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Some New Multiplicative Geometric-Arithmetic Indices

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Abstract

In this paper, we propose some new topological indices: second, third, fourth and fifth multiplicative geometric-arithmetic indices of a molecular graph. A topological index is a numeric quantity from the structural graph of a molecule. Here, we compute the fifth multiplicative geometric arithmetic index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8[p,q]$.

Key words: molecular graph, fifth multiplicative geometric-arithmetic index, nanostructures.
Mathematics Subject Classification: 05*C*05, 05*C*12, 05*C*35,

1. Introduction

In this paper, we consider only finite, simple and connected graph with a vertex set $V(G)$ and an edge set $E(G)$. A molecular graph or a chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of a chemical graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numerical parameter mathematically derived from the graph structure. These indices are useful for establishing correlation between the structure of a molecular compound and its physico-chemical properties, see[1].

The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. Let $S_G(v)$ denote the sum of degrees of all vertices adjacent to a vertex v. The line graph $L(G)$ of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The subdivision graph $S(G)$ of a graph G is the graph obtained from G by replacing each of its edges by a path of length two. We refer to[2,3] for undefined term and notation.

We need the following results

Lemma 1[3]. Let G be a (p, q) graph. Then $L(G)$ has q vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.

Lemma 2[3]. Let G be a (p, q) graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.

One of the well-known and widely used topological index is the product connectivity index or Randiæ index introduced by Randiæ in[4].

Motivated by the definition of the product connectivity index and its wide applications, Kulli [5] introduced the first multiplicative geometric-arithmetic index of a graph G and it is defined as

$$GA_1II(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u) + d_G(v)}.$$

Recently many other multiplicative indices were studied, for example, in [6,7,8,9,10,11,12,13,14].

Motivated by the definition of the first multiplicative geometric-arithmetic index and by previous research on topological indices, we now propose the second, third, fourth and fifth multiplicative geometric-arithmetic indices of a graph as follows:

The second multiplicative geometric-arithmetic index of a graph G is defined as

$$GA_2II(G) = \prod_{uv \in E(G)} \frac{2\sqrt{n_u n_v}}{n_u + n_v}$$

where the number n_u of vertices of G lying closer to the vertex u than to the vertex v for the edge uv of a graph G.

The third multiplicative geometric-arithmetic index of a graph G is defined as

$$GA_3II(G) = \prod_{uv \in E(G)} \frac{2\sqrt{m_u m_v}}{m_u + m_v}$$

where the number m_u of edges of G lying closer to the vertex u than to the vertex v for the edge uv of a graph G.`

The fourth multiplicative geometric-arithmetic index of a graph G is defined as

$$GA_4II(G) = \prod_{uv \in E(G)} \frac{2\sqrt{\varepsilon(u)\varepsilon(v)}}{\varepsilon(u) + \varepsilon(v)}$$

where the number $\varepsilon(u)$ is the eccentricity of all vertices adjacent to a vertex u.

The fifth multiplicative geometric-arithmetic index of a graph G is defined as

$$GA_5II(G) = \prod_{uv \in E(G)} \frac{2\sqrt{S_G(u)S_G(v)}}{S_G(u) + S_G(v)}, \quad \text{where } S_G(u) = \sum_{uv \in E(G)} d_G(v).$$

In[14], Todeshine *et al*. introduced the first and second multiplicative Zagreb indices of a graph G and they are defined as

$$II_1(G) = \prod_{u \in V(G)} d_G(u)^2, \qquad II_2(G) = \prod_{uv \in E(G)} d_G(u)d_G(v)$$

In[15] the first multiplicative Zagreb index is defined as

$$II_1^*(G) = \prod_{uv \in E(G)} \left[d_G(u) + d_G(v)\right].$$

We now define a new version of multiplicative Zagreb indices as follows.

$$II_1^2(G) = \prod_{u \in V(G)} n_u^2, \qquad II_2^2(G) = \prod_{uv \in E(G)} n_u n_v, \quad II_1^{2*}(G) = \prod_{uv \in E(G)} \left(n_u + n_v\right).$$

Also we define a new version of multiplicative Zagreb indices as follows:

$$II_1^3(G) = \prod_{u \in V(G)} m_u^2, \qquad II_2^3(G) = \prod_{uv \in E(G)} m_u m_v, \quad II_1^{3*}(G) = \prod_{uv \in E(G)} \left(m_u + m_v\right).$$

We define another version of multiplicative Zagreb indices as follows:

$$II_1^4(G)=\prod_{u\in V(G)}\varepsilon(u)^2,\quad II_2^4(G)=\prod_{uv\in E(G)}\varepsilon(u)\varepsilon(v),\ II_1^{4*}(G)=\prod_{uv\in E(G)}\left[\varepsilon(u)+\varepsilon(v)\right].$$

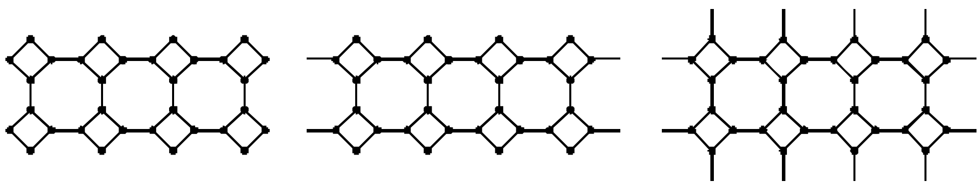
We also define another version of multiplicative Zagreb indices as follows:

$$II_1^5(G)=\prod_{u\in V(G)}S_G(u)^2,\ II_2^5(G)=\prod_{uv\in E(G)}S_G(u)S_G(v),\ II_1^{5*}(G)=\prod_{uv\in E(G)}\left[S_G(u)+S_G(v)\right].$$

In this paper; we determine the fifth multiplicative geometric arithmetic index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8\,[p,q]$.

2. 2D-lattice, nanotube, nanotorus of $TUC_4C_8\,[p,q]$:

We consider the graph of 2D-lattice nanotube and nanotorus of $TUC_4C_8\,[p,q]$ where p and q denote the number of squares in a row and the number of rows of squares respectively. These graphs are shown in Figure 1.



G_1 (a) H_1 (b) K_1 (c)

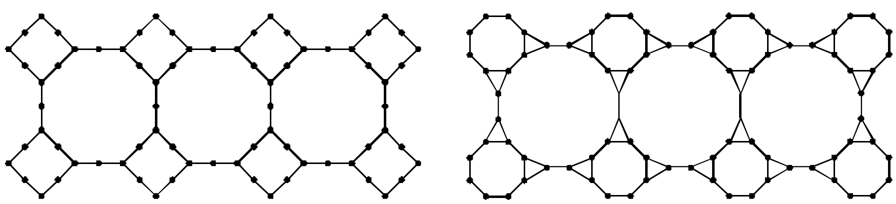
Figure 1

(a)2D-lattice of $TUC_4C_8[4, 2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8\,[4, 2]$ nanotorus

By algebraic method, we get $|V(G_1)| = 4pq$, $|E(G_1)| = 6pq - p - q$; $|V(H_1)| = 4pq$, $|E(H_1)| = 6pq - p$, $|V(K_1)| = 4pq$, $|E(K_1)| = 6pq$.

3. Results for 2D-Lattice of $TUC_4C_8\,[p,q]$:

The line graph of the subdivision graph of 2D-lattice of $TUC_4C_8[p,\ q]$ is shown Figure 2(b).



(a) (b)

Figure 2

(a) subdivision graph of 2D-lattice of $TUC_4C_8[4,2]$ (b) line graph of the subdivision graph of $TUC_4C_8\,[4, 2]$

Theorem 1. Let G be the line graph of the subdivision graph of 2D-lattice of $TUC_4C_8\,[p,q]$. Then

$$GA_5II(G)=\left(\frac{4\sqrt{5}}{9}\right)^8\times\left(\frac{4\sqrt{10}}{13}\right)^{4(p+q-2)}\times\left(\frac{12\sqrt{2}}{17}\right)^{8(p+q-2)}\qquad \text{if } p>1,\ q>1,$$

$$=\left(\frac{4\sqrt{5}}{9}\right)^4\times\left(\frac{4\sqrt{10}}{13}\right)^{4(p-1)}\times\left(\frac{12\sqrt{2}}{17}\right)^{4(p-1)}\qquad \text{if } p>1,\ q=1.$$

Proof: The 2D-lattice of $TUC_4C_8[p,q]$ is a graph G with $4pq$ vertices and $6pq - p - q$ edges. By Lemma 2, the subdivision graph of 2D-lattice of $TUC_4C_8\,[p,q]$ is a graph with $10pq - p - q$ vertices and $2(6pq - p - q)$ edges. Thus by Lemma 1, G has $2(6pq - p - q)$ vertices and $18pq - 5p - 5q$ edges. It is easy to see that the vertices of G are either of degree 2 or 3, see Figure 2. Therefore we have partition of the edge set of G as follows.

$S_G(u), S_G(v)\backslash uv\in E(G)$	(4, 4)	(4, 5)	(5, 5)	(5, 8)	(8, 9)	(9, 9)
Number of edges	4	8	$2(p+q-4)$	$4(p+q-2)$	$8(p+q-2)$	$2(9pq+10)-19(p+q)$

Table 1. Edge partition of G with $p>1$ and $q>1$.

$S_G(u), S_G(v)\backslash uv\in E(G)$	(4, 4)	(4, 5)	(5, 5)	(5, 8)	(8, 8)	(8, 9)	(9, 9)
Number of edges	6	4	$2(p-2)$	$4(p-1)$	$2(p-1)$	$4(p-1)$	$p-1$

Table 2. Edge partition of G with $p>1$ and $q = 1$.

Case 1. Suppose $p>1$ and $q>1$.

By algebraic method, we obtain $|V_4|=8$, $|V_5|=4(p+q-2)$ $|V_8|=4(p+q-2)$, and $|V_9| = 2(6pq-5p-5q+4)$ in G. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 1.

$$GA_5II(G) = \prod_{uv\in E(G)} \frac{2\sqrt{S_G(u)S_G(v)}}{S_G(u)+S_G(v)}$$

$$= \left(\frac{2\sqrt{4\times 4}}{4+4}\right)^4 \times \left(\frac{2\sqrt{4\times 5}}{4+5}\right)^8 \times \left(\frac{2\sqrt{5\times 5}}{5+5}\right)^{2(p+q-4)} \times \left(\frac{2\sqrt{5\times 8}}{5+8}\right)^{4(p+q-2)}$$

$$\times \left(\frac{2\sqrt{8\times 9}}{8+9}\right)^{8(p+q-2)} \times \left(\frac{2\sqrt{9\times 9}}{9+9}\right)^{2(9pq+10)-19(p+q)}$$

$$= (1)^4 \times \left(\frac{4\sqrt{5}}{9}\right)^8 \times (1)^{2(p+q-4)} \times \left(\frac{4\sqrt{10}}{13}\right)^{4(p+q-2)} \times \left(\frac{12\sqrt{2}}{17}\right)^{8(p+q-2)} \times (1)^{2(9pq+10)-19(p+q)}$$

$$= \left(\frac{4\sqrt{5}}{9}\right)^8 \times \left(\frac{4\sqrt{10}}{13}\right)^{4(p+q-2)} \times \left(\frac{12\sqrt{2}}{17}\right)^{8(p+q-2)}.$$

Case 2. Suppose $p>1$ and $q = 1$.

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 2.

$$GA_5II(G) = \prod_{uv\in E(G)} \frac{2\sqrt{S_G(u)S_G(v)}}{S_G(u)+S_G(v)}$$

$$= \left(\frac{2\sqrt{4\times 4}}{4+4}\right)^6 \times \left(\frac{2\sqrt{4\times 5}}{4+5}\right)^4 \times \left(\frac{2\sqrt{5\times 5}}{5+5}\right)^{2(p-2)} \times \left(\frac{2\sqrt{5\times 8}}{5+8}\right)^{4(p-1)}$$

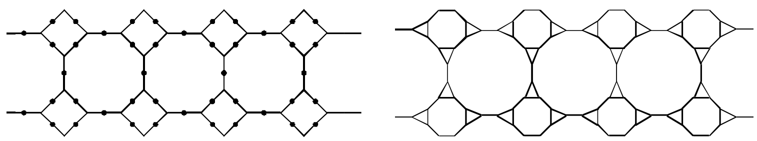
$$\times \left(\frac{2\sqrt{8\times 8}}{8+8}\right)^{2(p-1)} \times \left(\frac{2\sqrt{8\times 9}}{8+9}\right)^{4(p-1)} \times \left(\frac{2\sqrt{9\times 9}}{9+9}\right)^{(p-1)}$$

$$= (1)^6 \times \left(\frac{4\sqrt{5}}{9}\right)^4 \times (1)^{2(p-2)} \times \left(\frac{4\sqrt{10}}{13}\right)^{4(p-1)} \times (1)^{2(p-1)} \times \left(\frac{12\sqrt{2}}{17}\right)^{4(p-1)} \times (1)^{p-1}$$

$$= \left(\frac{4\sqrt{5}}{9}\right)^4 \times \left(\frac{4\sqrt{10}}{13}\right)^{4(p-1)} \times \left(\frac{12\sqrt{2}}{17}\right)^{4(p-1)}.$$

4. Results for $TUC_4C_8[p,q]$ nanotube :

The line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotube is shown in Figure 3(b)



(a) Figure 3 (b)

(a) Subdivision graph of TUC_4C_8 [4, 2] nanotube (b) line graph of subdivision graph of TUC_4C_8 [4, 2] nanotube.

Theorem 2. Let H be the line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then

$$GA_5II(H) = \left(\frac{4\sqrt{10}}{13}\right)^{4p} \times \left(\frac{12\sqrt{2}}{17}\right)^{8p}, \quad \text{if } p>1 \text{ and } q>1,$$

$$= \left(\frac{4\sqrt{10}}{13}\right)^{4p} \times \left(\frac{12\sqrt{2}}{17}\right)^{4p}, \quad \text{if } p>1 \text{ and } q=1.$$

Proof: The $TUC_4C_8[p,q]$ nanotube is a graph H with $4pq$ vertices and $6pq - p$ edges. By Lemma 2, the subdivision graph of $TUC_4C_8[p,q]$ nanotube is a graph with $10pq - p$ vertices and $12pq - 2p$ edges. Thus by Lemma 1, H has $12pq-2p$ vertices and $18pq - 5p$ edges. We see that in H, there are $4p$ vertices are of degree 2 and remaining all vertices are of degree 3. Therefore we have partition of the edge set of H as follows:

$S_H(u), S_H(v)\backslash uv\in E(H)$	(5, 5)	(5, 8)	(8, 9)	(9, 9)
Number of edges	$2p$	$4p$	$8p$	$18pq - 19p$

Table 3. Edge partition of H with $p>1$ and $q>1$.

$S_H(u), S_H(v)\backslash uv\in E(H)$	(5, 5)	(5, 8)	(8, 8)	(8, 9)	(9, 9)
Number of edges	$2p$	$4p$	$2p$	$4p$	p

Table 4. Edge partition of H with $p>1$ and $q = 1$

Case 1: Suppose $p>1$ and $q > 1$.

By algebraic method, we obtain $|V_5| = 4p$, $|V_8| = 4p$ and $|V_9| = 2(6pq - 5p)$ in H. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 3.

$$GA_5II(H) = \prod_{uv\in E(H)} \frac{2\sqrt{S_H(u)S_H(v)}}{S_H(u)+S_H(v)}$$

$$= \left(\frac{2\sqrt{5\times5}}{5+5}\right)^{2p} \times \left(\frac{2\sqrt{5\times8}}{5+8}\right)^{4p} \times \left(\frac{2\sqrt{8\times9}}{8+9}\right)^{8p} \times \left(\frac{2\sqrt{9\times9}}{9+9}\right)^{18pq-19p}$$

$$= (1)^{2p} \times \left(\frac{4\sqrt{10}}{13}\right)^{4p} \times \left(\frac{12\sqrt{2}}{17}\right)^{8p} \times (1)^{18pq-19p} = \left(\frac{4\sqrt{10}}{13}\right)^{4p} \times \left(\frac{12\sqrt{2}}{17}\right)^{8p}.$$

Case 2. Suppose $p>1$ and $q = 1$.

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 4.

$$GA_5II(H) = \prod_{uv\in E(H)} \frac{2\sqrt{S_H(u)S_H(v)}}{S_H(u)+S_H(v)}$$

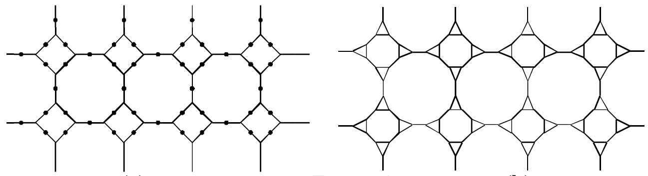
$$= \left(\frac{2\sqrt{5\times5}}{5+5}\right)^{2p} \times \left(\frac{2\sqrt{5\times8}}{5+8}\right)^{4p} \times \left(\frac{2\sqrt{8\times8}}{8+8}\right)^{2p} \times \left(\frac{2\sqrt{8\times9}}{8+9}\right)^{4p} \times \left(\frac{2\sqrt{9\times9}}{9+9}\right)^{p}$$

$$= (1)^{2p} \times \left(\frac{4\sqrt{10}}{13}\right)^{4p} \times (1)^{2p} \times \left(\frac{12\sqrt{2}}{17}\right)^{4p} \times (1)^{p}.$$

$$= \left(\frac{4\sqrt{10}}{13}\right)^{4p} \times \left(\frac{12\sqrt{2}}{17}\right)^{4p}.$$

5. Results for TUC_4C_8 [p,q] nanotorus :

The line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotorus is shown in Figure 4 (b).



(a) Figure 4 (b)

(a) subdivision graph of $TUC_4C_8[4,2]$ nanotorus

(b) line graph of subdivision graph of $TUC_4C_8[4,2]$ nanotorus.

Theorem 3. Let K be the line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanoturus. Then $GA_5II(K) = 1$.

Proof: Let K be the line graph of subdivision graph of $TUC_4C_8\,[p, q]$ nanotorus with $4pq$ vertices and $6pq$ edges. Then Lemma 2, the subdivision graph of $TUC_4C_8\,[p, q]$ nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Thus by Lemma 1, K has $12pq$ vertices and $18pq$ edges. We see easily that in K, $|V_9| = 12pq$ and we have edge partition based on the degree sum of neighbor vertices of each vertex as given in Table 5.

$S_K(u), S_K(v)\backslash uv\in E(K)$	(9, 9)
Number of edges	$18pq$

Table 5. Edge partition of K.

$$GA_5II(K) = \prod_{uv\in E(K)} \frac{2\sqrt{S_K(u)S_K(v)}}{S_K(u)+S_K(v)} = \left(\frac{2\sqrt{9\times 9}}{9+9}\right)^{18pq} = 1.$$

References

1. N. Trinajstić, Chemical Graph Theory, CRC Press, Boca Raton, FL (1992).
2. V.R. Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
3. F. Harary, *Graph Theory*, Addison Wesley, Reading, MA (1969).
4. M. Randić, On characterization of molecular branching, *Journal of the American Chemical Society, 97(23),* 6609-6615 (1975).
5. V.R. Kulli, Multiplicative connectivity indices of certain nanotubes, *Annals of Pure and Applied Mathematics, 12(2)* 169-176 (2016).
   DOI: http://dx.doi.org/10.22457/apam.v12n2a8.
6. V.R. Kulli, First multiplicative *K* Banhatti index and coindex of graphs, *Annals of Pure and Applied Mathematics, 11(2),* 79-82 (2016).
7. V.R. Kulli, Second multiplicative *K* Banhatti index and coindex of graphs, *Journal of Computer and Mathematical Sciences, 7(5),* 254-258 (2016).
8. V.R. Kulli, Multiplicative *K* hyper-Banhatti indices and coindices of graphs, *International Journal of Mathematical Archive, 7(6),* 60-65 (2016).
9. V.R. Kulli, On multiplicative *K*-Banhatti and multiplicative *K* hyper-Banhatti indices of *V*-Phenylenic nanotubes and nanotorus, *Annals of Pure and Applied Mathematics, 11(2),* 145-150 (2016).
10. V.R. Kulli, Multiplicative connectivity indices of nanostructures, *Journal of Ultra Scientist of Physical Sciencs,* A *29(1),* (2017) 1-10. DOI: http://dx.doi.org/10.22147/jusps-A/290101
11. V.R. Kulli, General multiplicative Zagreb indices of $TUC_4C_8[m,n]$ and $TUC_4[m,n]$ nanotubes, *International Journal of Fuzzy Mathematical Archive, 11(1),* 39-43 (2016). http://dx.doi.org/10.22457/ijfma.v11n1a6.
12. V.R. Kulli, Multiplicative connectivity indices of $TUC_4C_8[m,n]$ and $TUC_4[m,n]$ nanotubes, *Journal of Computer and Mathematical Sciences, 7(11),* 599-605 (2016).
13. V.R. Kulli, Some new multiplicative atom bond connectivity indices, *Annals of Pure and Applied Mathematics, 13(1),* 1-7 (2017).
14. R. Todeshine and V. Consonni, New local vertex invariants and descriptors based on functions of vertex degrees, *MATCH Commun. Math. Comput. Chem. 64,* 359-372 (2010).
15. M. Eliasi, A. Irammanesh and I. Gutman, Multiplicative versions of first Zagreb index, *MATCH Commun. Math. Comput. Chem. 68,* 217-230 (2012).
16. S.M. Hosamani, Computing sanskruti index of nanostructures, *J. Appl. Math. Comput.* DoI: 10.1007/s12190-016-1016-9.