent to some edges in F. The Edge domination number of G is the cardinality of smallest edge dominating set of G and is denoted by $\gamma'(G)$. This concept was introduced by Mitchell and Hedetniemi[6].

A dominating set D of a graph G is an accurate dominating set, if $V-D$ has no dominating set of cardinality $|D|$. The accurate domination $\gamma_a(G)$ is the minimum cardinality of an accurate dominating set. This concept was introduced by Kulli and Kattimani[11].

Throughout the paper we consider a graph with vertices $p \geq 4$.

2 Preliminary Notes:

We need the following results to prove further results.

Theorem 2.1[12] *If G is a* (p,q) *graph without isolated vertex then* $\frac{q}{\Delta(G)+1} \leq \gamma'(G)$.

In the next section we discuss results on Accurate edge domination number of a graph.

3 Accurate edge domination number of a graph:

In this we initiate the study of accurate edge domination is defined as below.

A edge dominating set F of a graph $G=(V,E)$ is an accurate edge dominating set, if $< E-F >$ has no edge dominating set of cardinality $|F|$. The accurate edge domination number $\gamma_{ae}(G)$ is the minimum cardinality of an accurate edge dominating set[10].

In this paper we study the graph theoretical properties of $\gamma_{ae}(G)$ and many bounds were obtained in terms of elements of G. For example, we consider the graph G in the figure 3.1. The Accurate edge dominating set of G is $A=\{3,5,7\}$. Therefore $\gamma_{ae}(G)=|A|=3$.

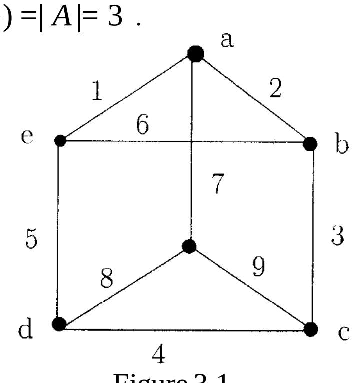

Figure 3.1

4 Results on accurate edge domination number Graph

Theorem 4.1 For any connected (p,q) graph G and if $n \geq 1$ is an integer then,

$$\gamma_{ae}[P_p] = \begin{cases} \frac{p+4}{3} & \text{if } p=3n+2 \\ \frac{p-1}{3} & \text{if } p=3n+1 \\ \frac{p+3}{3} & \text{if } p=3n+3 \end{cases}$$

Proof. Let $G = P_p(G)$ be a connected graph with $p \geq 4$. Let $E(P_p(G)) = \{e_1, e_2, ...e_q\}$ be the edge set of $P_p(G)$ and let $F = \{e_1, e_2, ...e_i / 1 \leq i \leq q\} \subseteq E(P_p(G))$ be the minimum edge dominating set of $P_p(G)$. Let $\{v_1, v_2, ...v_p\}$ be the vertex set of $P_p(G)$. Let K be the minimum edge dominating set of induced subgraph $< E(P_p(G)) - F >$. If $|F| \neq |K|$ then $A = F$ itself forms accurate edge dominating set of $P_p(G)$. Otherwise, consider $F_1 \subseteq (E(P_p(G) - F)$, Let $A = F \cup F_1$. If $< (E(P_p(G)) - A) >$ has no edge dominating set of cardinality $|A|$ then A forms the accurate edge dominating set with minimum cardinality. We have the following cases.

Case1. Suppose $p = 3n+2$.

Let $v_1, v_2, ...v_{3n+2}$ be the path $P_p(G)$ and $F = \{e_1, e_2, ...e_{n+1}\}$ forms minimum edge dominating set of $P_p(G)$. If K be the minimum edge dominating set of the induced subgraph $< E(P_p(G)) - F >$ then $|F| = |K|$, which is a contradiction. So we consider $e_l \in (E - F)$ such that $A = F \cup \{e_l\}$ forms the minimum accurate edge dominating set of $P_p(G)$.

Therefore $|A| = n+2$.

$\Rightarrow |A| = n+2 = \frac{p+4}{3}$.

Hence $\gamma_{ae}(G) = \frac{p+4}{3}$.

Case 2. Suppose $p=3n+1$.

Let $F = \{e_1, e_2, ...e_n\}$ be the edge dominating set of $P_p(G)$ and $K = \{e_1, e_2, ...e_{n+1}\}$ be the minimum edge dominating set of induced subgraph $< E(P_p(G)) - F >$. Clearly, $|F| \neq |K|$. So $A = F$ forms the minimum accurate edge dominating set of $P_p(G)$. Thus, $|A| = n$ but $n = \frac{p-1}{3}$.

Hence $\gamma_{ae}(G) = \frac{p-1}{3}$.

Case 3. Suppose $p = 3n+3$.

Let $F = \{e_1, e_2, \ldots e_{n+1}\}$ forms the edge dominating set of $P_p(G)$. If *K* be the minimum edge dominating set of the induced subgraph $E(P_p(G)) - F$ then $|F| = |K|$, which is a contradiction. So we consider $\{e_l\} \in (E - F)$ such that $A = F \cup \{e_l\}$ forms the minimum accurate edge dominating set of $P_p(G)$. Therefore $|A| = n + 2$.

$$\Rightarrow |A| = n + 2 = \frac{p+3}{3}.$$

Hence $\gamma_{ae}(G) = \dfrac{p+3}{3}$.

Theorem 4.2 For any connected (p,q) graph G, $\gamma'[G] \le \gamma_{ae}[G]$.

Proof. Let *G* be a connected graph. Let $F = \{e_i / 1 \le i \le q\} \subseteq E(G)$ be the minimum edge dominating set of *G* such that $|F| = \gamma'(G)$. Let *K* be minimum edge dominating set of induced subgraph $< E - F >$. If $|F| \ne |K|$ then *A=F* itself forms a minimum accurate edge dominating set of *G*. Otherwise, consider $F_1 \subseteq (E(P_p(G) - F)$, Let $A = F \cup F_1$. If $< (E(P_p(G)) - A) >$ has no edge dominating set of cardinality |*A*| then *A* forms the accurate edge dominating set with minimum cardinality. Clearly, every accurate edge dominating set is a edge dominating set. Hence $|F| \subseteq |A|$, Which gives $\gamma'[G] \le \gamma_{ae}[G]$.

Theorem 4.3 A caterpillar tree T with each cut vertex of degree greater than or equal to three then

$$\gamma_{ae}[T] = \begin{cases} s+1 & \text{if s is odd} \\ \dfrac{s}{2} & \text{if s is even} \end{cases}$$

Where s is the number of cut vertices in T.

Proof. Let *T* be a caterpillar tree of vertex set $V = \{v_1, v_2, \ldots v_p\}$ and let $C = \{v_1, v_2, \ldots v_r / 1 \le r \le p\} \subseteq V(T)$ be the cut vertex set of *T* with $deg(v_i) \ge 3$ for all $1 \le i \le r$ such that $|C| = s$. Let $E = \{e_1, e_2, \ldots e_q\}$ be the edge set of *T* and let $F = \{e_1, e_2, \ldots e_t / 1 \le t \le q\} \subseteq E(T)$ be the minimum edge dominating set of *T*. If $< E - F >$ has no edge dominating set of cardinality |*F*| then *A* = *F* it self forms minimum accurate edge dominating set *T*. Otherwise, consider $F_1 \subseteq (E(T) - F)$ such that $A = F \cup F_1$ forms the minimum accurate edge dominating set of *T* if $< E - (F \cup F_1) >$ has no edge dominating set of cardinality $F \cup F_1$ and $|A| = \gamma_{ae}(T)$. We have the following cases

Case 1. Suppose |*C*| is odd.

Let $\{e_1, e_2, .. e_j / 1 \le j \le q\} \in A$. It follows that $|A| = |C| + 1. \Rightarrow |A| = s + 1$. Hence $\gamma_{ae} = s + 1$.

Case 2. Suppose |*C*| is even.

Let $\{e_1, e_2, .. e_j / 1 \le j \le q\} \in A$. It follows that $|A| = |C|. \Rightarrow |A| = s$. Hence $\gamma_{ae}(G) = s$.

Theorem 4.4 Let G be a star if and only if $\gamma_{ae}(G) = (p - q) + 1$.

Proof. Suppose G be a star. Let $F=\{e_1\}\subseteq E(G)$ be the minimum edge dominating set of G and also $K=\{e_2\}$ be the edge dominating set of $<E(G)-F>$. But $|F|=|K|$. So consider an edge $e_3\in(E-F)$ such that $A=F\cup\{e_3\}$ forms the minimum accurate edge dominating set in G and induced subgraph $<E(G)-(F\cup\{e_3\})>$ has no edge dominating set of cardinality $|(F\cup\{e_3\})|$. We observe that $|A|=2=\gamma_{ae}(G)$.

Then $1+p+q-|A|=p+q+1-2$

$\Rightarrow 1+p+q-|A|=p+q-1$

since G is a star so $1 = p - q$, above equality becomes

$1+p+q-|A|=p+q-(p-q)$

$\Rightarrow 1+p+q-\gamma_{ae}(G)=2q$

Hence $\gamma_{ae}(G)=1+(p-q)$.

Conversely, Suppose $\gamma_{ae}(G)=1+(p-q)$.

From the above equality if $p-q=1$, it implies that $\gamma_{ae}(G)=2$.

Hence G is a star.

Theorem 4.5 For any connected (p, q) graph with $\Delta(G)\geq 3$ *then* $\gamma_{ae}(G)\leq q-\alpha_1+2$ *and equality holds for a star.*

Proof. Let G be connected graph with $\Delta(G)\geq 3$. Let $M=\{e_1,e_2...e_f\}$ be the set of all end edges. Suppose $B=\{e_1,e_2...e_k\}\subseteq(E(G)-M)$ be the set of edges such that $d(e_i,e_j)\geq 2$ for some $1\leq i\leq f, 1\leq j\leq k$ then $M\cup R$ where $R\subseteq B$ be the minimum set of edges which covers all the vertices in G such that $|M\cup R|=\alpha_1(G)$. Let $F=\{e_1,e_2...e_j/1\leq j\leq q\}$ be the minimum number of edges covers all the edges of G. Suppose induced subgraph $<E(G)-F>$ has no edge dominating set of cardinality $|F|$ then $A = F$ itself is a $\gamma_{ae}(G)$. Suppose $<E(G)-F>$ has edge dominating set of cardinality $|F|$ then consider $F_1\subseteq(E(G)-F)$ such that $A=F\cup F_1$ forms minimum accurate edge dominating set of G if induced subgraph $<E(G)-(F\cup F_1)>$ has no edge dominating set of cardinality $|F\cup F_1|$. Since $P\geq 4$ we have a connected graph G with at least 3 edges. It follows that $A\cup(M\cup R)\subseteq E(G)$.

$\Rightarrow |A|\leq|E(G)|-|M\cup R|+2$. Hence $\gamma_{ae}(G)\leq q-\alpha_1+2$.

Suppose G is a Star. Clearly, $|E(G)|=|M\cup R|=\alpha_1(G)$. From Theorem 4.4 $\gamma_{ae}(G)=2$. Therefore it follows that $\gamma_{ae}(G)=|E|-|M\cup R|+2$. It implies $\gamma_{ae}(G)=q-\alpha_1+2$. Hence $\gamma_{ae}(G)=q-\alpha_1+2$.

Corollary 4.6 For any connected (p, q) graph then $\gamma_{ae}(G)\leq q-\gamma'(G)+2$.

Theorem 4.7 For any connected $(p\geq 4,q)$ *graph then* $\lfloor\frac{q}{\Delta(G)+1}\rfloor\leq\gamma_{ae}(G)\leq\lceil\frac{q\Delta(G)}{\Delta(G)+1}\rceil+2$.

Proof. It is known that from theorem[7], $\frac{q}{\Delta(G)+1}\leq\gamma'(G)$ and since $\gamma'[G]\leq\gamma_{ae}[G]$. Clearly, we see that the lower bound $\lfloor\frac{q}{\Delta(G)+1}\rfloor\leq\gamma_{ae}(G)$ holds. By the Corollary 4.6,

$$\gamma_{ae}(G)\leq q-\gamma'(G)+2$$

$$\gamma_{ae}(G)\leq q-\frac{q}{\Delta(G)+1}+2$$

$$\gamma_{ae}(G) \le \frac{q(\Delta(G)+1)-q}{\Delta(G)+1}+2$$

$$\gamma_{ae}(G) \le \frac{q\Delta(G)}{\Delta(G)+1}+2$$

Hence $\gamma_{ae}(G) \le \lceil \frac{q\Delta(G)}{\Delta(G)+1} \rceil + 2$

This achieves the upper bound.

Theorem 4.8 For any wheel $p \ge 4$, $\gamma_{ae}[(W_p(G)] = \lceil \frac{p+1}{2} \rceil$.

Proof. Let $W_p(G)$ be a wheel with $p-1$ vertices on the cycle and a single vertex at the center. Let $V(W_p(G)) = \{c, v_1, v_2, ...v_{p-1}\}$ where c is the center vertex and $\{v_1, v_2, ...v_{p-1}\}$ vertices on the cycle. Let $E(W_p(G)) = \{e_1, e_2, ...e_i, e_{i+1}, ...e_q\}$ be the edge set of $W_p(G)$ where $\{e_1, e_2, ...e_i\}$ be the $p-1$ edges incident to the single vertex c as well as vertices lies on cycle and $\{e_{i+1}, e_{i+2}, ...e_q\}$ be the edges incident to the vertices lies on the cycle. Let $F = \{e_1\} \cup \{e_{i+1}, e_{i+2}, ...e_q\}$ be the minimum edge set covers all the edges of the wheel and let K be the minimum edge dominating of set of $< E(W_p(G)) - F >$. If $|F| \ne |K|$ then A=F is the accurate edge dominating set of $W_p(G)$. Otherwise, consider $F_1 \subseteq F$ such that $A = F \cup F_1$ forms the minimum accurate edge dominating set of $W_p(G)$ if $< E - (F \cup F_1) >$ has no edge dominating set cardinality $|F \cup F_1|$. clearly, $|A| = \frac{p+1}{2}$. $\Rightarrow$ $\gamma_{ae}[(W_p(G)] = \frac{p+1}{2}$.

Hence $\gamma_{ae}[(W_p(G)] = \lceil \frac{p+1}{2} \rceil$.

Theorem 4.9 For any connected (*p*, *q*) *graph G then* $\gamma_{ae}(G) + diam(G) \le p + \gamma^{'}(G)$.

Proof. Let $M = \{e_1, e_2, ...e_t / 1 \le t \le q\} \subseteq E(G)$ be the minimum set of edges which constitute the longest path between any two distinct vertices $v_1, v_2 \in V(G)$ such that $d(v_1, v_2) = diam(G)$. Let $F = \{e_i / 1 \le i \le q\}$ be the minimum edge dominating set of G such that $|F| = \gamma^{'}(G)$ and let $A = \{e_j / 1 \le j \le q\}$ be the minimum accurate edge dominating set of G such that $|A| = \gamma_{ae}(G)$. By the definition of $\gamma_{ae}(G)$, $< E(G) - A >$ has no edge dominating set of cardinality |*A*|. We discuss in the following cases.

Case 1. Suppose edge dominating set *F* itself forms minimum accurate dominating set of *G*, then $< E(G) - F >$ has no edge dominating set of cardinality |*F*|. Thus |*A*|=|*F*| it follows that $d(v_1, v_2) \le p$. It implies $|A| \cup d(v_1, v_2) \le p \cup |F|$. Hence $\gamma_{ae}(G) + diam(G) \le p + \gamma^{'}(G)$.

Case 2. Suppose cardinality of edge dominating set *F* is same as cardinality of edge dominating set of $< E(G) - F >$. Then consider $F \cup F_1$ be the minimum edge dominating set of G and its cardinality is different from the cardinality of edge dominating set of $< E(G) - (F \cup F_1) >$. Then $A = F \cup F_1$ is accurate edge dominating set of *G*. Clearly, it follows that $|F \cup F_1| \cup d(v_1, v_2) \le p \cup |F|$. It implies $|A| \cup d(v_1, v_2) \le p \cup |F|$. Hence $\gamma_{ae}(G) + diam(G) \le p + \gamma^{'}(G)$.

Theorem 4.10 For any connected (p, q) graph G, $\lceil \frac{\gamma_{ae}(G)}{2} \rceil \leq \beta_0(G)+3$.

Proof. Let $R = \{v_1.v_2,...v_l / 1 \leq l \leq p\} \subseteq V(G)$ be the maximum set of vertices such that $d(u,v) \geq 2$ and $N(u) \cap N(v) = x$ for all $u,v \in S$ and $x \in (V(G)-S)$. Clearly, $|S| = \beta_0(G)$. Let $A = \{e_1, e_2,...e_j / 1 \leq j \leq q\}$ be the minimum accurate edge dominating set of *G* and $< E(G)-A >$ has no edge dominating set of cardinality |*A*|. Clearly, $\frac{|A|}{2} \leq |R|+3$. Hence $\lceil \frac{\gamma_{ae}(G)}{2} \rceil \leq \beta_0(G)+3$.

Theorem 4.11 Let G be a connected graph with $\delta(G) \geq 2$ *and* $diam(G) = 2$ *then* $\gamma_{ae}(G) \leq 2\delta(G)-1$.

Proof. Let *G* be a connected graph with $p \geq 4$. Let $K = \{e, h\} \subseteq E(G)$ be the edges in which constitute the diametral path in G such that $diam(G) = 2$ and let $v_1 \in V(G)$ such that $deg(v_1) = \delta(G)$. We have the following cases.

Case 1. Suppose $\delta(G) = 2$. We observe that $F = \{e_1, e_2\} \subseteq E(G)$ be the $\gamma'(G)$ such that $|F| = 2$. Let $A = \{\{e_1, e_2\} \cup \{e_l\} / 3 \leq l \leq q\} \subseteq E(G)$ be the accurate edge dominating set of *G*. By the theorem 4.2, we have $\gamma'(G) \leq \gamma_{ae}(G)$. Therefore $2 \leq \gamma_{ae}(G) \leq 3$. Thus $|A| \leq 2deg(v_1)-1$. It implies $|A| \leq 2\delta(G)-1$. Hence $\gamma_{ae}(G) \leq 2\delta(G)-1$.

Case 2. Suppose $\delta(G) > 2$. We observe that $F = \{e_1, e_2,...e_t / 1 \leq t \leq q\} \subseteq E(G)$ be the $\gamma'(G)$ such that $|F| \geq 2$. A set $A = \{e_1, e_2,...e_l / 1 \leq l \leq q\}$ is the accurate edge dominating set of *G* if $< E(G)-A >$ has no dominating set of cardinality |*A*|. Thus it follows that

$$|A| + \delta(G) - 1 \leq 3\delta(G) + 1$$

$$|A| \leq 2\delta(G) - 1$$

Hence $\gamma_{ae}(G) \leq 2\delta(G)-1$.

Theorem 4.12 For any complete graph $G = K_p$, $\gamma_{ae}[K_p] \geq \lceil \frac{p}{2} \rceil$.

Proof. Let $V(G) = \{v_1, v_2,...v_p\}$ be the vertex set of *G*. The distance between any two vertices is exactly one that is $dim(G) = 1$ and $\Delta(G) = p-1 = \delta(G)$ or $\Delta(v_i) = p-1$ for all $1 \leq i \leq p$ then *G* is complete graph. Let $F = \{e_1, e_2,...e_l / 1 \leq l \leq q\} \subseteq E((G))$ be the minimum edge dominating set of *G*. Let *F'* be the minimum edge dominating set of induced subgraph $< E(G)-F >$. If $|F| \neq |F'|$ then *A*=*F* itself forms accurate edge dominating set of *G*. Otherwise, consider $F_1 \subseteq (E(G)-F)$, Let $A = F \cup F_1$. If $< (E(G)-A) >$ has no edge dominating set of cardinality |*A*| then *A* forms the accurate edge dominating set with minimum cardinality.

Then $(1+diam(G))\gamma_{ae}(G) \geq \Delta(v_i)+1$

$$\Rightarrow \gamma_{ae}(G) \geq \frac{\Delta(v_i)+1}{1+diam(G)}$$

$$\Rightarrow \gamma_{ae}(G) \geq \frac{\Delta(v_i)+1}{1+diam(G)} = \frac{p-1+1}{2}$$

Hence $\gamma_{ae}[K_p(G)] \geq \lceil \frac{p}{2} \rceil$.

Theorem 4.13 For any cycle C_p then $\gamma_{ae}(G) = \lfloor \frac{p}{2} \rfloor + 1$.

Proof. Let C_p be a cycle with $p \geq 4$ and it is 2-regular graph that is $deg(v_i) = 2 = \Delta(C_p)$ for all i. Let $F = \{v_i v_j / i \neq j, 1 \leq i \leq p, 1 \leq j \leq p\} \subseteq E(C_p)$ be the edge dominating set of C_p. Let K be the minimum edge dominating set of induced subgraph $< E(G) - F >$. If $| F | \neq | K |$ then A=F itself forms accurate edge dominating set of G. Otherwise, consider $F_1 \subseteq (E(G) - F)$, Let $A = F \cup F_1$. If $< (E(G) - A) >$ has no edge dominating set of cardinality $|A|$ then A forms the accurate edge dominating set with minimum cardinality.

Since $G = C_p$ is a 2-regular graph then degree of each vertex is two and the number of vertices is equal to number of edges. We observe that $A = \{e_1, e_2, ... e_{\lfloor \frac{p}{2} \rfloor + 1}\}$ forms the minimum accurate edge dominating set of C_p such that $| A | = \gamma_{ae}(G)$. From Theorem[7], $\frac{q}{\Delta(G)+1} \leq \gamma'(G)$ it implies that $\frac{p}{2+1} \leq \gamma'(G)$.

But $\frac{p}{2+1} \leq \frac{p}{2}$. Therefore $\frac{p}{2} \geq \gamma'(G)$.

By the theorem 4.2, we have $\gamma'(G) \leq \gamma_{ae}(G)$ so above inequality becomes

$\gamma'(G) \leq \gamma_{ae}(G) = \lfloor \frac{p}{2} \rfloor + 1$

Hence $\gamma_{ae}(C_p) = \lfloor \frac{p}{2} \rfloor + 1$.

Theorem 4.14 Let G be a 3-regular graph then $\gamma_{ae}(G) \leq \lceil \frac{q}{2} \rceil$.

Proof. Since $G = (p = 2n, q = 3n+3)$ for $n \geq 1$ is 3-regular graph with $deg(v_i) = 3$ for all i and each edge adjacent to exactly four edges in G that is $deg(e_j) = 4$ for all j. Let $V = \{v_1, v_2, ... v_{2n}\}$ be the vertex of G and $E = \{e_1, e_2, ... e_{3n+3)}\}$ be the edge set of G. Let $A = \{e_t / 1 \leq t \leq q\}$ be the accurate edge dominating set of G such that $| A | = \gamma_{ae}(G)$. It follows that $q \geq 2 | A |$. It implies that $| A | \leq \lceil \frac{q}{2} \rceil$. Hence $\gamma_{ae}(G) \leq \lceil \frac{q}{2} \rceil$.

Theorem 4.15 For any spider tree T and diam (T) ≤ 10, $\gamma_{ae}(T) \leq diam(T) + \Delta(T)$.

Proof. Let T be a spider. Let $v_1 \in V(T)$ be the head vertex such that $\Delta(v_1) \geq 3$ and $\{v_2, v_3, ... v_p\} \in (V(T) - v_1)$ such that $\Delta(v_i) \leq 2$ for all $2 \leq i \leq p$. Let $F = \{e_1, e_2, ... e_t / 1 \leq t \leq q\}$ be the minimum edge dominating set of T and there exits at least one edge $x \in F$ incident to v_1. Also $R = \{e_1, e_2, ... e_d / 1 \leq d \leq q\}$ be the set of all edges constitute the longest path in T such that $| R | = diam(T) \leq 10$. Suppose $< E - F >$ has no edge dominating set of cardinality $|F|$ then A=F forms $\gamma_{ae}(T)$. Otherwise, $A = F \cup \{e_1, e_2, ... e_s\}$ forms minimum accurate edge dominating set of T. Clearly,

$|A| \leq deg(v_1) \cup diam(T)$. Hence $\gamma_{ae}(T) \leq diam(T) + \Delta(T)$.

Theorem 4.16 Let G be a connected $(p \geq 4, q)$ *graph such that G and* $\overline{G}$ *are connected graph then,*

1) $\gamma_{ae}(G) + \gamma_{ae}(\overline{G}) \leq 2q$.

2) $\gamma_{ae}(G).\gamma_{ae}(\overline{G}) \geq \lfloor \frac{p}{3} \rfloor$.

5 Conclusion

In this paper we discussed the accurate edge domination number of a connected graph. Also we obtained the relationship between edge domination number, edge covering number, maximum and minimum degree, maximum independent number and diameter of accurate edge domination number of a graph.

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