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On Tricomplex Representation Methods of A Matrix Trace Inequality

K. GUNASEKARAN[1], K. KANNAN[2] and M. RAHAMATHUNISHA[3],

[1,3]Ramanujan Research Centre, P.G. and Research Department of Mathematics, Government Arts College (Autonomous), Kumabakonam - 612002 Tamil Nadu India

[2]Department of Mathematics and Statistics University of Jaffna, Jaffna, Sri Lanka

Corresponding Author Email:- srnisha.phdpmu@gmail.com

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Abstract

In this paper, on tricomplex representation methods of a matrix trace inequality is introduced. A matrix trace inequality for positive semi definite Hermitian matrices A and B, $0 \leq tr\,(A*B)^n \leq (trA)^n{}_*(trB)^n$ is established, where n is an integer. The above inequality improves the result given by Yang (J Math. Anal. Appl. 250 (2000), 372-374).

Key words: Product of quaternion matrix**,** positive semi definite, positive definite, Hermitian matrix.

1. Introduction

Recently, there has been substantial interest in matrix trace inequalities for positive semi definite Hermitian matrices of the same order. See, for example[1,2,3,4,5,6]. Our main purpose in this paper is to prove the following results.

2. Definitions :

Definition 2.1: Positive Semi definite.

A matrix A is said to positive semi definite if there exists $x \neq 0$ such that $Ax \geq 0$.

Definition 2.3: Positive definite.

A matrix A is positive definite there exists $x \neq 0$ such that $Ax > 0$.

Note:

A Positive definite matrix is semi positive definite, but the converse need not true.

3. Lemmas and Theorem :

Lemma 3.1 : If A and B are positive semi definite Hermitian matrices of the same order, then

$$0 \leq tr(A*B) \leq (trA) * (trB) \qquad (1)$$

Proof : For any quaternion matrix $A \in H^{mxn}$, A can be uniquely represented as

$A = A_0+A_1j+A_2k$, $B=B_0+B_1j+B_2k$ and $A*B = A_0B_0 +A_1B_1j+A_2B_2k$

here, $tr(A*B) \;=\; tr(A_0B_0 +A_1B_1j+A_2B_2k)$

$= \ tr(A_0B_0) + tr(A_1B_1j) + tr(A_2B_2k)$

$\leq \ tr(A_0)tr(B_0)+tr(A_1)tr(B_1j) + tr(A_2)tr(B_2k)$ [since $tr(AB)\leq tr(A)tr(B)$]

$= \ (trA)*(trB)$

Since A and B are positive semi definite Hermitian matrices.

So, $tr(A*B) \geq 0$

Thus $0 \leq tr(A*B) \leq (trA) * (trB)$

The Proof is complete.

Lemma 3.2 : If A and B are positive semi definite Hermitian matrices of the same order, then for n = 1,2,…, $(A*B)^n{}_* B$ and $(B*A)^n{}_* B$ are positive semi definite Hermitian Matrices.[4]

*Proof:*Let A and B be Hermitian matrices of the same order. If A is positive semi definite Hermitian matrices then $(A*B)^{2n}*A = (A*B)^n* A*(B*A)^n$ is clearly positive semi definite Hermitian Matrices. If B is positive semi definite Hermitian matrices.

Then, $(A*B)^{2n+1}*A = (A*B)^n*A*B*A*(B*A)^n$ is clearly positive semi definite Hermitian matrices. Therefore, if both A and B are positive semi definitie Hermitian matrices $(A*B)^k* A$ is positive semi definite Hermitian matrix, k= 1,2…

The proof is complete.

Theorem 3.3 : Let A and B be positive semi definite Hermitian matrices of the same order; then for n = 1,2…

$$0 \leq tr(A*B)^{2n} \leq (trA)^2* (trA^2)^{n-1}*(trB^2)^n \qquad (2)$$

$$0 \leq tr(A*B)^{2n+1}\leq (trA)*(trB)*(trA^2)^n*(trB^2)^n \qquad (3)$$

Proof : From Lemma 3.1 and 3.2, and the equality $tr(A*B) = tr(B*A)$ for any square matrices A,B of the same order, we obtain

$$tr(A*B)^{2n} = tr(A*(B*A))^{2n-1}*B) < (trA)*(tr(B*A)^{2n-1}*B) \qquad (4)$$

$$tr((B*A)^{2n-1}*B) = tr(B*(A*B)^{2(n-1)} A*B)$$

$$= tr((A*B)^{2(n-1)}*A*B^2)$$

$$\leq tr((B*A)^{2(n-1)}*A)*trB^2$$

$$= tr(A*(B*A)^{2(n-1)}*B*A)*trB^2$$

$$= tr((B*A)^{2(n-1)-1} B*A^2)*trB^2 \leq tr((B*A)^{2(n-1)-1}*B)* trA^2*trB^2 \qquad (5)$$

By (4) and (5) we obtain

$$tr((B*A)^{2n-1}*B) \leq tr((B*A)^{2(n-1)-1}*B)*trA^2*trB^2$$

which means

$$tr((B*A)^{2n-1}*B) \leq tr(B*A*B)*(trA^2)^{n-1}*(trB^2)^{n-1}$$

$$= tr(A*B^2)*(trA^2)^{n-1}*(trB^2)^{n-1}$$

$$\leq trA*(trA^2)^{n-1}*(trB^2)^n \qquad (6)$$

From (4) and (6) we obtain (2).

Similarly,

$$tr(A*B)^{2n+1} = tr(A*(B*A)^{2n}*B) \leq trA*tr((B*A)^{2n}*B) \qquad (7)$$

$$tr((B*A)^{2n}*B)=tr(B*(A*B))^{2n-1}*A*B)$$

$$= tr((A*B)^{2n-1}*A*B^2) \leq tr((A*B)^{2n-1}*A)*trB^2=tr(A*(A*B)^{2(n-1)}*B*A)* trB^2$$

$$= tr((B*A)^{2(n-1)}*B*A^2)*trB^2 \leq tr((B*A)^{2(n-1)}*B)*trA^2*trB^2 \qquad (8)$$

By (7) and (8) we obtain

$$Tr((B*A)^{2n}*B) \leq trA^2*trB^2*tr((B*A)^{2(n-1)}*B),$$

which means

$$\mathrm{tr}((B*A)^{2n}*B) \leq (\mathrm{tr}A^2)^n*\mathrm{tr}B^2)^n*\mathrm{tr}B \quad (9)$$

from (7) and (9) we obtain (3).

The proof is complete.

Corollary 3.4:

If A and B are defined in theorem 3.3, then

$$0 \leq \mathrm{tr}(A*B)^n \leq (\mathrm{tr}A)^n* (\mathrm{tr}B)^n \quad (10)$$

Remark : By Lemma 3.1, $0 \leq \mathrm{tr}A^2 \leq (\mathrm{tr}A)^2$, $0 \leq \mathrm{tr}B^2 \leq (\mathrm{tr}B)^2$, this together with (2) and (3), we obtain (10)

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